

Iterative learning security control for discrete-time systems subject to deception and DoS attacks

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Malicious cyber-attacks often occur in cyberspace during the signal transmission process. It might result in data dropout and system failure. This problem then draws attention to secure control, and a number of security control protocols are proposed [1–3].

In reality, the co-existence of multiple cyber-attacks increases the difficulty of the theoretical analysis. Furthermore, some sufficient conditions for achieving security are difficult to check when the dimension of the model is high. As a result, the applicable approaches are needed for analyzing and obtaining some concise conditions.

In contrast to the traditional tracking control methods, iterative learning control (ILC) technology provides an effective approach to achieve system targets. Such a control method has been applied in urban road networks [4] and security analysis [5]. Until now, the research on ILC still attracts the interest of the control science and engineering communities.

When deception and DoS attacks occur alternately, it is difficult to obtain the actual output of the system as the attacks may falsify the observations. To achieve system security, an output estimation formula under the DoS and deception attacks is proposed in this study. Furthermore, the appropriate ILC algorithms are designed based on the output estimation formula.

The following are the main contributions of this study: (1) The system security problem is investigated under the deception and DoS attacks. A novel output estimation formula is proposed to evaluate the unavailable measurement of the real output. (2) System security is achieved by using ILC strategies, and our schemes ensure that the system trajectory is bounded. (3) Compared with the existing literature, the present ILC strategies are simple and can be easily checked in practice.

Notations. $x = [x_1, \dots, x_n]^T$ and $y = [y_1, \dots, y_n]^T$ are two real vectors, and symbol T denotes transpose. The partial order relation \prec is defined as $x \prec y$ if and only if $x_i \leq y_i$ for all $i \in \{1, \dots, n\}$. $\|x\|$ represents the norm of vector x .

$\|A\|$ denotes the matrix norm of matrix A . The zero matrix is defined as \mathbf{O} , and the unit matrix is described by I . $\mathbb{E}(\cdot)$ is the mathematical expectation of a stochastic vector. $\mathcal{T} = \{1, \dots, l\}$ is the time interval, and l is the fixed terminal time.

Problem formulation. A discrete-time system is

$$\begin{cases} x_k(t+1) = Ax_k(t) + Bu_k(t), & t \in \mathcal{T} \setminus \{l\}, \\ y_k(t) = Cx_k(t) + Du_k(t), & t \in \mathcal{T}, \end{cases} \quad (1)$$

where k represents the k th iteration process. $x_k(t)$, $u_k(t)$, $y_k(t)$ are the state vector, input vector, and output vector, respectively. Matrices A , B , C , and D in (1) are with appropriate dimensions. Furthermore, two output modes are considered in this study, i.e., $D = \mathbf{O}$ and $D \neq \mathbf{O}$.

Inspired by [2], we consider the following impact pattern of random deception and DoS attacks:

$$\hat{y}_k(t) = y_k(t) + \alpha_{k,t}\beta_{k,t}\mu_k(t) + \alpha_{k,t}(1 - \beta_{k,t})\nu_k(t), \quad (2)$$

where $\hat{y}_k(t)$ is the received signal by the observer subject to attacks. $\mu_k(t)$ and $\nu_k(t)$ denote the deception attack and the DoS attack, respectively. The deception attack is described as

$$\mu_k(t) = -y_k(t) + \xi_k(t), \quad (3)$$

where $\xi_k(t)$ is a bounded signal, $\|\xi_k(t)\| \leq \xi_b$, $\xi_b > 0$. The DoS attack mode is

$$\nu_k(t) = -y_k(t). \quad (4)$$

Bernoulli stochastic variables $\alpha_{k,t}$ and $\beta_{k,t}$ in (2) are mutually uncorrelated and satisfy

$$\text{Prob}\{\alpha_{k,t} = 1\} = \mathbb{E}(\alpha_{k,t}) = \bar{\alpha}, \quad 0 \leq \bar{\alpha} < 1,$$

$$\text{Prob}\{\beta_{k,t} = 1\} = \mathbb{E}(\beta_{k,t}) = \bar{\beta}, \quad 0 \leq \bar{\beta} < 1,$$

where $\bar{\alpha}$ and $\bar{\beta}$ are two known constants. Moreover, α_{k_1,t_1} and α_{k_2,t_2} are mutually independent for all $k_1 \neq k_2$ and $t_1, t_2 \in \mathcal{T}$, and so is $\beta_{k,t}$. From (2)–(4), one obtains

$$\hat{y}_k(t) = (1 - \alpha_{k,t})y_k(t) + \alpha_{k,t}\beta_{k,t}\xi_k(t). \quad (5)$$

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Define the tracking error as $e_k(t) = y_d(t) - y_k(t)$, where $y_d(t)$ is the desired output. However, the real output series $\{y_k(t)\}$ might not be directly obtained due to cyber attacks. Thus, the tracking target is difficult to be achieved. Then, the desired output is applied to compensate for the incorrect output. The estimation output is designed as

$$\check{y}_k(t) = \hat{y}_k(t) + \alpha_{k,t} y_d(t), \quad (6)$$

and the available tracking error is $\check{e}_k(t) = y_d(t) - \check{y}_k(t)$. For system (1), the ILC strategy is constructed as

$$u_{k+1}(t) = u_k(t) + \mathcal{K}_1 \check{e}_k(t) + \mathcal{K}_2 \check{e}_k(t+1), \quad (7)$$

where \mathcal{K}_1 and \mathcal{K}_2 are learning gains, and the initial input $u_0(t)$ is chosen arbitrarily.

Definition 1. Let the desired security level be specified as $\sigma > 0$. The repetitive discrete-time system (1) is said to have the σ -secure in the meaning sense if the inequality $\lim_{k \rightarrow \infty} \mathbb{E}(\|e_k(t)\|) \leq \sigma$ holds for $t \in \mathcal{T}$.

Our purpose is to analyze the security of the system in line with Definition 1 and ILC protocol (7). The following assumptions are required.

Assumption 1. Each iteration's initial state is fixed, i.e., $x_k(0) = x_{k+1}(0)$.

Assumption 2. For any desired output $y_d(t)$ and system (1) with $D \neq \mathbf{O}$, there exist desired state $x_d(t)$ and bounded desired input $u_d(t)$ such that

$$\begin{cases} x_d(t+1) = Ax_d(t) + Bu_d(t), & t \in \mathcal{T} \setminus \{l\}, \\ y_d(t) = Cx_d(t) + Du_d(t), & t \in \mathcal{T}. \end{cases} \quad (8)$$

Robust convergence and boundedness analysis. The following section studies the robust convergence and boundedness of system trajectories.

Theorem 1. Letting Assumptions 1 and 2 hold, ILC strategy (7) with $\mathcal{K}_2 = \mathbf{O}$ is applied to system (1), where $D \neq \mathbf{O}$. Then, the σ -secure is achieved if the inequality $\|I - \mathcal{K}_1 D\| < 1$ holds.

Theorem 2. Letting Assumptions 1 and 2 hold, ILC strategy (7) with $\mathcal{K}_2 = \mathbf{O}$ is applied to system (1), where $D \neq \mathbf{O}$. If the inequality $\|I - \mathcal{K}_1 D\| < 1$ holds, one has $\sup_{k \in \mathbb{Z}_+, t \in \mathcal{T}} \mathbb{E}(\|x_k(t)\|) \leq \mathcal{B}_x$, $\sup_{k \in \mathbb{Z}_+, t \in \mathcal{T}} \mathbb{E}(\|u_k(t)\|) \leq \mathcal{B}_u$, $\sup_{k \in \mathbb{Z}_+, t \in \mathcal{T}} \mathbb{E}(\|y_k(t)\|) \leq \mathcal{B}_y$, $\sup_{k \in \mathbb{Z}_+, t \in \mathcal{T}} \mathbb{E}(\|e_k(t)\|) \leq \mathcal{B}_e$, where $\mathcal{B}_x \geq 0$, $\mathcal{B}_u \geq 0$, $\mathcal{B}_y \geq 0$, and $\mathcal{B}_e \geq 0$ are finite constants.

Theorem 3. Letting Assumption 1 hold, ILC strategy (7) with $\mathcal{K}_1 = \mathbf{O}$ is applied to system (1), where $D = \mathbf{O}$. Then, the σ -secure is achieved if the inequality $\|I - CB\mathcal{K}_2\| < 1$ holds.

Theorem 4. Letting Assumption 1 hold, ILC strategy (7) with $\mathcal{K}_1 = \mathbf{O}$ is applied to system (1), where $D = \mathbf{O}$. If the inequality $\|I - \mathcal{K}_2 CB\| < 1$ holds, one has $\sup_{k \in \mathbb{Z}_+, t \in \mathcal{T}} \mathbb{E}(\|x_k(t)\|) \leq \tilde{\mathcal{B}}_x$, $\sup_{k \in \mathbb{Z}_+, t \in \mathcal{T}} \mathbb{E}(\|u_k(t)\|) \leq \tilde{\mathcal{B}}_u$, $\sup_{k \in \mathbb{Z}_+, t \in \mathcal{T}} \mathbb{E}(\|y_k(t)\|) \leq \tilde{\mathcal{B}}_y$, and $\sup_{k \in \mathbb{Z}_+, t \in \mathcal{T}} \mathbb{E}(\|e_k(t)\|) \leq \tilde{\mathcal{B}}_e$, where $\tilde{\mathcal{B}}_x \geq 0$, $\tilde{\mathcal{B}}_u \geq 0$, $\tilde{\mathcal{B}}_y \geq 0$, and $\tilde{\mathcal{B}}_e \geq 0$ are finite constants.

The proofs of theorems are provided in Appendixes A–D. Appendix E contains a simulation example.

Remark 1. Compared with one type of attack in [1, 5], the co-existence of two attacks in this study increases the difficulty of the theoretical analysis. Therefore, it is difficult to obtain the actual output of the system because the two attacks might falsify the observations. As a result, ILC strategies are proposed to address this difficulty.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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