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# Tracking performance limitations of MIMO discrete-time networked control systems with multiple constraints

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Dear editor,

Networked control systems (NCSs) have attracted widespread attention in some fields because they offer the advantages of reduced cabling, greater resource sharing, and ease of installation and maintenance [1]. However, in an unreliable network environment, network constraints can significantly limit the performance of NCSs.

In existing studies on the optimal tracking performance of NCSs, the modeling of the communication channel is relatively simple. In engineering practice, NCSs with multiple communication constraints are frequently encountered. The study of tracking performance and the design of optimal controllers are particularly complex, and exact expressions for the optimal tracking performance of NCSs with multiple communication constraints have not yet been explicitly given. Information is passed over a communication link and is therefore necessarily limited by the bandwidth; the limited bandwidth will lead to the occurrence of a network-induced delay. When the time delay in the channel is too great, the data packet dropouts will be more serious. To improve the signal-to-noise ratio (SNR) of the channel, it is common practice to amplify the received signal; as almost all input signals are affected by interference, this leads to the amplification of the interfering signal as well. The quantizer has good robustness and anti-interference capabilities, so the design and application of the quantizer can optimize the performance of NCSs to a certain degree. The application of encoders and decoders in NCSs is also extremely important, since encoding and decoding the signal can effectively reduce signal distortion. Based on the above considerations, this study comprehensively investigates the tracking performance of NCSs under the influence of encoding-decoding, bandwidth, quantization, time delay, packet dropouts, and additive white Gaussian noise (AWGN), and it differs from general models that only focus on the uplink or downlink [2, 3]. This study considers the dual-channel model, which is better in line with the reality of engineering than the research on the optimal tracking performance of general NCSs. The main contributions of this study are outlined below.

(1) This study constructs a novel model of NCSs by fully considering various communication constraints such as time delay, packet-dropouts, bandwidth, and quantization.

(2) Difference from the existing studies on tracking performance limitations of NCSs, the communication constraints are integrated into both uplink and downlink channels, which thus can be widely used in many occasions of networked control.

(3) Because the channel input is required to satisfy the power constraint [4], a new performance index is proposed by considering the trade-off between tracking performance and channel input power constraint. Accordingly, by frequency-domain analysis methods, the exact expression of tracking performance limitations of NCSs with multiple constraints is derived.

The preliminaries and problem formulation are provided in Appendix A.

**Problem 1.** For the given reference input signal $r$, in order to track $r$ as accurately as possible, the optimal controller must be designed. In addition, the channel input is required to satisfy the power constraint $\mathrm{E}\left\{y^{\mathrm{T}}(t)y(t)\right\} < \Gamma$, where E is the expectation operator and $\Gamma > 0$. Therefore, we define the tracking performance index as

$$J := (1-\varepsilon)\mathrm{E}\left\{\|\tilde{e}\|_2^2\right\} + \varepsilon\left\{\mathrm{E}\,\|\tilde{y}\|_2^2 - \Gamma\right\}, \tag{1}$$

where $0 \leqslant \varepsilon < 1$ represents a trade-off between tracking

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error and channel input power constraint.

**Theorem 1.** For the multi-input multi-output (MIMO) NCSs as shown in Figure 1, we assume that the plant $G(z)$ has non-minimum phase (NMP) zeros $s_i \in \overline{D}^c, i = 1, \ldots, N_z$, with direction $\eta_i$, and unstable poles $p_i \in \overline{D}^c, i = 1, \ldots, N_p$, with direction $\omega_i$. The optimal tracking performance of NCSs with multiple constraints is

$$J^* = J_1^* + J_2^* + J_3^* - \varepsilon\Gamma,$$

where

$$\begin{aligned}
J_1^* \ = \ & (1-\varepsilon)\sum_{i,j=1}^{N_z}\frac{(1-s_i)(1-s_j)(1-|s_i|^2)(1-|s_j|^2)}{(1-\bar{s}_i)(1-\bar{s}_j)(\bar{s}_i s_j - 1)} \\
& \times \prod_{h=1}^{i}|g(s_{h-1})|^2\sum_{l=1}^{m}\alpha_l^2\cos^2\angle(\eta_i, e_l) \\
& +\varepsilon(1-\varepsilon)\sum_{i=1}^{N_z}|s_i|^2\sum_{l=1}^{m}\alpha_l^2\cos(\angle\eta_i, e_l), \\
J_2^* \ = \ & \frac{1}{(1-\alpha)^2}\sum_{i,j=1}^{N_z}\frac{(s_i-1)(1-s_j)(|s_i|^2-1)(1-|s_j|^2)}{(\bar{s}_i-1)(\bar{s}_j-1)(\bar{s}_i s_j-1)} \\
& \times\varsigma_i^{\mathrm{H}}(s_i)^{\tau}H_i^{\mathrm{H}}\Lambda A^{-\mathrm{H}}(s_i)F^{-\mathrm{H}}(s_i)M^{-\mathrm{H}}(s_i)N_m^{\mathrm{H}}(s_i)(s_j)^{\tau} \\
& \times N_m(s_j)M^{-1}(s_j)F^{-1}(s_j)A^{-1}(s_j)\Lambda H_j\varsigma_j\varsigma_j^{\mathrm{H}}G_jG_i^{\mathrm{H}}\varsigma_i, \\
J_3^* \ = \ & \sum_{i=1}^{N_p}(|p_{N_p+1-i}|^2-1)\|W\zeta_{N_p+1-i}\|_2^2 \\
& +\sum_{i,j=1}^{N_z}\frac{(1-s_i)(1-s_j)(1-|s_i|^2)(1-|s_j|^2)}{(1-\bar{s}_i)(1-\bar{s}_j)(\bar{s}_i s_j-1)} \\
& \times\eta_i^{\mathrm{H}}(s_i)^{\tau}E_i^{\mathrm{H}}(s_j)^{\tau}E_j\eta_j\eta_j^{\mathrm{H}}T_jW \\
& \times(\theta-B^{-1}(s_j))(\theta-B^{-1}(s_i))^{\mathrm{H}}WT_i^{\mathrm{H}}\eta_i.
\end{aligned}$$

The proof of Theorem 1 can be found in Appendix B and a numerical example is given in Appendix C.

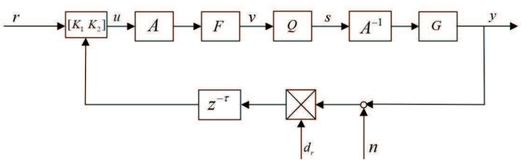

**Figure 1** Networked control system with multiple constraints.

**Remark 1.** According to Theorem 1, the controller parameter $Q$ can be obtained from

$$\inf_{Q\in RH_\infty}\left\|\left(\Delta_i^{\mathrm{T}}\begin{bmatrix}\sqrt{1-\varepsilon}\psi \\ 0\end{bmatrix}+\Delta_o Q\right)U\right\|_2^2 = 0,$$

where $\Delta_i^{\mathrm{T}}$ and $\Delta_o$ have been defined in the above. Similarly, the controller parameter $R$ can be obtained from

$$\inf_{R\in RH_\infty}\frac{1}{(1-\alpha)^2}\left\|R_2(z)-N_mR\tilde{N}_m\right\|_2^2 = 0,$$

where $R_2(z)$ is as defined above. Through the Youla parameterization in (6), the two-degrees-of-freedom (TDOF) controllers $K_1(z)$ and $K_2(z)$ can be designed. It is well known that the optimal tracking performance index in Theorem 1 is obtained by designing the optimal TDOF controller. For example, in [5], the TDOF control framework is applied to linear parameter-varying systems. According to the simulation results, it can be found that the anti-interference capability and target tracking performance of the system have been significantly improved.

**Remark 2.** This study constructs a novel model of NCSs by fully considering various communication constraints, and an exact expression for the optimal tracking performance of MIMO NCSs with multiple communication constraints is given using the frequency-domain analysis method. There are certain advantages of the frequency-domain analysis method compared with the state-space method [6, 7]. For example, Ref. [6] carried out stability analysis with the help of the Lyapunov function, and the results show that the tracking error can converge to a bounded region. However, the state-space method is more capable than the frequency-domain analysis method in terms of dealing with factors such as non-linearity and strong coupling.

*Conclusion.* In this study, the tracking performance of MIMO discrete-time NCSs is studied. We found that the forward channel is affected by encoding-decoding, quantization, and bandwidth, while the feedback channel is constrained by time delay, packet dropouts, and AWGN. Using frequency-domain analysis methods and some special decomposition techniques, an exact expression for the optimal tracking performance of NCSs was obtained by designing the optimal TDOF controller. The results highlighted the influence of network constraints and the essential characteristics of the plant on the tracking performance.

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**Supporting information** Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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