SCIENCE CHINA
Information Sciences

• LETTER •

September 2023, Vol. 66 190208:1–190208:2
https://doi.org/10.1007/s11432-022-3776-5

Special Topic: Control, Optimization, and Learning for Networked Systems

Leaderless output sign consensus of heterogeneous multi-agent systems over switching signed graphs

Yihan MENG, Hongwei ZHANG* & Aiguo WU

School of Mechanical Engineering and Automation, Harbin Institute of Technology, Shenzhen 518055*, China*

Received 28 September 2022/Revised 8 January 2023/Accepted 19 April 2023/Published online 17 August 2023

Citation Meng Y H, Zhang H W, Wu A G. Leaderless output sign consensus of heterogeneous multi-agent systems over switching signed graphs. Sci China Inf Sci, 2023, 66(9): 190208, https://doi.org/10.1007/s11432-022-3776-5

Distributed control of multi-agent systems (MASs) has drawn much attention due to its myriad uses in fields such as power systems, transportation systems, and vehicle formation, to name a few. Productive outcomes have been obtained, with most of them centering on MASs over cooperative relationships [1]. However, in practice, antagonistic and cooperative relations coexist within many MASs. When dealing with hostile interactions, one turns to signed graphs, in which positive edges represent cooperative interactions and negative edges represent antagonistic interactions.

Most studies about MASs over non-negative graphs concentrate on consensus behavior, whereas the collective behaviors investigated for signed graphs are more convoluted. One such behavior called the bipartite consensus [2] is based on structurally balanced signed graphs. However, being structurally balanced is a rather demanding condition to be satisfied since a simple alteration of the signs of some edges would disrupt the balance. By using the Perron-Frobenius property, researchers unveiled that MASs over a certain subclass of structurally unbalanced graphs, i.e., the eventually positive signed graphs, can display unanimity of opinions [3], which is also known as sign consensus [4].

For some MASs, agents inherently have distinct dynamics. Recent work [5] on sign consensus of heterogeneous MASs employed a leader-follower architecture to obtain the synchronization. Nonetheless, a group can be leaderless. Furthermore, due to different limitations in certain practical scenarios, the communication network can be dynamically changing. Therefore, leaderless output sign consensus (LOSC) of heterogeneous MASs over switching signed digraphs is difficult and requires further study. We briefly summarize the motivation of this study here, and a more comprehensive literature review is provided in Appendix A.

This study addresses LOSC concerns of heterogeneous MASs over switching signed graphs. We use a distributed "sign observer" design technique. The "sign observer" calculates a virtual leader, which is not pre-specified but completely dependent on the interaction of the agents. Based on the estimation and by using the output regulation approach, we propose two controllers that drive the output signals of the MAS to reach LOSC.

Problem formulation. Consider a heterogeneous MAS over a switching signed digraph $\mathcal{G}(\mathcal{A}_{\delta(t)})$ as follows:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i \\ y_i &= C_i x_i, \quad i = 1, 2, \ldots, N, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and $y_i \in \mathbb{R}^p$ are the state, input, and output of agent i, respectively. Moreover, each (A_i, B_i, C_i) is assumed to be both controllable and observable. The purpose of this study is to develop a distributed control law, such that the MAS eventually achieves LOSC. The LOSC problem is defined as follows.

Problem 1 ([4]). Consider the MAS (1) over a signed graph $\mathcal{G}(\mathcal{A}_{\delta(t)})$. LOSC is said to be reached, if there exists a nontrivial trajectory $z(t) \in \mathbb{R}^p$ such that

$$\begin{aligned} &\lim_{t\to\infty} \left[\operatorname{sgn}\left(y_{i_l}(t)\right) - \operatorname{sgn}\left(z_l(t)\right)\right] = 0, \ \forall l \in L_1, \\ &\lim_{t\to\infty} \left(y_{i_l}(t) - z_l(t)\right) = 0, \ \forall l \in L_2, \end{aligned}$$

where $z_l(t)$ and $y_{i_l}(t)$ are the lth entries of $z(t)$ and $y_i(t)$, respectively; $L_1 = \{l \mid \lim_{t\to\infty} z_l(t) \neq 0, \ l = 1, 2, \ldots, p\}$, $L_2 = \{l \mid \lim_{t\to\infty} z_l(t) = 0, \ l = 1, 2, \ldots, p\}$, $L_1 \cup L_2 = \{1, 2, \ldots, p\}$, and $L_1 \cap L_2 = \emptyset$.

Regarding the solvability of Problem 1, we make the following assumption on the signed graph.

Assumption 1. The switching graph $\mathcal{G}(\mathcal{A}_{\delta(t)})$ is assumed to be jointly eventually positive in any time interval $[t_k, t_{k+1})$.

The definition of jointly eventual positivity in Assumption 1 as well as other preliminaries on graph theory is obtained in Appendix C.

Distributed "sign observer" design. We solve the LOSC problem by employing a so-called distributed "sign observer". The definition of "sign observer" is provided in Appendix E. The "observer" observes the dynamics and the states of the virtual leader and is designed as follows:

$$\dot{S}_i = \mu_1 \left(-\sigma_i(t) S_i + \sum_{j=1}^{N} a_{ij}(t) S_j \right), \tag{2a}$$

* Corresponding author (email: hwzhang@hit.edu.cn)

$$\dot{\zeta}_i = \frac{1}{v_{r_i}} S_i \zeta_i + \mu_2 \left(-\sigma_i(t)\zeta_i + \sum_{j=1}^{N} a_{ij}(t)\zeta_j \right), \quad (2b)$$

where $S_i \in \mathbb{R}^{p\times p}, \zeta_i \in \mathbb{R}^p$, v_{r_i} is the ith entry of the integrated adjacency matrix $\bar{\mathcal{A}}$'s right eigenvector v_r corresponding to its spectral radius, and

$$\sigma_i(t) = \frac{\sum_{j=1}^{N} a_{ij}(t) v_{r_j}}{v_{r_i}}.$$

It is shown by Lemmas 6 and 7 in Appendix E that the designed distributed "sign observer" (2) eventually achieves sign consensus.

Virtual leader construction. By Lemma 7, we have that $\zeta_i(t) \to v_{r_i} \mathrm{e}^{S^* t}\hat{\zeta}^*$ as $t \to \infty$. In analogy to the state trajectory of a general linear system $\dot{x} = Ax$, which is $x(t) = \mathrm{e}^{At}x(0)$, we develop the virtual leader as follows:

$$\dot{\theta} = S^*\theta, \quad (3)$$

where $\theta \in \mathbb{R}^p$ and $S^* \in \mathbb{R}^{p\times p}$ are the state and system matrix of the virtual leader, respectively. By Lemmas 6 and 7, we know that the value of S^* depends on the initial condition of the MAS, the topology of the switching signed graph, and the nature of "observer" (2a); furthermore, the initial condition of η is provided as $\hat{\zeta}^*$. In this way, "observer" (2a) calculates the system matrix of the virtual leader, and "observer" (2b) observes the state of the virtual leader.

Controller design. Based on the distributed "sign observer", we are ready to create a state feedback controller and an output feedback controller. Both controllers can drive the output signals of a heterogeneous MAS (1) to reach LOSC.

We suggest the following state feedback control law:

$$u_i = K_i x_i + K_{\zeta_i}(t)\zeta, \quad (4a)$$

$$K_{\zeta_i}(t) = \frac{1}{v_{r_i}} \Upsilon_{2i}(t) - K_i \Upsilon_{1i}(t), \quad (4b)$$

where K_i is such that $A_i + B_i K_i$ is Hurwitz, Υ_{1i} and Υ_{2i} are generated by Lemma 5 in Appendix D. The following theorem demonstrates that this designed state feedback controller realizes LOSC of system (1).

Theorem 1. Consider MAS (1). Suppose Assumption 1 holds. For large enough scalars $\mu_1,\ \mu_2,\ \mu_3 \in \mathbb{R}^+$, LOSC is reached by control law (2), (D4), and (4).

Proof. See Appendix F.

In practice, the state of system (1) may not be obtained directly. As a result, we design the following observer-based output feedback controller to solve this problem:

$$u_i = K_i \xi_i + K_{\zeta_i}(t)\zeta_i, \quad (5a)$$

$$\dot{\xi}_i = A_i \xi_i + B_i u_i + L_i(y_i - C_i \xi_i), \quad (5b)$$

where K_i and $K_{\zeta_i}(t)$ are selected as in the state feedback controller design, L_i is such that $A_i - L_i C_i$ is Hurwitz, and Eq. (5b) is a Luenberger observer in which $\xi_i \in \mathbb{R}^{n_i}$ observes the state x_i of system (1). LOSC of system (1) by this output feedback controller is examined in the following theorem.

Theorem 2. Consider MAS (1). Suppose Assumption 1 holds. For large enough scalars $\mu_1,\ \mu_2,\ \mu_3 \in \mathbb{R}^+$, LOSC is reached by control law (2), (D4), and (5).

Proof. See Appendix G.

Simulation. We take into account a leaderless MAS with six agents. Each agent has the form of (1). The communication network of the MAS switches among four signed graphs with the switching signal $\delta(t)$. We select the scalars μ_1, μ_2, and μ_3 to be large enough that the conditions of Lemmas 7 and 5 are satisfied and the matrices K_i and L_i with proper values such that $A_i + B_i K_i$ and $A_i - L_i C_i$ are Hurwitz. The output trajectories of the MAS by the two designed controllers eventually have the same sign at the same instant; i.e., LOSC is obtained. The particular design and detailed results of this simulation can be observed in Appendix H.

Conclusion. This study examined the LOSC problem of heterogeneous MASs over signed switching digraphs. We remove the common requirement that a practical leader should exist for the consensus of heterogeneous MASs and enable the communication network to be dynamically changing. A virtual leader is induced by a distributed "sign observer", and by using the output regulation approach, the MAS can achieve LOSC. However, the selection of the switching signed graphs is quite demanding and requires further investigation.

Acknowledgements This work was supported by Guangdong Basic and Applied Basic Research Foundation (Grant No. 2023A1515011981).

Supporting information Appendixes A–H. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Yu W W, Wang H, Hong H F, et al. Distributed cooperative anti-disturbance control of multi-agent systems: an overview. Sci China Inf Sci, 2017, 60: 110202

2 Pan L L, Shao H B, Xi Y G, et al. Bipartite consensus problem on matrix-valued weighted directed networks. Sci China Inf Sci, 2021, 64: 149204

3 Altafini C, Lini G. Predictable dynamics of opinion forming for networks with antagonistic interactions. IEEE Trans Automat Contr, 2015, 60: 342–357

4 Jiang Y, Zhang H, Chen J. Sign-consensus over cooperative-antagonistic networks with switching topologies. Int J Robust Nonlinear Control, 2018, 28: 6146–6162

5 Sun Z, Zhang H, Lewis F L. Output sign-consensus of heterogeneous multiagent systems over fixed and switching signed graphs: an observer-based approach. Int J Robust Nonlinear Control, 2021, 31: 5849–5864