

# Distributed model-free adaptive predictive control of traffic lights for multiple interconnected intersections

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Over the last few decades, urban traffic congestion has been an out-of-control issue globally. Several well-known micromodeling traffic control strategies have been developed for traffic light control. Previous work [1] proposed a distributed feedback controller motivated by back-pressure to maximize the network throughput. However, these model-based control methods may not realize the specified performance if the traffic model is not precise enough. As numerous valuable traffic data are generated daily, data-driven control [2–5] would be more suitable for urban traffic control than model-based control.

For multi-intersection networks, a deep reinforcement learning method is used [2] for large-scale traffic signal control, but its computational speed and reliability may not be satisfied. Compared with this method, the proposed approach in this study, distributed estimation and distributed model-free adaptive predictive control (DED-MFAPC) algorithm, has a faster computational speed and is computationally tractable. Some groups [3, 4] proposed data-driven distributed adaptive coordination control algorithms for the calculation of the green time for each phase by balancing the multidirectional queuing length. Different from the method used elsewhere [3, 4], the proposed DED-MFAPC approach embeds predictive control into the data-driven method to control the interconnected multi-intersection networks. Predictive control is an advanced control method where the control inputs are derived by model predicting and solving the optimization problem. However, predictive control for large-scale systems is difficult to implement, specifically in large-scale interconnected systems, due to the difficulty in determining the interconnected influences at the prediction moments. Different from the existing decentralized estimation and decentralized model-free adaptive control method [5], DED-MFAPC as a distributed predictive control method requires the neighboring subsystems to share some information for each subsystem to optimize its objective function. After the subsystems solve their optimization prob-

lems, they share the intermediate solutions with their neighboring subsystems, which then solve the optimization problems with the received information.

In this study, the multi-intersection network is regarded as a group of one-way road systems. First, the multi-intersection network is divided into  $N$  interconnected subsystems, each of which comprises one intersection and two incoming links with traffic streams entering it. Thus, each subsystem  $i$  has a local state  $y_i \in \mathbb{R}^2$  with the density of vehicles on two roads at the intersection and a local control input  $u_i \in \mathbb{R}^2$  with the green time for each phase. Subsystem  $j$  is a neighbor of subsystem  $i$  if the outflow of  $j$  is the inflow of  $i$ . Let  $\mathcal{N}_i$  be the set of the neighbors of subsystem  $i$ . After the traffic network decomposition, the outflow of each subsystem is controllable and considered as the control input, while the inflow of subsystem  $i$  is uncontrollable, but controlled by the neighboring subsystems, and considered the interconnected influence.

From the dynamics of multiple interconnected subsystems detailed in Appendix A, one has

$$y_i(k+1) = f_i(y_i(k), V_i(k)), \quad (1)$$

where  $y_i(k) \in \mathbb{R}^2$  represents the density of vehicles of subsystem  $i$ ,  $V_i(k) = [u_i^T(k), z_i^T(k)]^T$  refers to the augmented control input vector with  $u_i(k) = [u_{i1}(k), u_{i2}(k)]^T \in \mathbb{R}^2$  being the control input and  $z_i(k) = [z_{j_1 i}(k), \dots, z_{j_{|\mathcal{N}_i|} i}(k)]^T \in \mathbb{R}^{|\mathcal{N}_i|}$  being the interconnected influences from neighboring subsystems,  $|\mathcal{N}_i|$  refers to the cardinality of  $\mathcal{N}_i$ , and  $f_i(\cdot)$  is an unknown nonlinear function relating the vehicle density and the augmented control input with the output.

**Theorem 1** ([5]). For nonlinear system (1) satisfying Assumptions 1 and 2 given in Appendix B, when  $\Delta V_i(k) \neq 0$ , there exists a pseudogradient  $\phi_i(k)$  such that the subsystem (1) can be transformed into the compact-form dynamic linearized (CFDL) data model,

$$y_i(k+1) = y_i(k) + \phi_i^T(k) \Delta V_i(k), \quad (2)$$

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where  $\Delta V_i(k) = V_i(k) - V_i(k-1)$ ,  $\phi_i(k) = [\Phi_i(k), \varphi_i^T(k)]^T$ , and  $\Phi_i(k), \varphi_i(k) = [\varphi_{j_1 i}(k), \dots, \varphi_{j_{|\mathcal{N}_i|} i}(k)]^T$ ,  $j_h \in \mathcal{N}_i$ , are the pseudogradients caused by  $u_i(k)$  and the interconnected influences  $z_i(k)$ , respectively.

Next, the MFAPC data model can be obtained by embedding the  $M$ -step ahead rolling horizon framework into the CFDL data model (2),

$$\mathbf{y}_i(k+1) = E(k)y_i(k) + A_i(k)\Delta \mathbf{V}_i(k), \quad (3)$$

where  $\mathbf{y}_i(k+1) = [y_i^T(k+1), \dots, y_i^T(k+M)]^T$ ,  $\Delta \mathbf{V}_i(k) = [\Delta V_i(k), \dots, \Delta V_i(k+M-1)]^T$ ,  $E(k) = [1, \dots, 1]^T \in \mathbb{R}^M$ , and

$$A_i(k) = \begin{bmatrix} \phi_i^T(k) & 0 & \cdots & 0 \\ \phi_i^T(k) & \phi_i^T(k+1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \phi_i^T(k) & \phi_i^T(k+1) & \cdots & \phi_i^T(k+M-1) \end{bmatrix}. \quad (4)$$

Notably, the variation of the output of subsystem  $i$  can be ascribed to the variation of  $u_i(k)$  and the interconnected influences from neighboring subsystems,

$$A_i(k)\Delta \mathbf{V}_i(k) = B_i(k)\Delta U_i(k) + C_i(k)\Delta Z_i(k), \quad (5)$$

where

$$\begin{aligned} \Delta U_i(k) &= [\Delta u_i(k), \Delta u_i(k+1), \dots, \Delta u_i(k+M-1)]^T, \\ \Delta Z_i(k) &= [\Delta z_i(k), \Delta z_i(k+1), \dots, \Delta z_i(k+M-1)]^T, \\ \Delta u_i(k) &= u_i(k) - u_i(k-1), \\ \Delta z_i(k) &= z_i(k) - z_i(k-1), \end{aligned}$$

$$B_i(k) = \begin{bmatrix} \Phi_i(k) & 0 & \cdots & 0 \\ \Phi_i(k) & \Phi_i(k+1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_i(k) & \Phi_i(k+1) & \cdots & \Phi_i(k+M-1) \end{bmatrix},$$

$$C_i(k) = \begin{bmatrix} \varphi_i^T(k) & 0 & \cdots & 0 \\ \varphi_i^T(k) & \varphi_i^T(k+1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_i^T(k) & \varphi_i^T(k+1) & \cdots & \varphi_i^T(k+M-1) \end{bmatrix}.$$

As seen elsewhere (4), the system pseudogradients in  $A_i(k)$  are unknown. The estimate of  $A_i(k)$  and the flow chart of the DED-MFAPC scheme can be found in Appendixes C and D, respectively.

**DED-MFAPC traffic light controller design.** To avoid uneven traffic flow distribution with severely congested roads and to balance the traffic flow, a cost function for each subsystem  $i$  to optimize is adopted as follows:

$$\begin{aligned} \min_{U_i(k)} P_i &= \sum_{j=1}^M \|y_i^*(k+j) - y_i(k+j)\|_2^2 \\ &+ \sum_{j=0}^{M-1} \xi \lambda_i \|\Delta u_i^2(k+j)\|_2^2, \end{aligned} \quad (6)$$

$$\text{s.t. } y_i(k+1) = y_i(k) + \phi_i^T(k)\Delta \mathbf{V}_i(k), \quad \forall i \in \mathcal{N}, \quad (7)$$

$$u_{im}(k+j) \in [u_{im}^{\min}, u_{im}^{\max}], \quad m = 1, 2, j = 0, \dots, M-1, \quad (8)$$

$$u_{i1}(k+j) + u_{i2}(k+j) + L_i = T, \quad j = 0, \dots, M-1, \quad (9)$$

where  $y_i^*(k+j)$  denotes the expected vehicle density of subsystem  $i$  at the prediction step  $j$ , which relies on the received information from neighboring subsystems,  $U_i(k) = [u_i^T(k),$

$u_i^T(k+1), \dots, u_i^T(k+M-1)]^T$ ,  $\lambda_i > 0$  is a weighting constant to punish the excessive changes of the green time,  $\xi$  is a factor such that the two items in (6) are of the same order of magnitude,  $L_i$  is the yellow light time at subsystem  $i$ , and we assume that all the intersections have the same cycle time  $T$ . Note that  $\|x\|_2$  is the 2-norm of a vector  $x$ .

Applying the optimization condition  $\frac{\partial P_i}{\partial U_i(k)} = 0$ , where  $\frac{\partial P_i}{\partial U_i(k)}$  represents the partial derivative of  $P_i$  with respect to  $U_i(k)$ ,

$$\begin{aligned} \Delta U_i(k) &= [B_i^T(k)B_i(k) + \xi \lambda_i I]^{-1} B_i^T(k) \\ &\times [\mathbf{y}_i^*(k+1) - E(k)y_i(k) - C_i(k)\Delta Z_i(k)], \end{aligned}$$

where  $\mathbf{y}_i^*(k+1) = [y_i^{*T}(k+1), \dots, y_i^{*T}(k+M)]^T$ . Subsequently, the control input can be obtained,

$$u_i(k) = u_i(k-1) + g^T \Delta U_i(k), \quad (10)$$

where  $g = [1, 0, \dots, 0]^T$ . Considering the cycle time and the minimum and maximum green time constraints on the control input, the following equation is needed to be calculated alternately until constraints (8) and (9) are satisfied:

$$u_{im}(k+j) = \begin{cases} u_{im}^{\min}, & \text{if } u_{im}(k+j) < u_{im}^{\min}, \\ u_{im}(k+j), & \text{if } u_{im}^{\min} \leq u_{im}(k+j) \leq u_{im}^{\max}, \\ u_{im}^{\max}, & \text{if } u_{im}(k+j) > u_{im}^{\max}, \end{cases}$$

$$u_{im}(k+j) = \frac{u_{im}(k+j)}{u_{i1}(k+j) + u_{i2}(k+j)}(T - L_i), \quad m = 1, 2.$$

**Case study.** A nine-intersection network is exploited for illustration of the validity and superiority of the proposed DED-MFAPC method in VISSIM. Appendix E presents the details of the experiment results.

**Conclusion.** In this study, a novel data-driven approach, DED-MFAPC, was proposed for traffic light control of multi-intersection networks. In the future, the fuel economy and emissions of vehicles and the case when the traffic lights and vehicle dynamics are simultaneously optimized are worthy of investigation. Another future direction would be calculating the offset of the signals and the cycle time by the data-driven method.

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**Supporting information** Appendixes A–E. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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