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Secure cooperative output regulation for linear parameter-varying systems under DoS attacks: a resilient observer approach

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Abstract This paper investigates the cooperative output regulation problem of heterogeneous linear parameter-varying multi-agent systems under denial-of-service (DoS) attacks. The matrix and state of the exosystem are taken to be unknown to the followers. Moreover, with the assumption that only a directed spanning tree exists in the communication topology, a resilient observer is proposed to exponentially estimate the global information, which can also be effective under DoS attacks with a certain intensity. Afterward, to avoid using the exact value of the exosystem matrix, a distributed online algorithm is proposed, through which the linear parameter-varying output regulation equation is asymptotically solved. Based on the resilient observer and the solution of the output regulation equation, a distributed regulator is proposed to achieve asymptotic cooperative output regulation for all agents. Some numerical simulations are conducted to verify the effectiveness of the proposed observer, algorithm, and controller.

Keywords cooperative output regulation, heterogeneous multi-agent systems, DoS attacks, linear parameter-varying systems

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1 Introduction

In the last two decades, the cooperative control problem of multi-agent systems (MASs) has attracted considerable attention owing to the potential practical applications of the systems in numerous fields, including mobile robot consensus [1], vehicle platoon formation [2], and the decentralized control of large-scale systems [3]. While the abovementioned studies focused on the cooperative control problem of homogeneous systems, agents are heterogeneous, and only a part of the system states can be measured. Under such circumstances, consider the control problem of output regulation is more encompassing. The output regulation problem of a single plant has been studied since the 1970s. Davison [4] and Isidori et al. [5] first studied this problem, and inspired by their studies, some other studies have been conducted [6, 7]. As an extensional part of the single-unit output regulation problem, the cooperative output regulation problem of MASs aims to force the outputs of a group of agents to track a reference signal while resisting some external disturbances [8], where both the reference signal and the disturbance are issued from the "exosystem".

In the earliest studies on the cooperative output regulation problem, the matrix and state of the exosystem were assumed to be known to each follower [9], which is impractical under a large MAS scale. A distributed observer approach, in which each follower constructs its own distributed observer to estimate the exosystem state, was first proposed by Huang et al. [10]. In their approach, while the exact value of the exosystem state is replaced with its estimated value, the matrix of the exosystem is still

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required. Further improvement was made in [11], in which the authors proposed a distributed observer to estimate both the matrix and state of the exosystem. However, in [9–11], the communication topology of MASs was assumed to be invariant, and the information transmission among agents was continuous. In such cases, even a short-time break in communication topology in MASs may lead to the invalidation of the controller.

Owing to the frequent information transmission among the agents, the information collection and exchange channels in the MASs are vulnerable to attacks, which implies that the possibility of cyber-attacks cannot be ignored. Cyber-attacks can be roughly divided into two main categories: deception attacks [12, 13] and denial-of-service (DoS) attacks [14, 15]. DoS attacks mainly impact two scenarios: the closed-loop information collection in the distributed actuator of each agent and the information exchanging process in communication channels among agents. MAS consensus may collapse owing to the failure of even one distributed controller under attack; therefore, the cyber-security problem of MASs has attracted the researchers' attention. Regarding the first scenario, some fundamental and important progress in the secure control of cyber-physical systems under DoS attacks was achieved in [16, 17]. The authors in [18] proposed Takagi-Sugeno fuzzy cooperative control rules against actuator attacks. In [19], the authors considered the secure-state consensus of MASs when the distributed controllers suffered part-time failure due to the attacks. Furthermore, in [20], the authors studied the actuator fault when DoS attacks occurred in double-integrator MASs. In [21], the sampling control method was proposed for discrete-time systems under DoS attacks. Regarding the second scenario, in [22], the authors studied the state consensus of MASs with communication channel failures under two DoS attack types. In [23], a secure distributed fixed-time state consensus control strategy for high-order MASs was proposed. In [24], the control problem of communication channels broken by DoS attacks in switched discrete-time systems was investigated. In [25], Markov jump systems were considered, and the DoS attacks were assumed to be identical during one operating stage and underwent jumping in the next stage. In [21, 26], all communication channels were assumed to be interrupted during DoS attacks, while in many practical applications, the attacks impacted only partial channels. The authors considered periodic attacks in [27], but the aperiodic cases have not been considered. In [28, 29], authors provided formulations according to undirected graphs. The consideration of directed topology scenarios remains challenging. Thus, considering an applicable attack model in consensus problems of MASs is another motivation for the present study.

As shown above, although the state consensus of MASs under DoS attacks has been widely studied, studies on the secure cooperative output regulation problem under DoS attacks are few. In [29], the authors solved the output regulation problem for partial-actuator-fault systems under undirected graphs. The impact of DoS attacks was handled by a predictor to maintain the positive-definite property of a matrix, which is relevant to the undirected topology. Under a directed communication topology, the predictor would lose its effectiveness. Secure output regulation problems were also investigated in [30]; the DoS attacks were assumed to occur in specific nodes but not communication channels. In [12], the authors studied the secure formation problem via the output regulation theory; however, both the system matrix and the state of the leader were assumed to be prescribed. Moreover, the aforementioned studies all focused on linear time-invariant systems, but in practice, the systems are very likely to exhibit time-varying dynamics [31]. For time-varying systems, it is more challenging to construct the controller, as the controller gain may not be constant [32]. As a member of linear time-varying system settings, the linear parameter-varying system is regarded as the bridge connecting the linear and nonlinear systems. Linear parameter-varying systems have wide real-world applications, such as four-wheel vehicles, airplanes, and mass-spring-damper systems [33, 34]. The cooperative output regulation problem for linear parameter-varying MASs was investigated in [35]; however, the matrix and state of the exosystem are still needed for regulator design, and the information transmission among agents is also assumed to be continuous without considering cyber-attacks.

To the best of the authors' knowledge, there is no work on the cooperative output regulation problem of linear parameter-varying systems under DoS attacks with directed graphs. In this paper, the authors propose a strategy based on a resilient observer to solve this problem. The contributions of this paper are as follows:

(1) The communication topology is assumed to be directed. Different from many existing studies on secure consensus control problems with undirected graphs [28–30], only a directed spanning tree is required in this paper.

(2) The DoS attacks considered in this paper can be unknown and injected by multiple adversaries

under a directed communication topology. Moreover, when DoS attacks are injected, there is no need to maintain a directed spanning tree.

(3) In existing related studies [12, 31], the secure observer only estimated the exosystem state, but the matrix of the exosystem was assumed to be known. To overcome the difficulties caused by the inability to acquire the matrix and state of the exosystem, a distributed observer is proposed to exponentially estimate the exosystem information, and the proposed observer is resilient to DoS attacks with a certain intensity.

(4) Different from previous studies [12, 29, 30], which considered linear time-invariant networked systems, this paper considers linear parameter-varying systems. An online algorithm is proposed to solve the linear parameter-varying output regulation equation, based on which both the state-feedback and output-feedback regulators are proposed to achieve the cooperative output regulation of linear parameter-varying MASs.

The rest of this paper is organized as follows: Section 2 presents the preliminaries and problem formulation; Section 3 presents the adequate resilient distributed observer, online algorithms, and distributed controllers against DoS attacks; Section 4 presents a numerical example to indicate the effectiveness of the proposed methodology. Finally, Section 5 presents the conclusion and future work.

Notations. In this paper, $\mathbb{R}$ denotes the field of real numbers. $M \in \mathbb{R}^{a\times b}$ means that M is a matrix of dimension $a \times b$. M^{T} is the transpose matrix of M and M^{-1} stands for the inverse matrix. Let I_p be the $p \times p$ identity matrix. $\mathbf{1}_n$ is the n-dimension vector with all elements identical to 1. For matrices M and N, $M > 0$ and $M > N$ represents that M and $M - N$ are positive definite, respectively. $M^{[j]}$ means the jth column of matrix M and M_{pq} means the element in the pth row and qth column of matrix M. For a matrix $P = [p_1, \ldots, p_m] \in \mathbb{R}^{n\times m}$ with vectors $p_i \in \mathbb{R}^n$ where $i = 1, \ldots, m$, let $\mathrm{vec}(P) = (p_1^{\mathrm{T}}, \ldots, p_m^{\mathrm{T}})^{\mathrm{T}}$ and $W_m^n(P) = [p_1, \ldots, p_m]$. diag$(\cdot)$ represents the block diagonal matrix. $\|\cdot\|$ stands for the Euclidean norm for vectors or the induced 2-norm for matrices. Operation $\otimes$ represents the Kronecker production. $\sigma_{\max}(M)$ and $\sigma_{\min}(M)$ denote the maximum and minimum eigenvalue of matrix M, respectively. We say a matrix $Q(t)$ is asymptotically (exponentially) stable if the corresponding unforced linear system $\dot{\xi}(t) = Q(t)\xi(t)$ is asymptotically (exponentially) stable.

2 Preliminaries and problem formulation

2.1 Graph theory

A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ consists of a node set $\mathcal{V}$, an edge set $\mathcal{E}$, and the corresponding adjacency matrix $\mathcal{A} = [a_{ij}]$. If the connections in $\mathcal{G}$ are directional, $\mathcal{G}$ is called a directed graph. In this paper, let the directed graph $\mathcal{G}$ denote the communication network topology among N follower agents. If two nodes p and q are connected and node p can receive the information sent by node q, then $a_{pq} = 1$; otherwise, $a_{pq} = 0$. For a directed graph, a path between nodes k_1 and k_p is a finite sequence of directed edges in the form of $(k_1, k_2), (k_2, k_3), \ldots, (k_{p-1}, k_p)$, where all edges are distinct. If node p can reach the other nodes through a directional path, we say that a directed spanning tree exists in the directed graph, where root node p is called the root. $\mathcal{N}_p$ denotes the neighboring set of node p. $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ denotes the Laplacian matrix of graph $\mathcal{G}$, with $l_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$, and $l_{ij} = -a_{ij}$, for $i \neq j$.

$\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$, where $\bar{\mathcal{V}} = \{0\} \cup \mathcal{V}$. Node 0 represents the exosystem, and all other nodes indexed by $1, \ldots, N$ denote the agents. Let $\bar{\mathcal{L}}$ denote the Laplacian matrix of $\bar{\mathcal{G}}$. $\mathcal{D} = \mathrm{diag}\{a_{10}, \ldots, a_{N0}\}$, and $H = \bar{\mathcal{L}} + \mathcal{D}$. If node i can receive information from the exosystem, $a_{i0} = 1$; otherwise, $a_{i0} = 0$.

2.2 Mathematics preliminaries

Lemma 1 ([36]). If a directed graph $\mathcal{G}$ with N nodes contains a directed spanning tree, then the matrix H is a nonsingular M-matrix with its eigenvalues all having positive real parts, and there exists a positive-definite diagonal matrix $M = \mathrm{diag}\{\mu_1^{-1}, \mu_2^{-1}, \ldots, \mu_N^{-1}\} \in \mathbb{R}^{N\times N}$ such that $\Omega = HM + MH^{\mathrm{T}} > 0$.

Lemma 2 ([11]). Given a system with dynamics

$$\dot{x}(t) = -aPx(t) + P_1(t)x(t) + P_2(t), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $a > 0$ is a positive constant, $P \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, $P_1(t) \in \mathbb{R}^{n\times n}$ and $P_2(t) \in \mathbb{R}^n$ are continuous and bounded time-varying matrix and vector, respectively. If $\lim_{t\to\infty} P_1(t) = 0$ and $\lim_{t\to\infty} P_2(t) = 0$, then for any $a > 0$ and $x(0)$, $\lim_{t\to\infty} x(t) = 0$.

2.3 Problem formulation

The exosystem is given by

$$\dot{\omega} = S\omega, \tag{2}$$

where $\omega \in \mathbb{R}^p$ is the exosystem state, and $S \in \mathbb{R}^{p\times p}$ is the exosystem matrix.

The heterogeneous linear parameter-varying agents can be modeled as

$$\dot{x}_i = A_i(\vartheta_i(t))x_i + B_i(\vartheta_i(t))u_i + E_i(\vartheta_i(t))\omega, \tag{3a}$$

$$e_i = C_i(\vartheta_i(t))x_i + D_i(\vartheta_i(t))u_i + F_i(\vartheta_i(t))\omega, \tag{3b}$$

$$y_{mi} = C_{mi}(\vartheta_i(t))x_i + D_{mi}(\vartheta_i(t))u_i + F_{mi}(\vartheta_i(t))\omega, \tag{3c}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $e_i \in \mathbb{R}^r$, and $y_{mi} \in \mathbb{R}^{y_i}$ are the state, input, regulated output, and measurable output of the ith agent, respectively; matrices in (3) are all in compatible dimensions. In this paper, the linear parameter-varying matrices are affine functions that depend on the scheduling variable $\vartheta_i(t) \in \mathbb{R}^{q_i}$. Matrix $\phi_i(\vartheta_i(t))$ is defined as

$$\phi_i(\vartheta_i(t)) = \phi_{i0} + \sum_{j=1}^{q_i} \vartheta_{ij}(t)\phi_{ij}, \tag{4}$$

and it is located in a convex hull restricted by q_i vertices $\phi_{i0} + \phi_{ij}$ $(j = 1, \ldots, q_i)$,

$$\phi_i(\vartheta_i(t)) = \sum_{j=1}^{q_i} \vartheta_{ij}(t)(\phi_{i0} + \phi_{ij}), \quad \sum_{j=1}^{q_i} \vartheta_{ij}(t) = 1. \tag{5}$$

Remark 1. The regulated output e_i and measurable output y_{mi} play different roles in output regulation problems. For the regulated output, which constitutes the control objective of this paper, the output regulation problems aim at finding proper control laws to force the regulated output to zero. The measurable output is used as the feedback to design output-feedback controllers.

Some basic assumptions are made for further discussion.

Assumption 1. Graph $\bar{\mathcal{G}}$ contains a directed spanning tree, and the leader node is the root.

Assumption 2. S in (2) has no eigenvalues with negative real parts.

Assumption 3. $B_i(\vartheta_i(t))$, $D_i(\vartheta_i(t))$, $C_{mi}(\vartheta_i(t))$, and $D_{mi}(\vartheta_i(t))$ are parameter-independent, thus $B_i(\vartheta_i(t)) = B_i$, $D_i(\vartheta_i(t)) = D_i$, $C_{mi}(\vartheta_i(t)) = C_{mi}$, and $D_{mi}(\vartheta_i(t)) = D_{mi}$.

Remark 2. Assumption 2 is a standard assumption in output regulation problems [8, 10, 11]. It is adopted to exclude trivial cases. The components of the exogenous signals corresponding to the models associated with the eigenvalues of S with negative real parts will exponentially decay to zero and not affect the asymptotic behavior of the closed-loop systems. Assumption 3 is quite often adopted in linear parameter-varying systems. In [37–39], it is pointed out that for linear parameter-varying systems with parameter-dependent $B_i(\vartheta_i(t))$, $D_i(\vartheta_i(t))$, $C_{mi}(\vartheta_i(t))$, and $D_{mi}(\vartheta_i(t))$, the variation caused by scheduling variable $\vartheta_i(t)$ can be eliminated through the pre- and post-filtering of control inputs u_i and measured outputs y_{mi}, which reformulates the former systems into an augmented system with parameter-independent input and output matrices.

The DoS attacks discussed in this paper can be unknown and launched by multiple adversaries. In terms of their impact on the communication topology, the attacks can be divided into the following two types.

Definition 1 (Connectivity-maintained attacks)**.** Under connectivity-maintained DoS attacks, even though some communication channels in MASs are disconnected, the communication topology still contains a directed spanning tree with the leader node as the root.

Definition 2 (Connectivity-broken attacks)**.** Under connectivity-broken DoS attacks, the information interchanges between agents are arbitrarily invalid; thus, the communication topology may not possess a directed spanning tree among attack duration.

In this paper, both connectivity-maintained and connectivity-broken attacks are considered. Let $\tau_1(t)$ denote the connectivity-broken attack duration set, $\tau_2(t)$ denote the connectivity-maintained attack duration set, and $\tau(t) = \tau_1(t) \bigcup \tau_2(t)$ denote the full duration set of attacks. The attack duration and frequency are given as follows.

Definition 3 (Attack duration). $T_\tau(t_1,t_2)$ $(t_2>t_1)$ denotes the duration of DoS attacks occurring in time interval $[t_1,t_2]$, and $T_\tau(t_1,t_2)$ satisfies the following equation:

$$T_\tau(t_1,t_2)\leqslant\frac{t_2-t_1}{\xi},$$

where $\xi>1$.

Definition 4 (Attack frequency). $N_\tau(t_1,t_2)$ $(t_2>t_1)$ denotes the number of DoS attacks occurring in time interval $[t_1,t_2]$. The attack frequency $f_\tau(t_1,t_2)$ is given as

$$f_\tau(t_1,t_2)=\frac{N_\tau(t_1,t_2)}{t_2-t_1}.$$

The control objective of this paper is summarized below.

Problem 1 (Secure heterogeneous cooperative output regulation problem). Consider MASs consisting of the exosystem (2) and N heterogeneous follower agents described by (3). The directed communication network topology of the MAS under DoS attacks is described as $\bar{\mathcal{G}}$. The resilient heterogeneous cooperative output regulation of MASs is achieved if the following two statements are satisfied.

(1) The states of both the overall closed-loop system and the controller are asymptotically stable at $\omega(t)=0$.

(2) Under DoS attacks, for arbitrary initial conditions $x_i(0)$ and $\omega(0)$, the regulated output of any follower will asymptotically decay to zero; that is, $\lim_{t\to\infty}e_i(t)=0$ for $i\in\mathcal{V}$.

3 Main results

3.1 Resilient distributed observer under DoS attacks

The distributed regulator design of the cooperative output regulation problem depends on the exosystem information. As the matrix and state of the exosystem are assumed to be unknown to the followers, a distributed observer to estimate them is needed. The resilient distributed observer under DoS attacks is proposed as

$$\dot{s}_i(t)=-\gamma_i\delta_i(t), \tag{6a}$$

$$\delta_i(t)=\sum_{j=1}^{N}a_{ij}^{\tau(t)}(s_i(t)-s_j(t))+a_{i0}^{\tau(t)}(s_i(t)-s_0), \tag{6b}$$

$$S_i(t)=W_p^p(s_i(t)), \tag{6c}$$

$$\dot{\xi}_i(t)=S_i(t)\xi_i(t)-d_i\tilde{\delta}_i(t), \tag{6d}$$

$$\tilde{\delta}_i(t)=\sum_{j=1}^{N}a_{ij}^{\tau(t)}(\xi_i(t)-\xi_j(t))+a_{i0}^{\tau(t)}(\xi_i(t)-\omega(t)), \tag{6e}$$

where γ_i and d_i are positive constants, and $a_{ij}^{\tau(t)}$ is the element of the time-varying Laplacian matrix affected by the DoS attacks $\tau(t)$. The definition of operator W_p^P can be found in Notations.

Remark 3. As the DoS attacks are injected into the communication channels of the MASs, information cannot be exchanged among the agents. The Laplacian matrix $\hat{\mathcal{L}}$ discussed herein is piecewise continuous, as the elements a_{ij} in $\hat{\mathcal{L}}$ are $a_{ij}^{\tau(t)}$.

Remark 4. Different from the observer proposed in [10, 11], observer (6) considers the time-varying Laplacian matrix related to unknown DoS attacks. The observer is called resilient because Eq. (6) can estimate the global information and defend against attacks from adversaries in the meantime.

Theorem 1. Given the MASs (3) under Assumptions 1 and 2, suppose the communication channels among the agents suffer from both connectivity-maintained and connectivity-broken attacks. Let $\lambda_{1\min}=\sigma_{\min}(\Omega)$, $\lambda_{2\max}=\sigma_{\max}(M)$ (matrices Ω and M can be founded in Lemma 1), $\lambda_{3\min}=\sigma_{\min}(H^{\tau_2(t)}+(H^{\tau_2(t)})^{\mathrm{T}})$, $\epsilon=d_{i\min}\frac{\lambda_{1\min}}{\lambda_{2\max}}$, and $\tilde{\mu}=\frac{\mu_{i\max}}{\mu_{i\min}}$. The following linear matrix inequality for $Q>0$ and $\alpha_2>0$ is solved as

$$\begin{bmatrix} QS+S^{\mathrm{T}}Q+(\alpha_2+2-\epsilon)Q & Q\\ Q & -I\end{bmatrix}<0; \tag{7}$$

select $\beta_2 > \epsilon - \alpha_2 - \lambda_{3\min}$, choose sufficiently large γ_i such that $\sigma_0 = \gamma_{i\min}\frac{\lambda_{1\min}(\hat{\rho}+\beta_2)}{\lambda_{2\max}(\alpha_2+\beta_2)} - ||S_0|| \geqslant \alpha_2$, and define $\phi = \frac{\sigma_0+\beta 2}{\sigma_0-\alpha_2}$ and $\psi = \max\{\tilde{\mu}, \phi\}$.

If there exist a positive constant $\hat{\rho} \in (0, \alpha_2)$ and a positive constant $\rho \in (0, \hat{\rho})$, such that the total duration of the connectivity-broken attacks $T_d(t_0, t)$ satisfies

$$T_d(t_0, t) \leqslant \frac{\alpha_2 - \hat{\rho}}{\alpha_2 + \beta_2}(t - t_0), \tag{8}$$

and the DoS attack frequency f_τ satisfies

$$f_\tau \leqslant \frac{\hat{\rho} - \rho}{\ln(\psi)}. \tag{9}$$

Then with the proposed resilient observer (6), the following two statements hold:

(1) For $i \in \mathcal{V}$, the estimated value of exosystem matrix S_i will exponentially converge to S.

(2) For $i \in \mathcal{V}$, the estimated value of exosystem state ξ_i will exponentially converge to ω with decaying rate ρ.

Proof. Let $\Gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_N\}$, $\bar{s}_i(t) = s_i(t) - s_0$, and $\bar{s}(t) = (\bar{s}_1^{\mathrm{T}}(t), \bar{s}_2^{\mathrm{T}}(t), \ldots, \bar{s}_N^{\mathrm{T}}(t))^{\mathrm{T}}$; then, Eq. (6a) is rewritten into the compact form as

$$\bar{s}(t) = -H^{\tau(t)} \otimes \Gamma \bar{s}(t). \tag{10}$$

Case 1. When $t \in \tau_1(t)$, there always exists a directed spanning tree in graph $\mathcal{G}$. The candidate Lyapunov function is as follows:

$$V_1(t) = \sum_{i=1}^{N} \mu_i^{-1} \bar{s}_i^{\mathrm{T}}(t) \bar{s}_i(t). \tag{11}$$

The time derivative of (11) gives

$$\begin{aligned} \dot{V}_1(t) &= -\bar{s}^{\mathrm{T}}[(H^{\tau_1(t)}M + M(H^{\tau_1(t)})^{\mathrm{T}}) \otimes \Gamma]\bar{s} = -\bar{s}^{\mathrm{T}}(\Omega^{\tau_1(t)} \otimes \Gamma)\bar{s} \\ &\leqslant -\frac{\gamma_{i\max}\lambda_{1\min}}{\lambda_{2\max}} V_1 \leqslant -\alpha_1 V_1, \end{aligned} \tag{12}$$

where $\lambda_{1\min} = \sigma_{\min}(\Omega)$, $\lambda_{2\max} = \sigma_{\max}(M)$, $\gamma_{i\max} = \max\{\gamma_i\}$, and $\alpha_1 = \frac{\gamma_{i\min}\lambda_{1\min}}{\lambda_{2\max}} > 0$.

Case 2. When $t \in \tau_2(t)$, the connectivity of digraph $\mathcal{G}$ is broken; thus, the directed spanning tree may be disconnected. The candidate Lyapunov function is as follows:

$$V_2(t) = \sum_{i=1}^{N} \bar{s}_i^{\mathrm{T}}(t) \bar{s}_i(t). \tag{13}$$

The time derivative of (13) yields

$$\dot{V}_2(t) = -\bar{s}^{\mathrm{T}}[(H^{\tau_2(t)} + (H^{\tau_2(t)})^{\mathrm{T}}) \otimes \Gamma]\bar{s} \leqslant -\gamma_{i\max}\lambda_{3\min} V_2 \leqslant -\beta_1 V_2, \tag{14}$$

where $\lambda_{3\min} = \sigma_{\min}(H^{\tau_2(t)} + (H^{\tau_2(t)})^{\mathrm{T}}) \geqslant 0$, $\beta_1 = \gamma_{i\max}\lambda_{3\min} \geqslant 0$. Combining (12) and (14) with (8) gives

$$||M||||\bar{s}(t)||^2 \leqslant \mathrm{e}^{-\frac{\alpha_1(\hat{\rho}+\beta_2)}{\alpha_2+\beta_2}(t-t_0)} V_1(t_0). \tag{15}$$

This implies that $||\bar{s}(t)||$ will exponentially approach 0; thus, the value estimation of the exosystem matrix of the ith agent will exponentially converge to S under the proposed observer.

Now the proof of the second part of Theorem 1 is given. Let $\bar{\xi}_i(t) = \xi_i(t) - \omega(t)$, $\bar{S}_i(t) = S_i(t) - S$, $\bar{S}(t) = \mathrm{diag}(\bar{S}_1(t), \bar{S}_2(t), \ldots, \bar{S}_N(t))$, and $\bar{\xi}(t) = (\bar{\xi}_1^{\mathrm{T}}(t), \bar{\xi}_2^{\mathrm{T}}(t), \ldots, \bar{\xi}_N^{\mathrm{T}}(t))^{\mathrm{T}}$.

Case 1. When $t \in \tau_1(t)$, the candidate Lyapunov function is chosen as

$$V_3(t) = \sum_{i=1}^{N} \mu_i^{-1} \bar{\xi}_i^{\mathrm{T}}(t) Q \bar{\xi}_i(t). \tag{16}$$

The time derivative of (16) gives

$$\dot{V}_3(t) \leqslant \bar{\xi}^{\mathrm{T}} M \otimes (QS + S^{\mathrm{T}}Q)\bar{\xi} - \bar{\xi}^{\mathrm{T}}(\Omega \otimes QD)\bar{\xi} + \bar{\xi}^{\mathrm{T}}(M \otimes 2Q\bar{S})\bar{\xi} + \bar{\xi}^{\mathrm{T}}(M \otimes Q^2)\bar{\xi} + \omega^{\mathrm{T}}\bar{S}^{\mathrm{T}}M\bar{S}\omega.$$

According to (15), there exists a fixed time T_1 such that, at $t \geqslant T_1$, $||\bar{S}|| \leqslant 1$, $\epsilon = d_{i\min}\frac{\lambda_{1\min}}{\lambda_{2\max}}$ and $\delta_0 = NV_1(t_0)||(\omega(t_0)||^2$, then

$$\dot{V}_3(t) \leqslant \bar{\xi}^{\mathrm{T}}(QS + S^{\mathrm{T}}Q + (2-\epsilon)Q + Q^2)\bar{\xi} + \delta_0 M_{i\min}^{\frac{1}{2}}\mathrm{e}^{-\sigma_0 t}.$$

Because $QS + S^{\mathrm{T}}Q + (\alpha_2 + 2 - \epsilon)Q + Q^2 < 0$, it follows that

$$\dot{V}_3(t) \leqslant -\alpha_2\bar{\xi}^{\mathrm{T}}(M \otimes Q)\bar{\xi} + \delta_0 M_{i\min}^{\frac{1}{2}}\mathrm{e}^{-\sigma_0 t} \leqslant -\alpha_2 V_3 + \delta_0 M_{i\min}^{\frac{1}{2}}\mathrm{e}^{-\sigma_0 t}.$$

Integrating both sides of this inequality, we have

$$\dot{V}_3(t) \leqslant \mathrm{e}^{-\alpha_2(t-T_1)}\Delta_1, \tag{17}$$

where $\Delta_1 = V_3(T_1) + \frac{1}{\sigma-\alpha_2}\delta_0 M_{i\min}^{\frac{1}{2}}\mathrm{e}^{-\sigma_0 T_1}$.

Case 2. When $t \in \tau_2(t)$, the candidate Lyapunov function is chosen as

$$\dot{V}_4(t) = \sum_{i=1}^{N}\bar{\xi}_i(t)Q\bar{\xi}_i(t). \tag{18}$$

The time derivative of (18) yields

$$\dot{V}_4(t) \leqslant \bar{\xi}^{\mathrm{T}}(I_p \otimes (QS + S^{\mathrm{T}}Q))\bar{\xi} - \bar{\xi}^{\mathrm{T}}[(H + H^{\mathrm{T}}) \otimes QD]\bar{\xi} + \bar{\xi}^{\mathrm{T}}(I_p \otimes 2Q\bar{S})\bar{\xi} + \bar{\xi}^{\mathrm{T}}(I_p \otimes Q^2)\bar{\xi} + \omega^{\mathrm{T}}\bar{S}^{\mathrm{T}}\bar{S}\omega.$$

At $t \geqslant T_1$,

$$\dot{V}_4(t) \leqslant \bar{\xi}^{\mathrm{T}}(I_p \otimes (QS + S^{\mathrm{T}}Q + (2-\lambda_{3\min})Q + Q^2))\bar{\xi} + \delta_0\mathrm{e}^{-\sigma t}.$$

As $\beta_2 > \epsilon - \alpha_2 - \lambda_{3\min}$, we have $QS + S^{\mathrm{T}}Q + (2 - \lambda_{3\min} - \beta_2)Q + Q^2 < 0$, then,

$$\dot{V}_4(t) \leqslant \bar{\xi}^{\mathrm{T}}\beta_2\bar{\xi} + \delta_0\mathrm{e}^{-\sigma_0 t} \leqslant \beta_2 V_4(t) + \delta_0\mathrm{e}^{-\sigma_0 t}. \tag{19}$$

By integrating both sides of (19), it can be converted into

$$V_4(t) \leqslant \mathrm{e}^{\beta_2(t-T_1)}\Delta_2, \tag{20}$$

where $\Delta_2 = V_4(T_1) + \frac{1}{\sigma_0+\beta_2}\delta_0\mathrm{e}^{-\sigma_0 T_1}$.

Summing up (17) and (20), let $\Xi\{V(\tau(t))\} = ||\bar{\xi}^2(t)||$, $t \in (t_{\tau_m}, t_{\tau_{m+1}})$ denote the estimated error of $\bar{\xi}$,

$$\Xi\{V(\tau(t))\} \leqslant \begin{cases} \mathrm{e}^{-\alpha_2(t-t_{\tau_m})}\Delta_1(t_{\tau_m}), & t \in \tau_1(t), \\ \mathrm{e}^{\beta_2(t-t_{\tau_m})}\Delta_2(t_{\tau_m}), & t \in \tau_2(t). \end{cases} \tag{21}$$

From (21), it can be seen that, even though the value of the estimated error $\bar{\xi}$ is continuous at the instant of the mth DoS attack, its convergence condition may be completely different. We have

$$\begin{aligned} \Xi\{V(\tau(t))\} \leqslant & \psi\mathrm{e}^{\beta_2 T_d(t_m,t)-\alpha_2 T_c(t_m,t)}\Xi\{V(t_m^-)\} \\ \leqslant & \psi\mathrm{e}^{\beta_2 T_d(t_{m-1},t)-\alpha_2 T_c(t_{m-1},t)}\Xi\{V(t_{m-1}^-)\} \\ \leqslant & \cdots \\ \leqslant & \psi^{N_{\tau(t)}(t_0,t)}\mathrm{e}^{\beta_2 T_d(t_0,t)-\alpha_2 T_c(t_0,t)}\Xi\{V(t_0)\} \\ = & \mathrm{e}^{ln(\psi)N_{\tau(t)}(t_0,t)}\mathrm{e}^{\beta_2 T_d(t_0,t)-\alpha_2 T_c(t_0,t)}\Xi\{V(t_0)\}. \end{aligned} \tag{22}$$

According to (8),

$$\beta_2 T_d(t_0,t) - \alpha_2 T_c(t_0,t) \leqslant -\rho(t-t_0)$$

with $T_d(t_0,t) + T_c(t_0,t) = t - t_0$. Because the total connectivity-broken attack duration $T_d(t_0,t)$ and attack frequency $f_{\tau(t)}$ satisfy (8) and (9), Eq. (22) can be converted into

$$||\bar{\xi}(t)||^2 \leqslant \mathrm{e}^{-\rho(t-t_0)}||\bar{\xi}^2(t_0)||;$$

thus, $\bar{\xi}(t)$ will exponentially converge to zero with decaying rate ρ, which indicates that the estimating value $\xi_i(t)$ of i-th agent will lead to the exponential estimation of the exosystem state $\omega(t)$.

Remark 5. Once conditions (8) and (9) are satisfied, Theorem 1 indicates that there is no restriction on when or in which channel the DoS attacks occur. As no more prior knowledge is required, the DoS attacks discussed in this paper can be launched by multiple unknown adversaries.

3.2 An online algorithm to solve the linear parameter-varying regulation equations

The solvability of the output regulation problems depends on the following regulator equations:

$$X_iS = A_i(\vartheta_i(t))X_i + B_iU_i(\vartheta_i(t)) + E_i(\vartheta_i(t)), \tag{23a}$$
$$0 = C_i(\vartheta_i(t))X_i + D_iU_i(\vartheta_i(t)) + F_i(\vartheta_i(t)). \tag{23b}$$

In (23), the real-time value of S is needed. In this paper, the value of the exosystem matrix S is not available to the followers. Owing to this restriction, we propose the following online algorithm to asymptotically solve the linear parameter-varying regulation equations, which only needs the estimated value S_i.

Lemma 3. Given matrix $J \in \mathbb{R}^{m\times n}$, vector $c \in \mathbb{R}^m$, rank$(J) =$ rank$(J, c) = h$, and $J\zeta^* = c$, if matrix $J(t) \in \mathbb{R}^{m\times n}$ is bounded and continuous such that $\lim_{t\to\infty} J(t) = J$ and $||J(t) - J|| \in \mathcal{L}^2$, an online algorithm to solve the linear equations $J\zeta^* = c$ can be designed as

$$\dot{\zeta}(t) = -kJ^{\mathrm{T}}(t)(J(t)\zeta(t) - c), \tag{24}$$

where scalar $k > 0$. For any initial condition $\zeta(0)$, $\zeta(t)$ under dynamic system (28c) will asymptotically reach the desired value; that is, $\lim_{t\to\infty} \zeta(t) = \zeta^*$.

Proof. Because rank$(J) = h$, according to singular value decomposition, there exists an orthogonal matrix $O \in \mathbb{R}^{n\times n} = [\, O_1 \;\, O_2 \,]$ such that,

$$O^{\mathrm{T}}J^{\mathrm{T}}JO = \begin{bmatrix} O_1^{\mathrm{T}}J^{\mathrm{T}} \\ O_2^{\mathrm{T}}J^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} JO_1 & JO_2 \end{bmatrix} = \begin{bmatrix} \bar{J}^{\mathrm{T}}\bar{J} & 0_{n\times(n-h)} \\ 0_{(n-h)\times n} & 0_{(n-h)\times(n-h)} \end{bmatrix},$$

where matrix $\bar{J}^{\mathrm{T}}\bar{J} \in \mathbb{R}^{h\times h}$ is positive definite, $O_1 \in \mathbb{R}^{n\times h}$, and matrix $O_2 \in \mathbb{R}^{n\times(n-h)}$. Letting $\hat{\zeta}^* = P^{\mathrm{T}}\zeta^*$, we have

$$\hat{\zeta}^* = \begin{bmatrix} O_1^{\mathrm{T}} \\ O_2^{\mathrm{T}} \end{bmatrix} \zeta^* = \begin{bmatrix} \hat{\zeta}_1^* \\ \hat{\zeta}_2^* \end{bmatrix}, \tag{25a}$$

$$J\zeta^* = JOO^{\mathrm{T}}\zeta^* = \begin{bmatrix} \bar{J} & 0_{m\times(n-h)} \end{bmatrix} \begin{bmatrix} \hat{\zeta}_1^* \\ \hat{\zeta}_2^* \end{bmatrix} = c. \tag{25b}$$

According to (25b), if $J\zeta^* = c$, $\bar{J}\hat{\zeta}_1^* = c$ and $\hat{\zeta}_2^*$ can be any arbitrary value.

$$\hat{\zeta}(t) = P^{\mathrm{T}}\zeta(t) = \begin{bmatrix} O_1^{\mathrm{T}}\zeta(t) \\ O_2^{\mathrm{T}}\zeta(t) \end{bmatrix} = \begin{bmatrix} \hat{\zeta}_1(t) \\ \hat{\zeta}_2(t) \end{bmatrix},\ \bar{\zeta}(t) = \hat{\zeta}(t) - \hat{\zeta}^*,\ \text{and}\ \bar{\zeta}_1(t) = \hat{\zeta}_1(t) - \hat{\zeta}_1^*,$$

$$\begin{aligned} \dot{\bar{\zeta}}_1(t) =& -k\bar{J}^{\mathrm{T}}\bar{J}\bar{\zeta}_1(t) + kO_1^{\mathrm{T}}(J^{\mathrm{T}}J - J^{\mathrm{T}}(t)J(t))O_1\bar{\zeta}_1(t) + kO_1^{\mathrm{T}}(J^{\mathrm{T}}J - J^{\mathrm{T}}(t)J(t))O_2\bar{\zeta}_2(t) \\ & - kO_1^{\mathrm{T}}J^{\mathrm{T}}(t)(J(t) - J)O\hat{\zeta}^*, \\ \dot{\hat{\zeta}}_2(t) =& kO_2^{\mathrm{T}}(J^{\mathrm{T}}J - J^{\mathrm{T}}(t)J(t))O\hat{\zeta}(t) + kO_2^{\mathrm{T}}(J^{\mathrm{T}}(t) - J^{\mathrm{T}})JO\hat{\zeta}^*. \end{aligned} \tag{26}$$

The Lyapunov function can be constructed as $V_3(t) = \bar{\zeta}^{\mathrm{T}}(t)\bar{\zeta}(t)$. It follows that

$$\begin{aligned} \dot{V}_3(t) =& -2k\bar{\zeta}^{\mathrm{T}}(t)O^{\mathrm{T}}J^{\mathrm{T}}(t)J(t)O\bar{\zeta}(t) - 2k\bar{\zeta}^{\mathrm{T}}(t)O^{\mathrm{T}}J^{\mathrm{T}}(t)(J(t) - J)O\hat{\zeta}^* \\ \leqslant& -k\bar{\zeta}^{\mathrm{T}}(t)O^{\mathrm{T}}J^{\mathrm{T}}(t)J(t)O\bar{\zeta}(t) + k\hat{\zeta}^{*\top}O^{\mathrm{T}}(J(t) - J)^{\mathrm{T}}(J(t) - J)O\hat{\zeta}^* \\ \leqslant& \|O\hat{\zeta}^*\|^2\|(J(t) - J)^{\mathrm{T}}(J(t) - J)\|, \end{aligned}$$

where $\|O\hat{\zeta}^*\|^2 \int_0^\infty \|(J(t) - J)^{\mathrm{T}}(J(t) - J)\|\mathrm{d}t$ is bounded. Thus, $V_3(t)$ is bounded, so $\bar{\zeta}(t)$ and $\hat{\zeta}(t)$ are bounded.

The time-varying items $kO_1^{\mathrm{T}}(J^{\mathrm{T}}J - J^{\mathrm{T}}(t)J(t))O_1$ and $kO_1^{\mathrm{T}}(J^{\mathrm{T}}J - J^{\mathrm{T}}(t)J(t))O_2\bar{\zeta}_2(t) - kO_1^{\mathrm{T}}J^{\mathrm{T}}(t)(J(t) - J)O\hat{\zeta}^*$ approach zero asymptotically. Based on Lemma 2, Eq. (26) leads to $\lim_{t\to\infty} \bar{\zeta}_1(t) = 0$. Because $\hat{\zeta}_2(t)$ is bounded and $\lim_{t\to\infty} \dot{\hat{\zeta}}_2(t) = 0$, $\hat{\zeta}_2(t)$ approaches a constant value asymptotically. Suppose that $\hat{\zeta}_2(t)$ approaches $\tilde{\zeta}_2$ and let $\hat{\zeta}_2^* = \tilde{\zeta}_2$; then, $\lim_{t\to\infty} \bar{\zeta}(t) = 0$, which indicates that $\lim_{t\to\infty} \zeta(t) = \zeta^*$ asymptotically.

Lemma 4. Consider the following distributed linear parameter-varying output regulation equation:

$$X_iS_i = A_i(\vartheta_i(t))X_i + B_iU_i(\vartheta_i(t)) + E_i(\vartheta_i(t)), \tag{27a}$$

$$0 = C_i(\vartheta_i(t))X_i + D_iU_i(\vartheta_i(t)) + F_i(\vartheta_i(t)), \tag{27b}$$

where $U_i(\vartheta_i(t)) = U_{i0}(t) + \sum_{j=1}^{q_i} \vartheta_{ij}(t)U_{ij}(t)$, $U_{i0}(t)$ and $U_{ij}(t)$ are the corresponding affine matrices of the linear time-varying matrix $U_i(\vartheta_i(t))$. Then, according to Lemma 3, the estimated solution pair $(\hat{X}_i(t), \hat{U}_i(\vartheta_i(t)))$ of (27) is proposed as

$$\hat{U}_i(\vartheta_i(t)) = \hat{U}_{i0}(t) + \sum_{j=1}^{q_i} \vartheta_{ij}(t)\hat{U}_{ij}(t), \tag{28a}$$

$$\begin{bmatrix} \hat{X}_i(t) \\ \hat{U}_{i0}(t) \\ \cdots \\ \hat{U}_{iq_i}(t) \end{bmatrix} = W_p^{n_i+m_i\times q_i}(\zeta_i(t)), \tag{28b}$$

$$\dot{\zeta}_i(t) = -k_iJ_i^{\mathrm{T}}(t)(J_i(t)\zeta_i(t) - c_i), \tag{28c}$$

$$J_i(t) = S_i^{\mathrm{T}}(t) \otimes J_{1i} - I_p \otimes J_{2i}, \tag{28d}$$

where

$$c_i = \mathrm{vec}\left(\begin{bmatrix} E_{i0} \\ F_{i0} \\ E_{i1} \\ F_{i1} \\ \vdots \\ E_{iq_i} \\ F_{iq_i} \end{bmatrix}\right), \quad J_{1i} = \begin{bmatrix} I_{n_i} & 0_{n_i\times m_i(q_i+1)} \\ 0_{(n_iq_i+rq_i+r)\times n_i} & 0_{(n_iq_i+rq_i+r)\times m_i(q_i+1)} \end{bmatrix}, \tag{29}$$

$$J_{2i} = \begin{bmatrix} A_{i0} & B_i & 0_{n_i\times m_i} & \cdots & 0_{n_i\times m_i} \\ C_{i0} & D_i & 0_{r\times m_i} & \cdots & 0_{r\times m_i} \\ A_{i1} & 0_{n_i\times m_i} & B_i & \cdots & 0_{n_i\times m_i} \\ C_{i1} & 0_{r\times m_i} & D_i & \cdots & 0_{r\times m_i} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{iq_i} & 0_{n_i\times m_i} & 0_{n_i\times m_i} & \cdots & B_i \\ C_{iq_i} & 0_{r\times m_i} & 0_{r\times m_i} & \cdots & D_i \end{bmatrix}. \tag{30}$$

Then, the proposed solution pair $(\hat{X}_i(t), \hat{U}_i(\vartheta_i(t)))$ of (27) will asymptotically converge to the solution pair $(X_i(t), U_i(\vartheta_i(t)))$ of (23).

Proof. Matrix $J_i(t)$ in (28d) is the estimate of matrix Z_i, and the time-varying vector $\zeta_i(t)$ in (28c) is the estimate of vector ζ_i^*. According to Theorem 1, $J_i(t)$ will asymptotically reach J_i such that $\lim_{t\to\infty} J_i(t) = J_i$. Considering Lemma 3, the estimation of vector ζ_i^* will asymptotically reach ζ_i^*; thus, $\lim_{t\to\infty} \zeta_i(t) = \zeta_i^*$. According to (28a), (28b), and (28c), we can conclude that $\lim_{t\to\infty} \hat{X}_i(t) = X_i$ and $\lim_{t\to\infty} \hat{U}_i(\vartheta_i(t)) = U_i(\vartheta_i(t))$. $K_{2i}(\vartheta_i(t)) = U_i(\vartheta_i(t)) - K_{1i}(\vartheta_i(t))X_i(t)$, which gives that $\hat{K}_{2i}(\vartheta_i(t))$ is the estimate of $K_{2i}(\vartheta_i(t))$; moreover, $\bar{K}_{2i}(\vartheta_i(t)) = \hat{K}_{2i}(\vartheta_i(t)) - K_{2i}(\vartheta_i(t))$; we can deduce that $\lim_{t\to\infty} \bar{K}_{2i}(\vartheta_i(t)) = 0$.

Remark 6. Owing to the linear combination characteristics of scheduling variable $\vartheta(t)$, the linear parameter-varying output regulation equation (27) can be extended to several linear algebraic equations. The exact value of S cannot be acquired for solving the precise solution of (23); instead, according to the exponentially estimated value S_i and the online algorithm proposed in Lemma 3, the asymptotically estimated solution of (27) is used for distributed regulator design.

3.3 Distributed linear parameter-varying controller design

According to the distributed resilient observer (6) and the online algorithm (28), two categories of linear matrix inequality (LMI)-based controllers are provided in this subsection. In the case that the full state information is available, a static state-feedback controller can be proposed as follows:

$$u_i(t) = K_{1i}(\vartheta_i(t))x_i(t) + K_{2i}(\vartheta_i(t))\xi_i(t). \tag{31}$$

A dynamic output-feedback controller is given below for cases in which the system states are (partially) unmeasurable:

$$\begin{aligned} \dot{\hat{x}}_i(t) =& K_{3i}(\vartheta_i(t))\hat{x}_i(t) + K_{4i}(\vartheta_i(t)\xi_i(t), \\ u_i(t) =& K_{1i}(\vartheta_i(t))\hat{x}_i(t) + K_{2i}(\vartheta_i(t))\xi_i(t). \end{aligned} \tag{32}$$

The following sufficient conditions are given to construct controllers (31) and (32).

Lemma 5. The regulated output e_i asymptotically converges to the origin for $i \in \mathcal{V}$ if:

(i) (State-feedback) There exists a positive-definite matrix $P_{1i} > 0$ such that

$$\begin{bmatrix} \hat{A}_{ij}^{\mathrm{T}} P_{1i} + P_{1i}\hat{A}_{ij} & B_i \\ B_i^{\mathrm{T}} & -2I \end{bmatrix} < 0, \tag{33}$$

where $\hat{A}_{ij} = A_{i0} + A_{ij}$. Then, $K_{1i} = B_i^{\mathrm{T}} P_{1i}^{-1}$ is selected, and the state-feedback controller (31) is proposed as

$$\begin{aligned} \hat{K}_{2i}(\vartheta_i(t)) &= \hat{U}_i(\vartheta_i(t)) - K_{1i}\hat{X}_i(t), \\ u_i(t) &= K_{1i}x_i(t) + \hat{K}_{2i}(\vartheta_i(t))\xi_i(t). \end{aligned} \tag{34}$$

(ii) (Output-feedback) LMI (33) is solvable, and in addition, there exists a positive-definite matrix $P_{2i} > 0$ such that

$$\begin{bmatrix} \hat{A}_{ij}^{\mathrm{T}} P_{2i} + P_{2i}\hat{A}_{ij} & C_{mi}^{\mathrm{T}} \\ C_{mi} & -2I \end{bmatrix} < 0. \tag{35}$$

Then, $J_{mi} = P_{2i}^{-1} C_{mi}^{\mathrm{T}}$ is selected, and the output-feedback controller (31) is proposed as

$$\begin{aligned} &\hat{K}_{2i}(\vartheta_i(t)) = \hat{U}_i(\vartheta_i(t)) - K_{1i}\hat{X}_i(t), \\ &\dot{\hat{x}}_i(t) = A_i(\vartheta_i(t))\hat{x}_i(t) + B_i u_i(t) + E_i(\vartheta_i(t))\xi_i(t) - J_{mi}[y_i(t) - C_{mi}(\vartheta_i(t))\hat{x}_i(t) \\ &\qquad - D_{mi}u_i(t) - F_{mi}(\vartheta_i(t))\xi_i(t)], \\ &u_i(t) = K_{1i}\hat{x}_i(t) + \hat{K}_{2i}(\vartheta_i(t))\xi_i(t). \end{aligned} \tag{36}$$

Proof. Because the state-feedback case can be regarded as a special case of the output-feedback controller, only the proof of condition (ii) will be given. Let $\overline{\xi}_i(t) = \xi_i(t) - \omega(t)$, $\tilde{x}_i(t) = \hat{x}_i(t) - X_i\omega(t)$, $\bar{x}_i(t) = x_i(t) - X_i\omega(t)$. Combining (2), (3), (6), and (32), the following equations hold:

$$\begin{aligned} \dot{\tilde{x}}_i(t) =& (A_i(\vartheta_i(t)) + B_iK_{1i} + J_{mi}C_{mi})\tilde{x}_i(t) - J_{mi}C_{mi}\bar{x}_i(t) + B_i\hat{K}_{2i}(\vartheta_i(t))\bar{\xi}_i(t) \\ &+ E_i(\vartheta_i(t))\bar{\xi}_i(t) + J_{mi}F_{mi}(\vartheta_i(t))\bar{\xi}_i(t) + B_i\bar{K}_{2i}(\vartheta_i(t))\omega(t), \end{aligned} \tag{37a}$$

$$\dot{\bar{x}}_i(t) = A_i(\vartheta_i(t))\bar{x}_i(t) + B_iK_{1i})\tilde{x}_i(t) + B_i\hat{K}_{2i}(\vartheta_i(t))\bar{\eta}_i(t) + B_i\bar{K}_{2i}(\vartheta_i(t))\omega(t). \tag{37b}$$

Letting $x_{ci}(t) = \mathrm{col}(\bar{x}_i(t), \tilde{x}_i(t))$, we can obtain that

$$\dot{x}_{ci}(t) = A_{ci}(\vartheta_i(t))x_{ci}(t) + B_{ci}(\vartheta_i(t))\bar{\xi}_i(t) + C_{ci}(\vartheta_i(t))\omega(t), \tag{38}$$

where $A_{ci}(\vartheta_i(t))$, $B_{ci}(\vartheta_i(t))$, and $C_{ci}(\vartheta_i(t))$ are described as

$$A_{ci}(\vartheta_i(t)) = \begin{bmatrix} A_i(\vartheta_i(t)) & B_iK_{1i} \\ -J_{mi}C_{mi} & A_{22}(\vartheta_i(t)) \end{bmatrix}, B_{ci}(\vartheta_i(t)) = \begin{bmatrix} B_i\hat{K}_{2i}(\vartheta_i(t)) \\ B_i\hat{K}_{2i}(\vartheta_i(t)) + J_{mi}F_{mi}(\vartheta_i(t)) + E_i(\vartheta_i(t)) \end{bmatrix}, \tag{39a}$$

$$C_{ci}(\vartheta_i(t)) = \begin{bmatrix} B_i\bar{K}_{2i}(\vartheta_i(t)) \\ B_i\bar{K}_{2i}(\vartheta_i(t)) \end{bmatrix}, \tag{39b}$$

and $A_{22}(\vartheta_i(t)) = A_i(\vartheta_i(t)) + B_iK_{1i} + J_{mi}C_{mi}$. A unitary matrix is given below:

$$T = \begin{bmatrix} I_{n_i} & 0_{n_i\times n_i} \\ I_{n_i} & I_{n_i} \end{bmatrix}. \tag{40}$$

Considering a Lyapunov transformation that $\hat{x}_{ci}(t) = T^{-1}x_{ci}(t)$, it follows that

$$\dot{\hat{x}}_{ci}(t) = T^{-1}A_{ci}(\vartheta_i(t))T\hat{x}_{ci}(t) + T^{-1}B_{ci}(\vartheta_i(t))\bar{\xi}_i(t) + T^{-1}C_{ci}(\vartheta_i(t))\omega(t). \tag{41}$$

According to Theorem 1, $\lim_{t\to\infty}\bar{\xi}_i(t) = 0$. Moreover, $\lim_{t\to\infty}\bar{K}_{2i}(\vartheta_i(t)) = 0$. Thus, the following equation is obtained:

$$\lim_{t\to\infty} T^{-1}B_{ci}(\vartheta_i(t))\bar{\xi}_i(t) + T^{-1}C_{ci}(\vartheta_i(t))\omega(t) = 0. \tag{42}$$

Then, $T^{-1}A_{ci}(\vartheta_i(t))T = \hat{A}_{ci}$ possesses the following structure:

$$\bar{A}_{ci}(\vartheta_i(t)) = \begin{bmatrix} A_i(\vartheta_i(t)) + B_iK_{1i} & B_iK_{1i} \\ 0 & A_i(\vartheta_i(t)) + J_{mi}C_{mi} \end{bmatrix}.$$

The solvability of LMIs (33) and (35) guarantees the exponential stability of $A_i(\vartheta_i(t)) + B_iK_{1i}$ and $A_i(\vartheta_i(t)) + J_{mi}C_{mi}$, which implies the asymptotic stability of $\bar{A}_{ci}(\vartheta_i(t))$. Then, it can be verified that Eq. (41) is input-to-state stable with respect to $T^{-1}B_{ci}(\vartheta_i(t))\bar{\xi}_i(t) + T^{-1}C_{ci}(\vartheta_i(t))\omega(t)$; by virtue of $\lim_{t\to\infty}\bar{\xi}_i(t) = 0$ and $\lim_{t\to\infty}C_{ci}(\vartheta_i(t)) = 0$, $x_{ci}(t)$ in (38) is asymptotically stable.

Moreover, considering (2), (3), (6), and (32), we have

$$\begin{aligned} e_i(t) =& C_i(\vartheta_i(t))\bar{x}_i(t) + C_i(\vartheta_i(t))X_i\omega(t) + D_iK_{1i}(\vartheta_i(t))\hat{x}_i + D_i\hat{K}_{2i}(\vartheta_i(t))\xi_i(t) + F_i(\vartheta_i(t))\omega(t) \\ =& C_i(\vartheta_i(t))\bar{x}_i(t) + D_iK_{1i}(\vartheta_i(t))\tilde{x}_i(t) + D_i\hat{K}_{2i}(\vartheta_i(t))\bar{\xi}_i + D_i\bar{K}_{2i}(\vartheta_i(t))\omega(t). \end{aligned}$$

With $\lim_{t\to\infty}\bar{x}_i(t) = 0$, $\lim_{t\to\infty}\tilde{x}_i(t) = 0$, $\lim_{t\to\infty}\bar{\xi}_i(t) = 0$, and $\lim_{t\to\infty}\bar{K}_{2i}(t) = 0$, thus, $\lim_{t\to\infty}e_i(t) = 0$.

Theorem 2. Consider the linear parameter-varying MASs consisting of the exosystem (2) and N follower agents modeled by (3) under Assumptions 1–3. Under controller (31) or (32), Problem 1 is solved.

Proof. This proof directly follows the same pattern as those of Theorem 1 and Lemma 5.

Remark 7. In linear time-invariant settings, LMIs (33) and (35) are equal to the controllability of (A_i, B_i) and observability of (C_{mi}, A_i), respectively. In the output-feedback controller (32), the auxiliary variable $\hat{x}_i(t)$ is the estimation of the unavailable system state $x_i(t)$. Thus, an observer is embedded in (32) to reconstruct the system state. Moreover, when the full state information is available, compared with the dynamic output-feedback controller (32), a static state-feedback controller (31) is preferred because it has less dimension.

4 Simulation results

In this section, some numerical simulations to verify the proposed distributed observer, online algorithm, and regulator for linear parameter-varying MASs for both attack-existing and attack-free cases are illustrated for system performance comparison. Systems with one leader and four followers of two-dimensional states are considered. The communication topology of the systems is piecewise constant owing to DoS attacks (Figure 1). For $t \in [1, 1.7) \cup [3, 3.4) \cup [4.7, 5) \cup [6, 6.8)$, the communication channels among the agents are under attack. The connectivity-maintained attacks occur at instants $t = 1$ s and $t = 6$ s with total duration $T_c = 1.5$ s, while the connectivity-broken attacks occur at instants $t = 3$ s and $t = 4.7$ s with total duration $T_d = 0.7$ s.

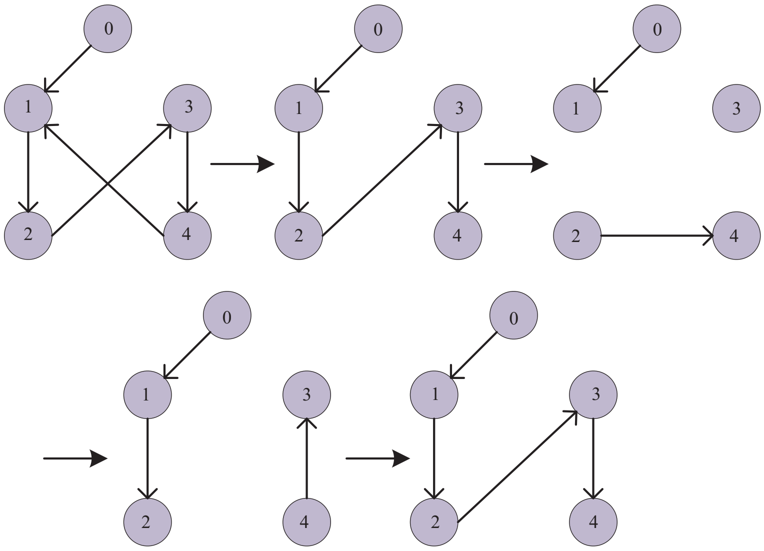

Figure 1 (Color online) Switching topology $\bar{\mathcal{G}}$ under DoS attacks.

The exosystem is given as

$$\dot{\omega}(t) = \begin{bmatrix} 0 & 0.82 \\ -0.82 & 0 \end{bmatrix} \omega(t).$$

The dynamics of agents 1–4 are

$$\begin{aligned} \dot{x}_i(t) &= A_i(\vartheta_i(t))x_i(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(t) + E_i(\vartheta_i(t))\omega(t), \\ e_i(t) &= C_i(\vartheta_i(t))x_i(t) + F_i(\vartheta_i(t))\omega(t), \\ y_{mi}(t) &= C_{mi}(\vartheta_i(t))x_i(t) + F_{mi}(\vartheta_i(t))\omega(t), \end{aligned}$$

where $A_i(\vartheta_i(t)) \in \mathbb{R}^{2\times 2}$, $E_i(\vartheta_i(t)) \in \mathbb{R}^{2\times 2}$, $C_i(\vartheta_i(t)) = C_{mi}(\vartheta_i(t)) \in \mathbb{R}^{1\times 2}$, $F_i(\vartheta_i(t)) \in \mathbb{R}^{2\times 2}$, and $F_{mi}(\vartheta_i(t)) \in \mathbb{R}^{2\times 2}$, $i = 1, 2, 3, 4$, are chosen the same as those in [31]. The scheduling variable $\vartheta_{ij}(t)$ for each agent is chosen as

$$\vartheta_{ij}(t) = m_{ij}(1 - \cos(n_{ij}t)),$$

where m_{ij} and $n_{ij} \in [0, 1]$ are selected to satisfy (5). $\omega(0)$ is chosen as $[3, 4.5]^{\mathrm{T}}$, and the initial states of each follower is chosen as $x_1(0) = [0, 1]^{\mathrm{T}}$, $x_2(0) = [2, 0.5]^{\mathrm{T}}$, $x_3(0) = [-0.7, 1.5]^{\mathrm{T}}$, and $x_4(0) = [0.7, 0.8]^{\mathrm{T}}$. The variables of the observer and online algorithm are initialized at the original point as $s_i(0) = 0$, $\xi_i(0) = 0$, and $\zeta_i(0) = 0$. Matrices K_{1i} and J_i are recommended as $K_{11} = [-0.144, 0.761]$, $K_{21} = [0.254, 0.192]$, $K_{31} = [-1.619, 0.304]$, $K_{41} = [-0.924, -1.612]$, $J_1 = \left[\begin{smallmatrix} 0.504 \\ -0.188 \end{smallmatrix}\right]$, $J_2 = \left[\begin{smallmatrix} 0.627 \\ -0.406 \end{smallmatrix}\right]$, $J_3 = \left[\begin{smallmatrix} 1.075 \\ -1.056 \end{smallmatrix}\right]$, and $J_4 = \left[\begin{smallmatrix} 0.070 \\ -0.728 \end{smallmatrix}\right]$. According to the communication topology described in Figure 1, we have $\sigma(H_{\min}^{\tau(t)}) = 0.1808$, $\lambda_{1\min} = 0.1292$, $\lambda_{2\max} = 1$, and $\epsilon = 4.1$. The simulation results are given in Figures 2–4.

Figure 2(a) shows that after $t = 8.9$, S is almost estimated, as the estimating errors $||s_i - s_0||$ for $i = 1, 2, 3, 4$ are less than 10^{-3}, which demonstrates the effectiveness of the first part of the observer (6). Figure 2(b) indicates that, the simulation results of the online algorithm proposed in Lemma 5 are given. $\zeta_i(t)$, $i = 1, 2, 3, 4$ can asymptotically track the desired value ζ^*; thus, the solution pair $(\hat{X_i}(t), U_i(\hat{\vartheta}(t)))$ can asymptotically converge to $(X_i(t), U_i(\vartheta(t)))$.

In Figure 3, the simulation results of the second part of the observer (6) show that with a given two-dimensional exosystem state $\omega(t) = [\omega_1(t), \omega_2(t)]^{\mathrm{T}}$, the desired trajectory can be obtained by estimating variables $\xi_{i1}(t)$ and $\xi_{i2}(t)$. The gray shadow blocks in Figure 3 indicate that the communication channels in observer (6) are under attack. The sudden interruption of the topology leads to discontinuity of the derivative of the estimated variables. This logically follows because the variable dynamics in (6) will be discontinuous as the $a_{ij}^{\tau(t)}$ value jumps at the injection instant of DoS attacks.

The solution of LMI in Theorem 1 is given as $\alpha_2 = 2$ and $Q = [0.0498, 0; 0, 0.0498]$; $\beta_2 = 2.1$, $\sigma_0 = 2.842$, and $\hat{\rho} = 0.5$ are set; then, the upper bound of the total duration of connectivity-broken attack T_d^* on

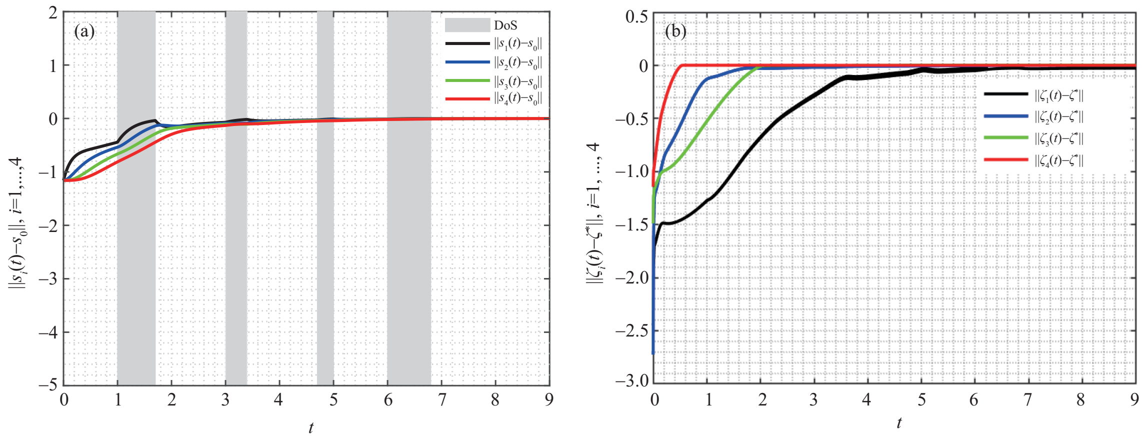

Figure 2 (Color online) Dynamics of $s_i(t)$ and $\zeta_i(t)$ under DoS attacks. (a) Estimation of the matrix S; (b) estimation of solutions of regulation equations.

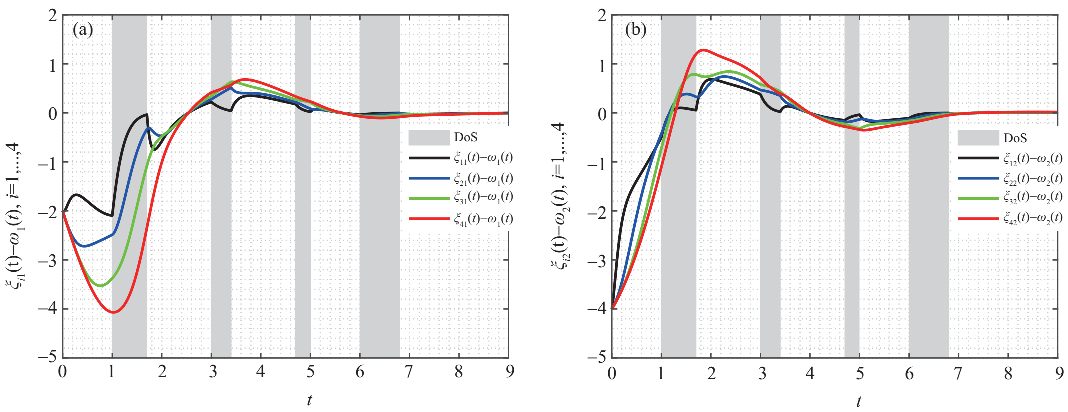

Figure 3 (Color online) Estimating errors of (a) the first element and (b) the second element in $\omega(t)$ under DoS attacks.

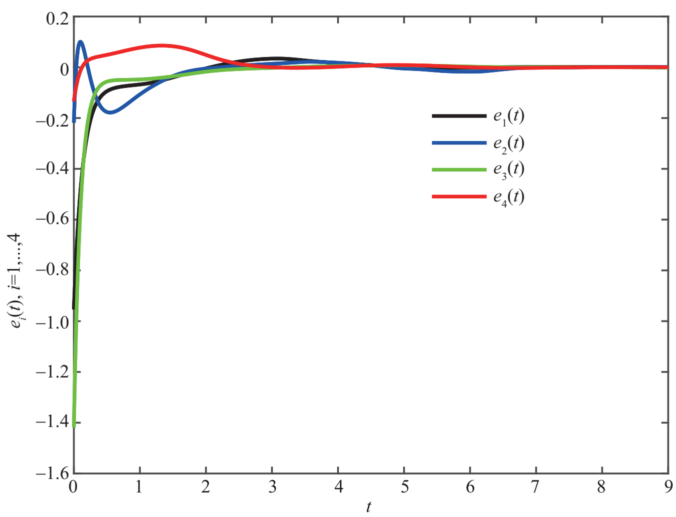

Figure 4 (Color online) Regulated outputs $e_i(t)$.

$t \in [0, 10]$ can be calculated as $T_d^* \leqslant 3.91$. The exact total time of connectivity-broken attacks is 0.7 s, and the exact attack frequency is 0.57; both are less than the theoretical upper bound, according to Theorem 1.

Figure 4 shows the simulation results of cooperative output regulation errors $e_i(t)$ for $i = 1, 2, 3, 4$. The cooperative output regulation is achieved if the regulated outputs satisfy $e_i(t) \leqslant 10^{-3}$. The simulation results show that after $t \geqslant 7.56$, the control objective is achieved, which verifies Theorem 2.

5 Conclusion

The cooperative output regulation problem of linear parameter-varying (LPV) systems under DoS attacks was studied. A resilient observer was proposed to estimate the matrix and state of the exosystem and to resist DoS attacks from adversaries. The LPV output regulation equation was solved based on an asymptotic online algorithm. According to the aforementioned results, a distributed LPV regulator was designed to achieve cooperative output regulation. Some numerical simulations of the proposed resilient observer, online algorithm, and regulator were given, and the observers under DoS attack-existing and attack-free cases were compared. The results verified the effectiveness of the method presented in this paper. Moreover, the application of the presented cooperative output regulation methodology for LPV MASs to the formation problem of vehicles and unmanned aerial vehicles is clarified. The applications will be further explored in a future study.

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