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Design and implementation of Chinese libration point missions

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Abstract Libration points are vital for lunar and deep space explorations because of their unique positions and dynamics. This paper first traces the development of relevant missions since ISEE-3 and then presents the details of the trajectory design and implementation of four Chinese libration point missions in the lunar exploration project: the two Sun-Earth libration point missions by CHANG'E-2 and CHANG'E-5 and the two lunar libration point missions accomplished by CHANG'E-5T1 and Queqiao. The orbit technologies for these libration point missions are also elaborated on regarding trajectory design, maneuvering, and tracking, as well as orbit determination. This paper is expected to provide a reference for future cislunar and deep space exploration.

Keywords libration point, three-body problem, lunar exploration, cislunar space, flight scheme

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1 Introduction

Libration points and libration point orbits (LPOs) are important for planning and implementing lunar and deep space exploration missions. The five libration points of the circular restricted three-body exhibit great value in space missions because of their unique positions and dynamics [1, 2].

The Sun-Earth L1 point is preferred for missions such as Sun and Earth observation, solar wind and space environment monitoring, coronal mass ejection warnings, and terrestrial planet and energetic space particle detection. The ISEE-3 [1] mission, later renamed ICE, was successfully launched on August 12, 1978, as the first Sun-Earth libration point mission (LPM) entering the Sun-Earth L1 halo orbit and observed solar wind, geomagnetism, and cosmic rays to study the interaction between the Earth's magnetic field and the solar wind. This mission was followed by Wind [3], SOHO [4] and ACE [5] around the Sun-Earth L1 LPO to measure the solar wind properties before it reaches the Earth, investigate the outer layer and interior structure of the Sun, and study matter comprising energetic particles from sources including solar wind and the interplanetary medium. In 2001, Genesis [6] was launched into the Sun-Earth L1 halo orbit to collect solar wind particles through the transfer trajectory designed according to the interplanetary superhighway (IPS) theory. Compared with the traditional transfer method, this trajectory saves a considerable amount of fuel, further proving that the LPO dynamics structure can save the energy required for deep space exploration. Later, the detectors of Deep Space Climate Observatory [7] and Laser Interferometer Space Antenna Pathfinder [8] were sent into the L1 LPO for solar and Earth observation for early warning of coronal mass ejections, climate monitoring, and technological tests of a gravitational wave observatory. In 2020, the CHANG'E-5 (CE5) service module was sent into the Sun-Earth L1 LPO after a lunar sampling [9].

The Sun-Earth L2 point is an ideal location for space observation because of its lack of interference from solar radiation, terrestrial infrared radiation, the residual atmosphere, and space debris. In addition, the energy required for L2 LPO maintenance is small, but the LPO amplitude can range from

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hundreds of thousands to millions of kilometers, making it suitable for missions such as cosmic microwave background measurement and radio, ultraviolet, and infrared astronomical observation [10–19]. Thus far, 15 Sun-Earth LPMs have been implemented, with all detectors deployed around the Sun-Earth L1 or L2 points and the orbit types, including Lissajous, halo, and the corresponding quasi-periodic ones. Moreover, the invariant manifolds of the Sun-Earth L1 and L2 LPOs can be used to construct low-energy transfer trajectories, allowing the probe to the Moon in a ballistic transfer mode while saving fuel for the transfer [20–28].

The Sun-Earth L3, L4, and L5 points can be used to monitor the Sun and the solar-terrestrial space environment, which can provide observation angles different from the Earth and other libration points [29]. They can also function as communication network nodes to provide services such as communication, navigation, and positioning for the missions of solar-terrestrial space and beyond.

The lunar libration points also have many merits for space missions. The lunar L1 and L2 points can be used for future lunar exploration, particularly for relaying the communication, positioning, and autonomous navigation of a lunar surface patrol to explore invisible areas, such as the lunar farside and poles, playing an important role in compensating for the lack of ground telemetry, tracking, and command (TT&C), relieving the pressure of ground TT&C resources, and improving the autonomous capability of the probe [1, 30, 31]. Meanwhile, L1 and L2 can be used for lunar exploration and sampling return missions and function as the transport hub of future lunar and deep space exploration missions [2] to achieve low-energy transfer, emergency orbit reconfiguring, rendezvous and docking, and the transfer center linking the Earth and deep space [6, 30]. The lunar L4 and L5 points, equidistantly located from the Earth and the Moon, are convenient to approach. In the 1970s, the "L5 Society" organization proposed the concept of establishing a space city at the point [32]. At the same time, L4 and L5 can be linked to the space-based TT&C network comprising facilities on the Earth, Moon, and other libration points to provide relay communication, autonomous navigation, and other services for missions of Earth-Moon space and the solar system. In addition, for the current controversial issue of Kordylewski clouds [33], a dedicated detector can be deployed on the lunar L4 and L5 LPOs to execute an "in-site" observation.

The use of the lunar libration points lags behind that of the Sun-Earth libration points. The first lunar LPM was realized in May 2008, i.e., ARTEMIS mission [34] (now called THEMIS-ARTEMIS), which flew the lunar L1 and L2 Lissajous orbits through the vicinity of the Sun-Earth L1 and L2 points. The ARTEMIS mission was followed by the lunar L2 flight of CHANG'E-5T1 (CE5T1) in 2014 [35–39] and the Queqiao (QQ) relay mission of CHANG'E-4 (CE4) in 2018 [40–44]. In 2022, the CAPSTONE mission headed into the lunar near-rectilinear halo orbit for the first time [27].

China has thus far completed four LPMs, including the LPOs of Sun-Earth L1 and L2 and the lunar L2 points, three of which are the extended missions of CHANG'E satellites or their modules after completing their respective primary missions and are characterized by maximizing the relevant resources. The prominent QQ is the lunar L2 LPM dedicated to relaying communication, and it achieved the world's first engineering application of the lunar LPO. This paper will review the details of designing and implementing these missions, particularly the pivotal roles of libration points and LPOs, which are expected to facilitate future space and lunar exploration missions.

2 Sun-Earth LPMs

China's initial LPM, Kuafu, was proposed in 2003, with the Sun-Earth L1 point adopted to investigate space weather [45]. In 2004, China launched the CHANG'E (CE) lunar exploration project implemented in three phases: lunar orbiting, soft landing, and sample return. Thus far, seven missions have been accomplished [46] (Table 1). During the missions, China implemented four LPMs, three of which use the lunar satellites (CE2, CE5T1, and CE5) after their primary missions, completing the virgin Sun-Earth and lunar LPMs. A more important achievement is made in another LPM that deploys the relay satellite QQ in the lunar L2 halo orbit for the CHANG'E-4 (CE4) mission, realizing the longstanding concept of relay communication with libration points for the first time in space research.

In 2011, CE2 started an extended flight to the Sun-Earth L2 point after completing its primary mission [16–19]; in 2020, the CE5 service module proceeded with its journey to the Sun-Earth L1 point after separating from the lunar sample module [9].

Table 1 China's lunar exploration project

Satellite	Launch date	Primary objective
CE1	October 24, 2007	Lunar circling orbit at an altitude of 200 km
CE2	October 1, 2010	Lunar circling orbit at an altitude of 100 km
CE3	December 2, 2013	Lunar soft landing
CE5T1	October 24, 2014	Lunar return, Earth re-entry, and soft landing
QQ	May 21, 2018	Relay communication
CE4	December 8, 2018	Lunar farside soft landing
CE5	November 24, 2020	Lunar sample return

Table 2 Constraints of orbit design and control of the CE2 LPM [16]

No.	Constraint	Parameters and requirements					
1	Initial lunar orbit	a (km)	e	i ($\circ$)	Ω ($\circ$)	ω ($\circ$)	M ($\circ$)
		1835.4	0.0092	83.90	260.34	325.51	311.15
2	Lunar orbit epoch	2011-05-29 02:00:00.000 (UTC)					
3	Satellite mass	1659.7 kg					
4	Propulsion configuration	Fuel sink thrusters and main thruster					
5	Maximum velocity increment by the remaining propellant	867 m/s					
6	Sunlight condition	Consecutive shadow less than 3 h					
7	Lunar escape time window	June 1–16, 2011					
8	TT&C condition	2.5 h before and 1 h after the lunar escape maneuver observable					
9	Target orbit type	Lissajous or halo					
10	Target orbit characteristics	The farthest distance from Earth less than 2 million km					
11	Telemetry condition	The angle between the Sun-satellite and Earth-satellite greater than 5°					
12	Mission deadline	December 2012					

2.1 CE2 LPM

CHANG'E-2, as the pioneering satellite of the second phase of China's lunar exploration project, undertakes critical tasks of verifying key technologies for a lunar soft landing and acquiring high-resolution images of the preselected landing areas for the CE3 mission. CE2 was launched at 18:59:57 (Beijing time) on October 01, 2010, from the Xichang launch site by a "Long March" 3C rocket. After separating from the rocket, the satellite entered the cislunar transfer trajectory with an inclination of 28.5° and an altitude of perigee and apogee of 212.8 and 35700 km, respectively. After a trajectory correcting maneuver on October 2 and a lunar brake on October 6 at the perilune, CE2 was captured by the Moon and eventually injected into a circling orbit of 100 km height following two more braking maneuvers on October 8th and 9th.

By April 01, 2011, CE2 had achieved all of its engineering objectives, including direct injection of a cislunar transfer trajectory, technology verification of X-band telemetry, and high-resolution imaging of a soft-landing area for CE3, and all of its research goals, including high-resolution 3D imaging of the lunar surface and detection of the lunar surface composition, thus concluding the first phase of the extended mission. The accomplished tasks included imaging the residual polar area and descending to 15 km to take photographs of Sinus Iridium for CE3. With the extended mission, the first full lunar image with a resolution of 7 m was obtained, as well as an image of Sinus Iridium with a resolution of better than 1.5 m.

The extended mission of the second phase, i.e., the CE2 LPM, is planned and implemented to further use the satellite and verify the flight technologies of libration points. Based on the status at the end of the first phase and the requirements of the second phase, the main constraints of orbit design and control for the CE2 LPM have been formulated, and these constraints include residual propellant, satellite mass, initial orbit, transfer time window, lighting conditions, and TT&C conditions (Table 2).

Under the constraints shown in Table 2, the transfer trajectory from the Moon to the Sun-Earth L2 point and the corresponding target orbit is designed, with a focus on the relationship between the velocity increment required for the lunar escape, transfer flight time and escape time. The results of analysis and design are shown in Figure 1 [16,47], where (a) is the relationship among the required lunar escape velocity

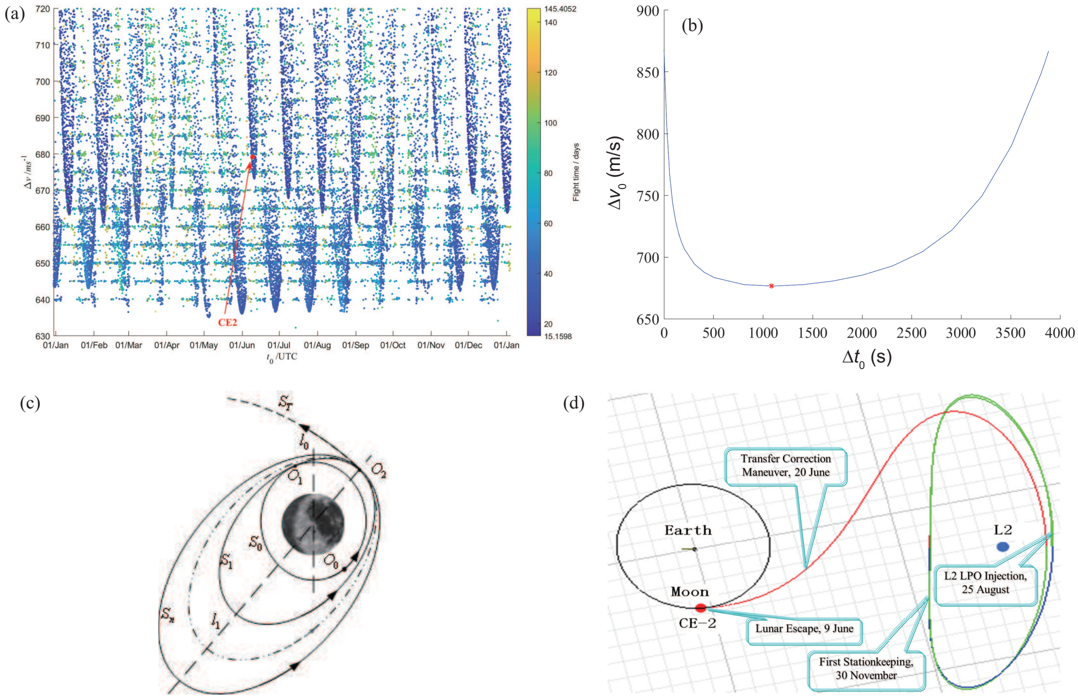

Figure 1 (Color online) CE2 Sun-Earth LPM. (a) Relationship among the required lunar escape velocity increment, transfer flight time, and escape time; (b) local relationship between the velocity increment required for the lunar escape and escape time; (c) scheme of lunar escape control; (d) transfer trajectory and target libration orbit.

increment, transfer flight time and escape time within one year, and (b) shows the velocity increment required for escape within one hour after 07:36:33 on June 9, 2011. As can be seen, the velocity increment within 07:36:33–08:41:15 (UTC) meets the energy requirements, the minimum of which is approximately 677 m/s, and the corresponding time is 07:54:30, shown as "*" in Figure 1(b).

Based on the results of analysis and design, the thruster capability, and control error information, the lunar escape control was divided into two maneuvers. Figure 1(c) shows the selection of a 5.3-hour phasing orbit resulting from the investigation of the scheme of lunar escape control. On June 8–9, 2011, the two escape maneuvers were executed with a velocity increment of approximately 381 and 332 m/s, respectively; on June 20, a trajectory correction maneuver (TCM) with a velocity increment of approximately 3.2 m/s was made; on August 25, the injection maneuver with a velocity increment of approximately 3.6 m/s was completed, resulting in the successful entry of the satellite into the Lissajous orbit around the Sun-Earth L2 point with an amplitude of approximately 28400, 88000, and 36700 km in the x, y, and z directions of the Sun-Earth synodic system, respectively, as shown in Figure 1(d). The maintenance of the target L2 orbit was implemented on November 30 to keep the satellite in the orbit until April 2012, when the next phase of the extended mission was initiated to fly by the asteroid Toutatis [48–50].

CE2 is China's first satellite to reach the Sun-Earth libration point region and the world's first satellite to transfer from a lunar orbit to the Sun-Earth libration point. The mission not only epitomizes the debut of China's LPM era but also accumulates valuable experience for the country in LPM design and implementation.

2.2 CE5 LPM

China launched the CE5 lunar probe on November 24, 2020, and successfully recovered the sample capsule on December 17, the second time a lunar soil sample has been obtained since the success of the Soviet Union in 1976. Meanwhile, the CE5 mission marked the successful completion of China's lunar exploration project, making China the third country to successfully implement lunar sampling and return. For the sample return, the CE5 probe comprises a sample capsule and a service module. The two parts are separated when CE5 reaches its predetermined height above the Earth, with the sample capsule returning to the landing site, and the service module is controlled to enter a large elliptical geocentric orbit. To maximize the use of mission resources, an extended mission of the Sun-Earth L1 flight is

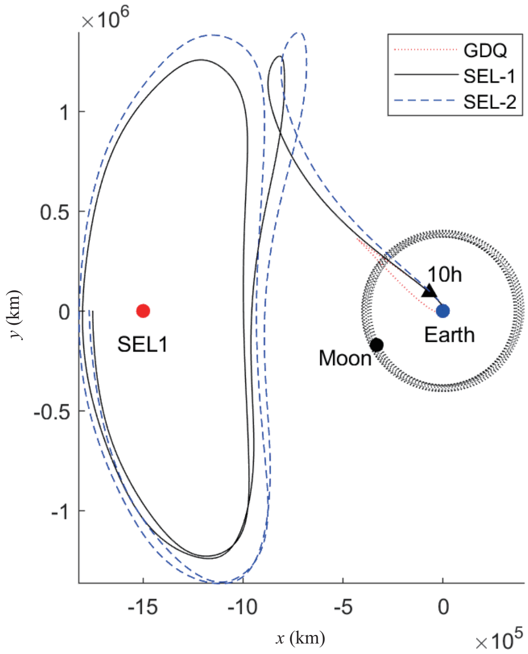

Figure 2 (Color online) Scheme of the CE5 Sun-Earth LPM.

implemented using the CE5 service module, which has verified the key technologies for subsequent lunar and deep space exploration missions.

The perigee of the CE5 service module is relatively low after separation from the sample capsule, and the module faces the potential hazard of being incinerated by the dense atmosphere. More dangers come from the fragments of the broken module, which could hit the sample capsule, despite the low probability of this event. To rescue the module and avoid the risk of capsule damage, a maneuver must be applied to raise the perigee of the module as soon as possible after separation, that is, a collision avoidance maneuver (CAM). Taking the case of increasing the perigee to 135 km as an example, the module enters a large elliptical geocentric orbit with a period of approximately 17.8 days, as shown by the trajectory labeled as “GDQ” in Figure 2 [9].

Further analysis shows that if the module is maneuvered to escape from the cislunar space, it will fly directly to the Sun-Earth L1 point with minimal energy cost. Therefore, the Lissajous orbit of the Sun-Earth L1 point is selected as the target orbit. The high-fidelity dynamics model is used to design the transfer trajectory of the module to the Sun-Earth L1 point, thus obtaining the corresponding Lissajous orbit, with the trajectory “SEL-1” shown in Figure 2. With the ground TT&C requirements considered, the TCM is executed 10 h after the perigee, denoted as ‘▲’ in Figure 2.

The transfer trajectory to the L1 point and the corresponding Lissajous orbit is shown as the trajectory “SEL-2” (Figure 2) when the perigee of the module is increased to 160 km. Here only 1–2 circles of the Sun-Earth Lissajous orbit are plotted as a schematic, and the flight time can be extended according to the actual mission requirements.

The flight scheme parameters of the CE5 LPM are shown in Table 3 [9], in which SEL-1 and SEL-2 are the cases of raising the perigee of the service module to 135 km and 160 km, respectively. Additionally, SEL-1 comprises two schemes of different maneuvers that are one-maneuver and two-maneuver schemes for comparison, shown as Nos. 1 and 2 in Table 3. For the one-maneuver scheme, only one single CAM is executed when the service module separates from the sample capsule, while the two-maneuver scheme adds a transfer trajectory maneuver after the CAM and at 10 h after perigee. SEL-2 only gives the one-maneuver scheme. The flight time in Table 3 shows that the module transfers from the perigee to the point crossing the xz-plane of the Sun-Earth synodic system and near point L1.

The CE5 LPM can not only realize the exploration of the Sun-Earth L1 point but also provide efficient time for planning, decision-making and implementing follow-up missions, such as further exploration of the Sun-Earth L2 point and/or cislunar space and/or other celestial bodies, return and re-entry to the Earth.

At 1:00 on December 17, 2020, the CE5 sample capsule and service module approached the Earth and

Table 3 Parameters of flight schemes of CE5 LPM

No.	Parameter	SEL-1	SEL-2
1	Altitude of perigee (km)	135	160
2	Location of maneuvers	(1) CAM after separation (2) CAM after separation & 10 h after perigee	CAM after separation
3	Velocity increment (m/s)	(1) 103, (2) 310	122
4	Flight time (days)	88	90
5	x-oriented amplitude of target Lissajous orbit ($\times 10^5$ km)	4.3	4.8
6	y-oriented amplitude of target Lissajous orbit ($\times 10^5$ km)	12.5	13.7
7	z-oriented amplitude of target Lissajous orbit ($\times 10^5$ km)	1.4	1.6

entered the atmosphere, and the two vehicles separated in approximately 20 min. To avoid the aforementioned risk and rescue the module, a CAM was performed to raise its perigee height to approximately 160 km. Meanwhile, to avoid capsule damage caused by the thruster plume of the module, a CAM was performed at a specific attitude of the module, which increased the velocity increment to 158 m/s, and the apogee of the module after the maneuver was approximately 43000 km. A TCM of approximately 142 m/s was executed in approximately 2 h after the CAM implementation to ensure that the module flew to the Sun-Earth L1 point after escaping from the Earth. The transfer trajectory has a semi-major axis of approximately 696000 km, along which two additional TCMs were executed to inject the module successfully into the Lissajous orbit of the L1 point at 13:29 on March 15, 2021.

3 Lunar LPMs

China achieved its first lunar LPM with CE5T1 in 2014 [35–39] and the world's first relay communication mission around the lunar L2 point with QQ, which was launched in 2018 to support the CE4 lunar farside exploration [40–44].

3.1 CE5T1 LPM

In 2014, after fulfilling the first two phases of lunar circling and soft landing, China made active preparations for the third phase of its lunar exploration project and implemented the CE5T1 mission of circumlunar return and re-entry testing to technologically prepare for its subsequent CE5 mission. CE5T1 was launched at the Xichang Launch Site on October 24, 2014. After completing a series of cislunar transfer, lunar swing-by, lunar transfer, and Earth re-entry, the sample capsule was successfully landed on November 1, with much propellant remaining in the CE5T1 service module by the end of the test. The CE5T1 extended mission was implemented to maximize the use of resources, and the mission consisted of completing the lunar L2 flight and returning to the Moon, verifying the corresponding TT&C technologies for the future lunar LPMs and the technologies of lunar rendezvous and docking for the CE5 lunar sampling return mission.

With flight time and energy considered, the transfer trajectory of CE5T1 to the lunar L2 point was designed to swing by the Moon, aimed at using lunar gravity to save energy. Originally proposed by Farquhar [51], the transfer mode is shown in Figure 3(a). Compared with the transfer mode of a direct flight from the Earth to the L2 point without the lunar swing-by, the total energy is decreased by at least 800 m/s despite one additional maneuver at perilune [51, 52]. Meanwhile, the flight time can be reduced by at least 70 days compared with the low-energy transfer based on the theory of a weakly stable boundary (WSB) or invariant manifold [53, 54].

The transfer trajectory of CE5T1 to the lunar L2 point is shown in Figures 3(b)–(d) [36, 37], and the flight path from the CAM to the lunar rendezvous in the geocentric J2000.0 coordinate system is shown in Figure 3(b). Notably, a CAM is necessary as CE5T1 and CE5 adopt the same structure and flight style of Earth re-entry. Most of the path in Figure 3(b) is a large elliptical geocentric orbit of 1.5 revolutions after three trajectory maneuvers, namely, trajectory a. If the factor of apogee altitude is considered in the CAM, the three trajectory maneuvers can be combined into one, and the consequent transfer trajectory has a shape very similar to a, shown as b in Figure 3(b). For both scenarios, the trajectories of the service module from the perilune to the L2 point and returning to the Moon are shown in Figures 3(c)

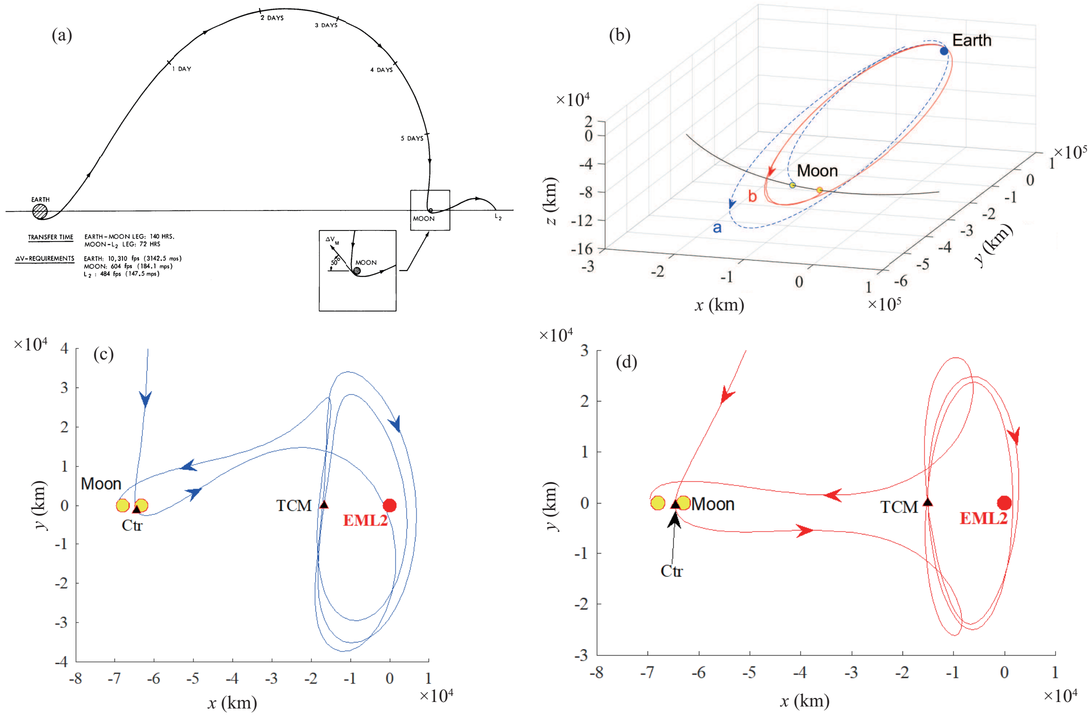

Figure 3 (Color online) Transfer style, transfer trajectory and target L2 orbit of CE5T1 LPM. (a) Transfer trajectory from the Earth to the lunar L2 point with a lunar swing-by; (b) transfer trajectory from the collision avoidance maneuver to the Moon of CE5T1; (c) Lunar L2 orbit and the trajectory of returning to the Moon of CE5T1 with three maneuvers; (d) Lunar L2 orbit and the trajectory of returning to the Moon of CE5T1 with one maneuver.

and (d), in which Ctr and TCM are the perilune brake and lunar return maneuver, respectively, and the coordinate system is the Earth-Moon synodic system centered on lunar L2 point.

CE5T1 adopted the lunar LPM trajectory *a* in Figure 3, and the flight course to L2 was as follows:

(1) The main thruster was ignited at approximately 500 s after the separation of the service module and the sample capsule; a CAM was then implemented, which raised the perigee of the module and prepared for the subsequent rendezvous with the Moon.

(2) When the CAM was completed, the service module entered a large elliptical geocentric orbit with a period of approximately 16 days.

(3) A trajectory maneuver was executed at apogee, and the control target was to increase the perigee altitude to 600 km.

(4) A trajectory maneuver was executed at perigee, and the control target was to adjust the conditions of lunar rendezvous; that is, the altitude and inclination of perilune were 200 km and 45°, respectively, with a consideration of the subsequent flight to the L2 point.

(5) The lunar brake was executed at perilune to decrease the orbital energy and guide the service module to the lunar L2 point.

The above maneuvers and the subsequent control of the lunar return are shown in Table 4 [36]. The entire process of the CE5T1 lunar LPM approximates 72.6 days, and the total velocity increment is approximately 983.9 m/s, except for the CAM.

The CE5T1 lunar LPM adopted the transfer style with the world's first lunar swing-by to reach the lunar libration point. It is also the second flight of the lunar libration point after THEMIS-ARTEMIS and verified the TT&C technology for future development and use of the lunar libration points, specifically providing insightful concepts while technologically preparing for the subsequent CE4 lunar farside soft-landing mission. Thus, CE5T1 can be considered the pioneering mission of QQ.

3.2 QQ LPM

In 2018, China implemented the CE4 mission, the objectives of which are to complete the first lunar farside soft landing and rover using the first lunar L2 relay satellite, as well as conduct astronomical observations and research on lunar low-frequency radio, detection and research on the topography, geomorphology,

Table 4 Flight events and control parameters of the CE5T1 lunar LPM

No.	Time (UTC)	Flight event	Velocity increment (m/s)
1	November 01, 2014 08:13:00	Maneuver 10 h after separation	26.0
2	November 9, 2014 06:42:00	Apogee maneuver	17.4
3	November 17, 2014 15:37:30	Perigee maneuver	26.4
4	November 21, 2014 08:00:00	TCM	1.9
5	November 23, 2014 15:11:41	Perilune maneuver	233.6
6	November 28, 2014 09:00:20	EML2 orbit injection	2.2
7	December 10, 2014 16:10:20	EML2 orbit maintenance 1	2.7
8	December 26, 2014 08:30:20	EML2 orbit maintenance 2	9.6
9	January 04, 2015 15:00:00	Maneuver for returning from the EML2 orbit	41.3
10	January 10, 2015 19:04:33	Lunar brake 1	219.7
11	January 11, 2015 17:35:39	Lunar brake 2	171.9
12	January 12, 2015 19:33:21	Lunar brake 3	231.2
Total	72.6 days		983.9

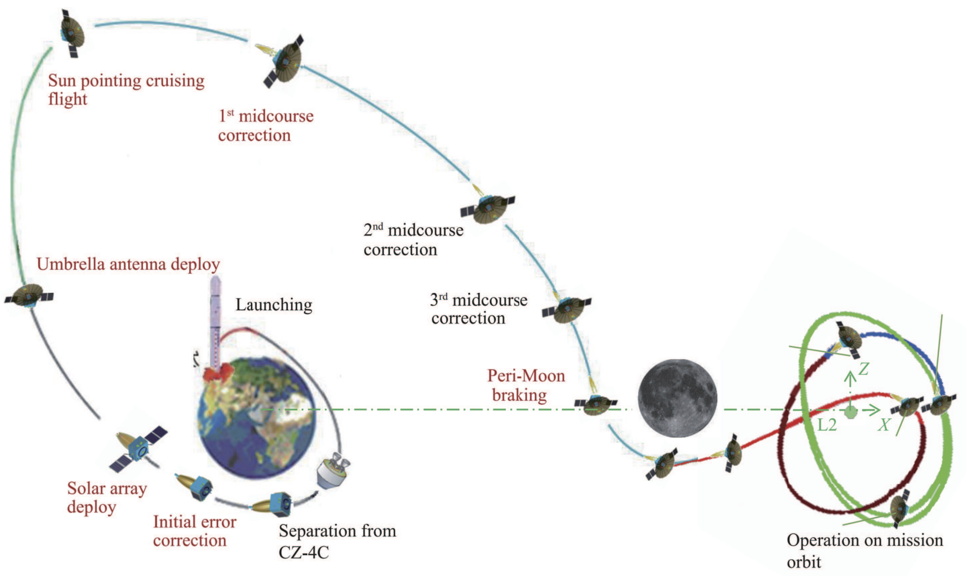

Figure 4 (Color online) Flight course of QQ.

surface structure, and mineral composition of the lunar farside patrol area. The chosen lunar farside soft landing site of CE4 is the Von Kármán impact crater in the Aitken Basin of the lunar South Pole. The diameter of the impact crater is approximately 186 km with a central coordinate of (44.8°S, 175.9°E).

Because of the relative motion of the Earth and the Moon, the CE4 lander and rover on the lunar farside cannot communicate directly with the ground TT&C network. Therefore, the CE4 relay communication satellite QQ is deployed at the lunar L2 point. According to the mission plan, QQ was first launched in 2018 before the CE4 spacecraft, and the process of launch, transfer, and deployment is shown in Figure 4 [41]. The CE4 spacecraft, a combination of lander and rover, will be launched after the QQ is injected into the lunar L2 orbit and passes various on-orbit performance tests. With the relay communication support of QQ, the lander will perform in-situ detection, and the rover will execute scientific detection at all planned target positions individually and then send the detection data back to the ground [40].

Therefore, QQ is a prerequisite for successfully implementing the CE4 farside soft-landing mission, thus making the trajectory design of QQ a decisive link to the CE4 mission. The QQ trajectory design covers a series of factors, including launch time, launch site, rocket, lunar soft-landing area, orbit characteristics, and TT&C condition, with the details listed in Table 5.

The design requirements in Table 5 are the determinant factors for the QQ mission orbit of the lunar L2 point and the transfer trajectory needed. A thorough analysis leads to the selection of a common halo orbit as the QQ mission orbit, to be more specific, the southern halo orbit with an amplitude of

Table 5 Requirements of the QQ trajectory design

No.	Requirement	Description
1	Mission time	Launch window: May–June 2018 Interval of lunar descent: from 2019-1-3 0:00 to 2019-1-5 12:00 (UTC)
2	Launch site	Xichang
3	Mass and fuel budget	Total mass less than 450 kg, velocity increment for transfer trajectory less than 367 m/s
4	Soft-landing site	Lunar location (176°E, 45°S)
5	TT&C network	USB station: Kashi, Qingdao, and Chile Deep space stations: Kashi, Jiamusi, and South America VLBI stations: Shanghai, Miyun, Kunming, and Urumqi
6	Telemetry and relay of L2 orbit	Angle of Earth-QQ-lander Angle of QQ-Earth-Moon Distance between QQ and lander Coverage of lander by QQ
7	Lighting	Consecutive shadow (umbra) less than 3 h

approximately 13000 km in the z-oriented direction of the lunar synodic coordinate system. For the transfer trajectory, the successful experience of the CE5T1 LPM is fully used as a reference, and the transfer mode with a lunar swing-by is adopted to achieve a trade-off between the flight time and energy required.

After the mission orbit and transfer trajectory are determined, the daily launch windows and corresponding transfer trajectories are studied and designed in May–June 2018, following the design requirements of the launch site and rocket in Table 5. The shapes of all transfer trajectories are similar to that in Figure 3(a). When all the conditions for design are met, the focus is shifted to optimizing the energy of trajectory controls, including the lunar brake and the maneuvers of transfer from the Moon to the lunar L2 mission orbit, L2 capture, and injection [44]. In May 2018, there are eight launch opportunities with transfer trajectory control energy of less than 300 m/s, a departure date range of May 17–25, a total velocity increment between 246 and 298 m/s, and a perilune time ranging from May 24 to 29. The situation in June is much similar to May, with a launch window of 14–22, requiring lower energy. Meanwhile, an analysis of the impact of launch time variation shows that constrained by the launch site conditions, the consecutive launch span within 24 h is approximately 4.5 h, and the corresponding trajectory control energy is 258–326 m/s.

At 5:28:50 on May 21, 2018, a CZ-4C rocket lifted off from the Xichang launch site with QQ and two microsatellites. Approximately 25.5 min after lift-off, the rocket released the satellites into the transfer trajectory scheduled. Figure 5 shows the entire flight course, comprising five phases: launch, cislunar transfer, L2 transfer, L2 capture, and L2 mission.

(1) The launch phase refers to the time from launch to satellite release. At the end of this phase, QQ and two microsatellites were directly sent into the cislunar transfer trajectory with a perigee of 200 km, an apogee of 40000 km, and an inclination of 28.5° by the CZ-4C rocket.

(2) The cislunar transfer phase is the time from the release of satellites to perilune. The flight time of the phase is approximately 112 h, during which 2–3 transfer TCMs are planned. Ultimately, the perilune has an altitude of approximately 100 km and an inclination of approximately 15°.

(3) The L2 transfer phase refers to the time from the Moon to the L2 vicinity, i.e., first crossing the xz-plane of the lunar synodic coordinate system outside L2. The flight time is approximately 3–4 days.

(4) The L2 capture phase is the time from the end of the L2 transfer phase to the injection of the L2 halo orbit, during which three maneuvers and three corresponding post-maneuver correction controls are planned.

(5) The L2 mission phase starts from the injection of the L2 halo orbit. In this phase, QQ enters the southern L2 orbit with a z-direction amplitude of 13000 km in the lunar synodic system. The mission period is scheduled to be three years.

As of December 2022, QQ has been in service for over four years, exceeding the initial design life of three years, and has provided indispensable relay communication services for the CE4 lunar farside soft-landing mission. Currently, the satellite is still in good condition.

CE4 executes the world's first soft-landing and patrol mission on the farside surface of the Moon, while QQ is the world's first relay communication around the libration point, realizing the envisioning of relay

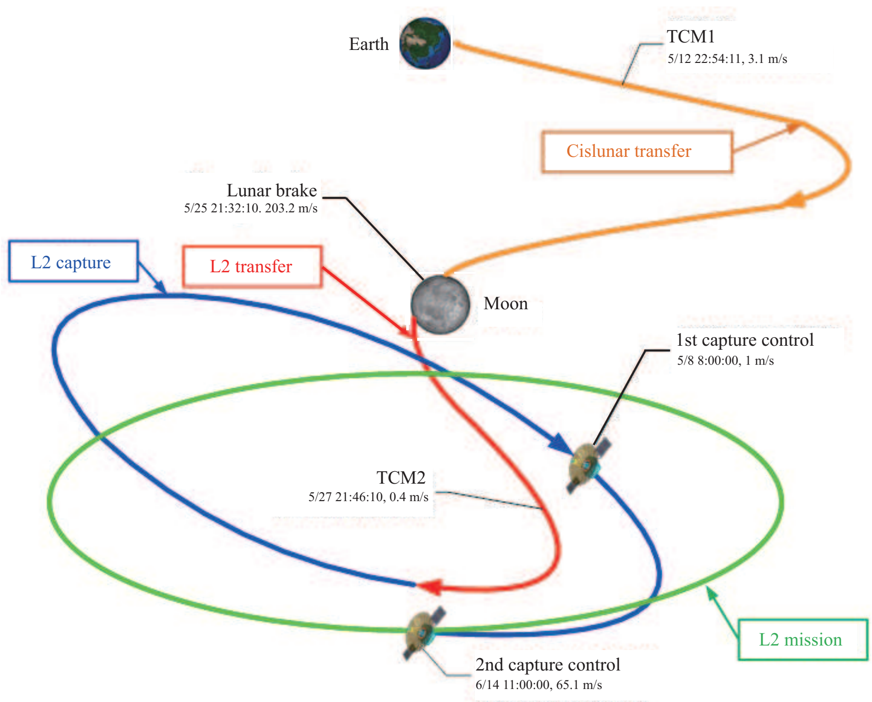

Figure 5 (Color online) Flight course of QQ.

communication using the lunar libration point for the first time in human space activities. This feat is of pioneering importance for further development and use of the cislunar space.

4 LPO technology

The orbit is a prerequisite for space missions, thus being the primary issue in space mission design. The LPOs are divided into the mission orbit libration points and the corresponding transfer trajectories. The position and dynamics characterized by libration points differentiate LPO from the common orbits circling celestial bodies and the transfer orbits between celestial bodies. Therefore, LPO and the corresponding technologies are vital to LPMs.

4.1 LPO design and control

The LPO selection is the first issue to be solved in LPM design, and the relevant issues are the tracking, measurement, maintenance, and control of the LPO, which are primary in the LPM implementation. These issues, in essence, have the same mechanical nature as multibody orbit dynamics.

(1) Theory of three-body dynamics. LPOs belong to three-body dynamics problems, first proposed by Euler and Lagrange, addressing the particular solutions of the circular restricted three-body problem (CR3BP) in the 18th century. The bounded motion around the libration point is composed of periodic orbits and quasi-periodic orbits, with the latter located on the central manifold of the former. Thus, the research on periodic orbits and quasi-periodic orbits has laid a theoretical foundation for the LPO problems. Of all the researchers, Poincaré, Strömgren, Moulton, Hénon, and many other scholars made groundbreaking work [55–62]. Consequently, Lyapunov, Halo, distant retrograde orbit (DRO) and other periodic orbits and n-period ($n = 2, 3, 4, \ldots$) orbits have been found, and the research tools of nonlinear dynamics, such as continuation and bifurcation, have been gradually developed [63].

Regarding the construction of periodic and quasi-periodic LPOs, Farquhar, Breakwell, Richardson, Gómez, Howell, and other scholars proposed analytical, numerical, and semi-analytical methods and obtained high-order analytical formulas for these orbits as well as effective numerical iterative algorithms [64–72]. For orbit dynamics models, in addition to the basic CR3BP model, the models include more dynamics factors, such as the Hill model, restricted Hill four-body model, elliptical restricted three-body problem, bicircular model, phase harmonic model, and JPL ephemeris model [73–76]. These models have provided not only solid theoretical support for LPOs but also many design tools, greatly facilitating the development of LPOs from theory to application.

(2) Design of mission orbit. The location should be the primary consideration of an LPM mission orbit, which is determined by the LPM objectives. As discussed in Section 1, the Sun-Earth L1 point is selected for a mission of solar observation and research, solar wind surveillance, forecast and warning, and space weather research; the Sun-Earth L2 point is appropriate for tasks such as space observation and cosmic science research. For communication, navigation, remote sensing, and other tasks in cislunar space, the lunar libration points are preferred, and the Sun-Earth L1/L2 points are also an option if the distance is not an obstacle.

Next, the type of mission orbit is as important as the location and should be a focus of the LPMs. There are numerous periodic and quasi-periodic orbits around the libration points. The typical periodic orbits include Lyapunov, Halo, Vertical, Axial, short period orbit, long period orbit, DRO, distant prograde orbit (DPO), Butterfly, and n-period orbits. There exist a great number of quasi-periodic orbits near the periodic orbits. It should be noted that some periodic orbits such as DRO and DPO are far from the libration points and centered on the secondary; therefore, they are more accurately referred to as three-body orbits. Furthermore, the Lissajous orbit is often used in the LPMs, along with the periodic and quasi-periodic orbits.

In the design of mission orbit type, amplitude, and phase, priority should not only be given to satisfying the mission requirements but also weighing the relevant mission cost. For example, for LPMs of the Sun-Earth and cislunar space, the design of mission orbit type, amplitude, and phase directly affects the conditions of ground tracking and measurement, lunar and Earth shadows, solar lighting, dynamic stability, launch requirements, and transfer and injection. These factors determine the cost of energy and time in the mission implementation.

(3) Design of transfer trajectory. The transfer trajectory of an LPM refers to the trajectory that departs from the celestial bodies and other space locations to orbits around libration points or that leaves the libration points to the bodies and locations or simply passes by the libration points. The primary goal of transfer trajectory design is heading to, departing from, or passing by the libration points with minimal fuel consumption and/or flight time.

Typical forms of LPM transfer trajectory are direct transfer, transfer with celestial body swing-by, and ballistic transfer or WSB transfer. Ideally, these three forms require two, three, and one maneuvers, respectively. The first transfer form requires the most energy and is rarely used in practice. The second transfer form is a trade-off between energy and flight time, suitable for most missions, with its typical transfer trajectory shown in Figure 3(a). Here, lunar gravity is used for the lunar L2 orbit. The third transfer form, proposed by Belbruno through a numerical method in 1987, is a low-energy cislunar transfer trajectory under solar gravity involving a new concept of lunar ballistic transfer (LBT), which was further developed as WSB theory [20]. On the basis of this theory, he designed the low-energy LBT trajectories for the Hiten mission in 1991 and the SMART-1 mission. By the end of the 20th century, Lo et al. proposed the IPS theory and designed the transfer trajectory of the Genesis mission [77] as well as the LBT trajectory of a lunar mission [22], theoretically proving that WSB and IPS transfer trajectories have identical dynamics properties. Compared with traditional methods of orbit design, the innovative methods fully use three-body and multibody dynamics, thereby minimizing the fuel required for transfer.

In terms of transfer form, CE2 and CE5 flying to the Sun-Earth libration points adopt the transfer trajectories with an energy requirement approximating that of a low-energy transfer trajectory. In contrast, CE5T1 and QQ flying to the lunar L2 point adopt the second transfer form by a lunar swing-by to achieve a compromise between energy and flight time. Regarding LBT using solar gravity, Hiten, SMART-1, GRAIL, BepiColombo, CAPSTONE (launched on June 28, 2022), and KPLO (launched on August 05, 2022) all adopt transfer trajectories of this type. As the transfer form can effectively conserve energy, it is of great value for lunar exploration missions with no stringent requirements on flight time or missions using small satellites with limited onboard propulsion capacity.

(4) LPO control. The LPO control comprises transfer trajectory maneuvering and mission orbit maintenance around the libration points. The former task, similar to the orbit control of general space missions, is relatively mature in theory and technology, leaving the latter task a focus of LPO control. In the 1960s, the stability and maintenance of the halo orbit around the collinear libration point were proposed followed by various control theories and methods of LPO maintenance, including the methods of target points, Floquet, and continuous circling [34, 38, 78–81].

The method of target points selects the expected state of the nominal orbit as the control target. With the feedback of deviation from the expected state, the satellite is controlled to fly along the nominal orbit. This method can realize a trajectory near the nominal one, but the energy cost is relatively high. Based on the Floquet theorem and the dynamic structure of LPOs, the Floquet method eliminates the state components corresponding to the unstable manifold of the current state, thus avoiding a flight path deviation from the expected trajectory with a relatively small energy cost. The continuous circling method uses the dynamics features of LPOs to control the satellite for multiple arrivals at the selected characteristic points of LPOs; thus, the satellite can be kept near the libration point for a long time. This method does not require a nominal trajectory and can efficiently reduce the energy consumption of LPO control in the case of a large trajectory deviation. Therefore, it was adopted in China's previous LPMs.

In addition to the control method, the impact of various perturbations on LPOs must be considered while implementing LPMs. For example, the attitude adjustment or maneuver of the satellite will cause small disturbances, possibly resulting in the satellite's gradual deviation from the nominal orbit due to the instability of the LPO. Therefore, the maintenance of mission orbit must include the attitude factor. For another example, the influence of the light pressure perturbation on the satellite of an LPM is also a disturbing factor that cannot be ignored. If the light pressure perturbation cannot be accurately measured and evaluated, it may increase the frequency and energy consumption of orbit maintenance.

4.2 LPO tracking and determination

An LPO satellite has the characteristics of a far distance from the celestial body, slow apparent motion, and weak gravitational constraint. Therefore, the tracking and orbit determination of LPOs, unlike the missions flying around the central body, involve orbit determination with a rather long tracking, difficulties of convergence, and large errors in the results. In the previous LPMs, after the injection of the halo orbit around the Sun-Earth L1 libration point in October 1978, the orbit determination accuracy of ISEE-3 is approximately 6 km, with an orbit calculation conducted every two weeks using a three-week tracking arc. SOHO adopts an orbit determination strategy similar to that of ISEE-3 and achieves an orbit determination accuracy of approximately 7 km. The orbit determination of MAP has a much better tracking condition than the previous missions, which benefits from the NASA large-aperture antenna. The orbit deviation of MAP is 2 km for the propagation of five weeks and 6.7 km for that of nine weeks, with a tracking arc of 14–72 days used according to different conditions [82]. ESA used the 35 m aperture deep space station at New Norcia to track Planck and obtained the two-way Doppler and ranging data. The accuracy of orbit determination is better than 2 km using an arc of 28 days [83]. In the THEMIS-ARTEMIS mission, the accuracy of the orbit determination of the first lunar LPO is better than 100 m using the joint tracking of DSN, USN, and BGS [84]. The tracking of China's lunar exploration missions mainly relies on ground USB facilities supplemented by VLBI stations and an international network of deep space stations. In the CE2 Sun-Earth LPM, the accuracy of orbit determination was approximately 2–10 km with a tracking arc of 4 weeks, using USB 3–6 h every day and VLBI twice every week [85]. In the CE5T1 lunar LPM, the accuracy of orbit determination was approximately 100 m with a tracking arc of a week, using USB 4 h every day and VLBI once every three days [86]. During the QQ lunar LPM, the accuracy of orbit determination can be better than 100 m using a tracking arc of USB and VLBI for 5–11 days [87].

In the TT&C of LPMs, special attention should be paid to the means of tracking. In the past, lunar and deep space missions relied on ground-based tracking facilities characterized by a high operation cost due to long-term tracking. In addition, the simultaneous tracking of multiple satellites could incur problems such as insufficient ground facilities and corresponding resources, demanding space-based tracking and orbit determination. In 2005, Hill proposed the space-based means of LiAISON based on the LPO for autonomous navigation and orbit determination, which adopted the integration of traditional intersatellite tracking and measurement with the unique three-body dynamics characteristics of LPOs, thereby achieving the autonomous navigation and orbit determination of LPOs [82, 88–90], while eliminating the deficiency of absolute positioning, which would not be available through intersatellite tracking among Earth satellites. In the future, it is expected to become the method for space-based navigation and orbit determination of LPMs with the most potential. The CAPSTONE launched in 2022 will soon verify the effectiveness of this navigation means.

5 Summary

China has thus far successfully implemented four libration point missions, including the Sun-Earth L1 and L2, and the lunar L2 points in the international missions. The three missions other than the Queqiao relay satellite mission are the extended missions of the CHANG'E satellites or their modules after completing their respective primary missions, maximizing the relevant resources. These missions have accumulated experience for future missions with verified key technologies for the design, TT&C, and implementation of libration point missions. In addition, the Queqiao lunar L2 relay mission, designed and implemented after only one lunar L2 flight test, is considered the first successful soft landing on the far side of the Moon with the first application of the lunar libration point.

China's libration point missions, distinct from previous libration point missions with a focus on space research, are mostly extended missions that were not considered in designing the primary missions, and the satellites rarely carry payloads for space detection in the LPOs. Thus, these extended missions are mainly intended to verify flight technologies and accumulate engineering experience. Another case in point is the Queqiao satellite, which was initially designed to provide relay communication services for the CHANG'E-4 mission.

Technologically, cislunar space should soon be a popular area for space missions, thus requiring increased use of LPOs, resonance orbits, and other three-body dynamics orbits. This requirement will further highlight the pivotal roles of libration points and LPOs, including inter-satellite autonomous navigation and orbit determination, space situation awareness, and trajectory design and control for cislunar missions. The details summarized from the four completed Chinese missions should provide a reference for future cislunar and deep space explorations.

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