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Demand response management of smart grid based on Stackelberg-evolutionary joint game

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Abstract We investigated the real-time pricing demand response management system of multiple microgrids and multiple power users. Accordingly, we have proposed a Stackelberg-evolutionary joint game framework to examine the real-time pricing scheme of multiple microgrids and multiple power users so as to establish equilibrium strategies. Both a non-cooperative game among multiple microgrids and a multi-population evolutionary game among multiple power users were considered. Furthermore, we constructed a Stackelberg game between microgrids and power users to reflect their sequential interaction, wherein the microgrids are leaders, and the power users are followers. We also proved the existence and uniqueness of the Stackelberg equilibrium. Furthermore, we proposed an iterative algorithm to compute the equilibrium strategy and demonstrate the convergence and effectiveness through numerical simulations, which demonstrated that the algorithm could achieve a balance between power supply and demand balance.

Keywords smart grid, real-time pricing, demand response management, Stackelberg game, multi-population evolutionary game

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1 Introduction

A smart grid coordinates the needs and functions of all stakeholders in power generation, power grid operation, terminal power consumption, and the power market. Through advanced information and automatic control technology, the smart grid could improve the reliability and stability of the system as much as possible while improving the operating efficiency of an individual part of the system and reducing the costs and the eventual impact on the environment [1–3]. Demand response refers to the fact that power users can change their consumption behaviors in accordance with the incentive signals or real-time price signals in the electricity market. It also ensured the power supply and demand balance while guaranteeing grid stability [4–6]. The demand response strategies can mainly be divided into incentive-based strategies and price-based strategies [7]. The real-time pricing scheme is the most direct and effective price-based demand response scheme deemed suitable for the highly competitive electricity market [8–10].

The game theory facilitates dealing with complex interaction problems and has emerged as a powerful tool for studying the equilibrium strategies of multiple decision-making subjects in the demand response of smart grids [11–13]. The existing results on demand-response management can be categorized into two main types: demand-side management for power users and electricity price strategy management for microgrids. In the former category, power users schedule the demand to reduce their electricity bills and improve their satisfaction function [14,15]. Lee and Kundur [16] proposed an evolutionary game approach to predict user strategy from real-time pricing. This approach can explain the dynamic strategies of consumers and design adaptable prices for them. Liu et al. [17] proposed a network evolutionary game

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model for studying the influence of user strategy on the deployment of privacy protection mechanisms. In the latter category, microgrids determine the appropriate electricity prices to maximize the revenue function [18]. Liu et al. [19] proposed a game theoretic non-cooperative distributed coordination control scheme that addresses multi-operator energy trading and facilitates a powerful control structure for microgrids. However, these past studies only considered games among microgrids or those among power users, which are all single-level games. In the real electricity market, the microgrids select the real-time prices to maximize their profit functions based on the demands of power users, while the power users select the demands to maximize their satisfaction functions based on the real-time prices of microgrids. Therefore, there exist interactions between microgrids and power users, which makes it more meaningful to consider two-level games [20]. Kamyab et al. [21] resolved the interactions between multiple microgrids and multiple users in a smart grid by modeling the demand response problem as two non-cooperative games: the supplier game and the client game. Chai et al. [22] proposed a two-level game to resolve the demand response management problem between microgrids and power users. At the higher level, a non-cooperative game was considered among microgrids. At the lower level, the evolutionary game was formulated for residential users.

In the real electricity market, the microgrids first announce real-time prices to power users based on the available power, and then the power users respond to real-time prices with optimal demands. Finally, the microgrids reset the real-time prices to maximize profit functions by feedback demands of the power users. As microgrids make decisions first, the strategies of microgrids and power users are sequential. Most results on the two-level games, where the strategies of microgrids reached the Nash equilibrium and the strategies of power users reached the Nash equilibrium or evolutionary equilibrium, did not consider the sequential strategies between microgrids and the power users [21, 22]. These results, therefore, cannot ensure that the interactions between microgrids and power users remain stable. To ensure the stability of this interaction, a Stackelberg game approach for demand response management was proposed previously [23]. Yao et al. [24] proposed a novel category-specific pricing strategy approach for managing demand response in microgrids. The proposed approach establishes the Stackelberg game between the microgrid market operator and power users to achieve peak shaving and valley filling. Aguiar et al. [25] proposed a network-constrained Stackelberg game framework to set energy prices for flexible users. The aggregator acts as a leader by setting the energy price for the users, while the users act as followers who adjust their demands based on the price. In the real electricity market, there exist interactions between microgrids and power users, as well as among power users. Hence, more attention should be paid to the strategies of the population rather than to those of individual power users. For the interactions among power users, most results considered a single population [17, 26]. However, for multiple overlapping sales areas and separate sales areas in the same microgrid, power users purchased electricity in different proportions across different sales areas. Meanwhile, we cannot calculate the total demand of power users in each microgrid based on the existing models.

Motivated by [22, 23], we have proposed a framework of Stackelberg-evolutionary joint game to study the real-time pricing scheme of multiple microgrids and multiple power users so as to achieve equilibrium strategies of microgrids and power users. Based on the sequential strategies between microgrids and power users, we constructed a Stackelberg game. In addition, we considered the power users in individual sales areas or overlapping sales areas as a population. Then, we constructed a multi-population evolutionary game among power users. Owing to the price competition among microgrids, we constructed a non-cooperative game model among microgrids. In the real electricity market, microgrids first announced real-time prices to power users, and then the power users responded to real-time prices with optimal demands. Subsequently, microgrids set optimal real-time prices to maximize their profit functions through the feedback demands of power users. When the microgrids and power users do not change their strategies, the Stackelberg equilibrium is achieved. In the traditional power grid, microgrids maintain a pre-determined electricity price for each period at the beginning of the day. Furthermore, power users have no incentive to change their demands based on the lack of interaction with the microgrid [27]. We considered the real-time pricing scheme of multiple microgrids and multiple power users with the assumption that the electricity consumption time in a day is divided into H periods. When compared with the fixed pricing algorithm in [28], we calculated the electricity price for specific periods in real time and announced it at the beginning of each period. The response of power users to the electricity price in the previous period influences the electricity price in the next one. When compared with [22], we considered the sequential strategies between microgrids and power users and accordingly constructed a Stackelberg game wherein the microgrids act as leaders and the power users as followers. Compared with [23], we considered the interactions among power users, with a focus on the strategies of the populations. Specifically, we regarded multiple separate sales areas and multiple overlapping sales areas as multiple populations to construct a multi-population evolutionary game among power users. Furthermore, the cost parameters of microgrids and the satisfaction parameters of the power users in [23] are fixed, while the parameters in this paper are time-varying. The microgrids set the time-varying electricity prices and induce power users to shift their demands from peak to off-peak periods.

The main contributions to the study are summarized as follows:

(1) We propose a framework of the Stackelberg-evolutionary joint game to study the real-time pricing scheme of multiple microgrids and multiple power users. This framework can reflect the changing relationship between supply and demand at each period and adjust the demands of power users according to real-time prices. When the real-time prices of microgrids are higher, the power users reduce their electricity consumption, thereby reducing the demand during peak periods. When the real-time prices are lower, the power users increase their electricity consumption, thereby increasing the demand during the off-peak periods.

(2) When there are multiple overlapping sales areas in the same microgrid, power users purchase electricity corresponding to different proportions in different sales areas. However, we cannot calculate the total demand of power users in each microgrid based on the existing models. Specifically, we regard multiple separate sales areas and multiple overlapping sales areas as multiple populations and construct a multi-population evolutionary game model to calculate the total demands of power users through the proportions of their purchasing electricity in multiple populations.

(3) For microgrids with overlapping sales areas, power users have different preferences for microgrids. However, we cannot calculate different satisfaction functions for power users based on the existing models. Then, we considered the different preference functions in the model of power users and constructed a Stackelberg game model between microgrids and power users.

(4) We proved the convergence of the evolutionary equilibrium. Moreover, we proved the existence and uniqueness of the Stackelberg equilibrium between microgrids and power users. Specifically, we proposed an iterative algorithm to calculate the equilibrium strategies and verify the convergence of the algorithm.

The rest of this article is organized as follows: Section 2 introduces a system model of microgrids and power users. Section 3 introduces a multi-evolutionary game among power users. Section 4 introduces the Stackelberg game between microgrids and power users. Section 5 sets the simulation parameters and discusses the simulation results. Finally, Section 6 concludes the paper.

2 System model

2.1 Microgrid model

We assumed that the electricity consumption time in a day is divided into H periods. In the microgrid model, to maximize the profit function $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ at time $t \in \{1, 2, \ldots, H\}$, the microgrid j needs to determine its optimal real-time price $P_{j,t}$ from the strategy set $\mathbf{\Gamma}_{j,t}=\{P_{j,t} : P_{\min,t} \leqslant P_{j,t} \leqslant P_{\max,t}\}$, where $P_{\min,t}$ and $P_{\max,t}$ are the lower and upper bounds at time t, respectively, of the real-time price. Then, we defined the optimization problem of the microgrid $j \in \{1, 2, \ldots, N\}$ at time t as

$$\max_{P_{j,t}\in\mathbf{\Gamma}_{j,t}} F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t}), \tag{1}$$

where $P_{j,t}$ is the real-time price of microgrid j at time t. $T_{j,t}$ is the production power of microgrid j at time t. $D_{j,t} = \sum_{i\in\boldsymbol{C}_{j,t}}(y^j_{i,t}B^j_{i,t})$ is the total demand of power users from the microgrid j at time t, $\boldsymbol{C}_{j,t}$ is the set of power users covered by the microgrid j at time t, $y^j_{i,t}$ is the proportion of power user i purchasing electricity from microgrid j at time t, and $B^j_{i,t}$ is the demand of the power user i from microgrid j at time t.

We have defined the profit function of the microgrid j at time t as

$$F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t}) = P_{j,t}\min(T_{j,t}, D_{j,t}) - \phi_{j,t}(T_{j,t}), \tag{2}$$

where $\phi_{j,t}(T_{j,t})$ is the cost function of the microgrid j at time t. $\phi_{j,t}(T_{j,t})$ is given by

$$\phi_{j,t}(T_{j,t}) = a_{j,t}T^2_{j,t} + b_{j,t}T_{j,t} + c_{j,t}, \quad j \in \{1, 2, \ldots, N\}, \tag{3}$$

where $a_{j,t} > 0$, $b_{j,t} > 0$ and $c_{j,t} \geqslant 0$ are the production cost constants of microgrid j at time t.

Remark 1. In this paper, we considered a quadratic cost function. The cost function represents the cost of generating power at each time point. Generally, the cost function is supposed to increase and be strictly convex; therefore, a quadratic function model is usually adopted in most of the related studies [22, 23, 28–30].

2.2 Power user model

In the real-time pricing demand response scheme, the demands of power users can be accurately predicted by analyzing the historical data on power consumption. If power user i chooses microgrid j at time t to consume $B^j_{i,t}$, and the real-time price is $P_{j,t}$, the power user needs to pay $P_{j,t}B^j_{i,t}$. Therefore, we have defined the satisfaction function of power user $i \in \{1, 2, \ldots, L\}$ from the microgrid $j \in \boldsymbol{M}_{i,t}$ at time t as

$$f^j_{i,t}(P_{1,t}, \ldots, P_{N,t}, \boldsymbol{B}_{i,t}) = \psi^j_{i,t}(P_{1,t}, \ldots, P_{N,t}, \boldsymbol{B}_{i,t}) - \lambda_{i,t}P_{j,t}B^j_{i,t}, \tag{4}$$

where $\boldsymbol{M}_{i,t}$ represents the set of microgrids covering power user i at time t. $\boldsymbol{B}_{i,t} = \{B^j_{i,t}, j \in \boldsymbol{M}_{i,t}\}$ is the demand of power user i at time t. $\psi^j_{i,t}(P_{1,t}, \ldots, P_{N,t}, \boldsymbol{B}_{i,t})$ is the preference function of power user i from the microgrid $j \in \boldsymbol{M}_{i,t}$ at time t. $\lambda_{i,t} \in (0, 1]$ is the dissatisfaction constant of power user i at time t.

We have defined the preference function of power user i from the microgrid $j \in \boldsymbol{M}_{i,t}$ at time t as

$$\psi^j_{i,t}(P_{1,t}, \ldots, P_{N,t}, \boldsymbol{B}_{i,t}) = \omega^j_{i,t}(P_{1,t}, \ldots, P_{N,t})B^j_{i,t} - \frac{\theta_{i,t}}{2}{B^j_{i,t}}^2, \tag{5}$$

where $\omega^j_{i,t}(P_{1,t}, \ldots, P_{N,t})$ is the preference degree function of power user i from the microgrid j at time t. $\theta_{i,t} > 0$ is the consumption constant of power user i at time t. To reflect the impact of the real-time prices and the number of microgrids on the response mechanism of power users, we have defined the preference degree function of power user i from the microgrid j at time t as

$$\omega^j_{i,t}(P_{1,t}, \ldots, P_{N,t}) = m_{i,t} - P_{j,t} + \sum_{l \in \boldsymbol{M}_{i,t} \setminus \{j\}} P_l, \tag{6}$$

where $m_{i,t}$ is the preference constant of power user i at time t. $\omega^j_{i,t}(P_{1,t}, \ldots, P_{N,t})$ is related to the real-time price of all microgrids at time t.

Considering the overlapping sales areas among microgrids, we have defined the satisfaction function of power user $i \in \{1, 2, \ldots, L\}$ at time t as

$$f_{i,t}(P_{1,t}, \ldots, P_{N,t}, \boldsymbol{B}_{i,t}) = \sum_{j \in \boldsymbol{M}_{i,t}} f^j_{i,t}(P_{1,t}, \ldots, P_{N,t}, \boldsymbol{B}_{i,t}). \tag{7}$$

In the power user model, the power user i needs to determine its optimal demand strategy from the strategy set $\boldsymbol{\Omega}_{i,t} = \{\boldsymbol{B}_{i,t} : B^j_{i,t} \geqslant 0, j \in \boldsymbol{M}_{i,t}; \sum_{j \in \boldsymbol{M}_{i,t}} B^j_{i,t} = K_{i,t}\}$, where $K_{i,t}$ is the total demand of power user i at time t. Each power user does not know about the demand strategies of other power users in multiple separate sales areas and multiple overlapping sales areas. Through a multi-population evolutionary game among the power users, we selected the optimal demand strategies in order to maximize the satisfaction functions.

In the real-time pricing scheme of multiple microgrids and multiple power users, when the power users' feedback demands to microgrids through smart meters instead of the optimal response demand curves, it is difficult for microgrids to predict the optimal response demands of power users for the certain real-time prices. The microgrid j needs to continuously measure the total demand of its power users $D_{j,t}$ at time t and adjust the real-time price according to the relationship between $D_{j,t}$ and its power production $T_{j,t}$ at time t. This ensures the balance of power supply and demand and maximizes the profits of microgrids. The power users determine the optimal demands based on the real-time prices announced by microgrids and maximize their satisfaction functions. We have considered a system model between multiple microgrids and multiple power users with partially overlapping sales areas. We regarded the multiple separate sales areas and multiple overlapping sales areas as multiple populations. Then, we constructed a Stackelberg game model between multiple microgrids and multiple power users, as well as constructed a multi-population evolutionary game model among the power users. As leaders, microgrids announce real-time prices to maximize their profit functions. As followers, power users select the appropriate total demands to maximize their satisfaction functions. Each power user is equipped with a smart meter. The power users can send the demands and consumption parameters to microgrids. Meanwhile, the power users can receive real-time price information feedback from microgrids.

3 Evolutionary game among power users

In the system model, there are multiple overlapping sales areas. The power users in each separate sales area or each overlapping sales area can be regarded as a population. Hence, there is a multi-population evolutionary game. In an evolutionary game, each player can observe and replicate the strategies of others in the same population. In other words, the strategies of different users are identical in one population [31,32]. Let $\chi = \{1, 2, \dots, X\}$ denote a set of populations. $I_x, x \in \chi$ is the set of power users in the population x. $J_x, x \in \chi$ is the set of microgrids in the population x.

3.1 Replicator dynamics

Replicator dynamics is a key concept in an evolutionary game. It can be modeled as a set of ordinary differential equations. We have described the dynamic process of power users in a population changing their strategies [33,34]. By designing the appropriate replication, the population gradually reached an evolutionary equilibrium. Let $y_{i,t}^j$ denote the proportion of power user i purchasing electricity from microgrid j at time t, where $0 \leqslant y_{i,t}^j \leqslant 1$,

$$\sum_{j=1}^{N} y_{i,t}^j = 1, \quad i = 1, \dots, L.$$

Then, the proportion of power user i at time t is $\boldsymbol{y}_{i,t} = (y_{i,t}^1, \dots, y_{i,t}^N)$. If power user i is covered by microgrid j alone, then $y_{i,t}^j = 1$. If power user i does not purchase power from microgrid j, then $y_{i,t}^j = 0$. Furthermore, the strategies of different users are identical in one population; namely, $y_{i,t}^j = y_t^{j,x}, i \in I_x$, and $j \in J_x$. If the range of real-time prices satisfies certain conditions, the results are more precise. Therefore, we made the following assumption.

Assumption 1. The range of the real-time price covering power user i at time t satisfies the following condition:

$$(|\boldsymbol{M}_{i,t}| - 1)(P_{\max,t} - P_{\min,t}) < \frac{\theta_{i,t} K_{i,t}}{\lambda_{i,t} + 2}, \tag{8}$$

where $|\boldsymbol{M}_{i,t}|$ represents the number of microgrids covering power users i at time t.

Lemma 1 ([23])**.** Considering the real-time price strategy $\{P_{i,t}, j = 1, 2, \dots, N\}$ at time t, $\boldsymbol{\Omega}_{i,t} = \{\boldsymbol{B}_{i,t} : B_{i,t}^j \geqslant 0, j \in \boldsymbol{M}_{i,t}; \sum_{j \in \boldsymbol{M}_{i,t}} B_{i,t}^j = K_{i,t}\}$ is a bounded closed set, and the satisfaction function $f_{i,t}(P_{1,t}, P_{2,t}, \dots, P_{N,t}, \boldsymbol{B}_{i,t})$ on the set $\boldsymbol{\Omega}_{i,t}$ about $\boldsymbol{B}_{i,t}$ is continuously differentiable, we define the optimal demand of power users $i \in \{1, 2, \dots, L\}$ from the microgrid $j \in \boldsymbol{M}_{i,t}$ at time t as

$$B_{i,t}^{j\ *} = \max\left\{\frac{m_{i,t} - (\lambda_{i,t} + 2)P_{j,t} + \sum_{l \in \boldsymbol{M}_{i,t}} P_l - \mu_{i,t}^*}{\theta_{i,t}}, 0\right\}, \tag{9}$$

where $\mu_{i,t}^*$ satisfies

$$\sum_{j \in \boldsymbol{M}_{i,t}} \max\left\{\frac{m_{i,t} - (\lambda_{i,t} + 2)P_{j,t} + \sum_{l \in \boldsymbol{M}_{i,t}} P_l - \mu_{i,t}^*}{\theta_{i,t}}, 0\right\} = K_{i,t}. \tag{10}$$

If Assumption 1 holds, then the unique optimal response strategy of power users i from the microgrid j at time t is given by

$$B_{i,t}^{j\ *} = \frac{(\lambda_{i,t} + 2)(\sum_{l \in \boldsymbol{M}_{i,t}} P_l - |\boldsymbol{M}_{i,t}| P_{i,t}) + K_{i,t}\theta_{i,t}}{|\boldsymbol{M}_{i,t}|\theta_{i,t}}. \tag{11}$$

As the strategies of different power users are identical in one population, we define the total demand of power users for microgrid j at time t as

$$D_{j,t} = \sum_{i \in \boldsymbol{C}_{j,t}} \left(y_{i,t}^{j} B_{i,t}^{j} \right), \quad j \in \{1, 2, \ldots, N\}. \tag{12}$$

When the microgrids determine their real-time prices, they cannot immediately obtain the total demands of power users. In an ideal situation, the microgrid sells all the production power. Then, we define the production power of microgrid j at time t as

$$\begin{aligned} {T_{j,t}}^{*} &= \arg\max_{T_{j,t}} P_{j,t} T_{j,t} - \phi_{j,t}(T_{j,t}) \\ &= \frac{P_{j,t} - b_{j,t}}{2a_{j,t}}, \quad j \in \{1, 2, \ldots, N\}. \end{aligned} \tag{13}$$

The satisfaction function of power users i from microgrid j in the population x has two possibilities.

If $T_{j,t} \geqslant D_{j,t}$, the demand can be satisfied, and the power user $i \in I_x$ from microgrid $j \in J_x$ can obtain $B_{j,t}^{j}$ at time t. Then, we define the satisfaction function as

$$\begin{aligned} f_t^{j,x} = &\frac{1}{|I_x|} \sum_{i \in I_x} \left(\omega_{i,t}^{j} (P_{1,t}, \ldots, P_{N,t}) B_{i,t}^{j} \right) \\ &- \frac{1}{|I_x|} \sum_{i \in I_x} \frac{\theta_{i,t}}{2} \left(B_{i,t}^{j} \right)^2 - \frac{1}{|I_x|} \sum_{i \in I_x} \left(\lambda_{i,t} P_{j,t} B_{i,t}^{j} \right), \end{aligned} \tag{14}$$

where $|I_x|$ represents the number of power users in the population x.

If $T_{j,t} < D_{j,t}$, the demand cannot be satisfied, and power users $i \in I_x$ from microgrid $j \in J_x$ can only obtain $B_{i,t}^{j} \left(T_{j,t} / D_{j,t} \right)$. Then, we define the satisfaction function at time t as

$$\begin{aligned} f_t^{j,x} = &\frac{1}{|I_x|} \sum_{i \in I_x} \left(\omega_{i,t}^{j} (P_{1,t}, \ldots, P_{N,t}) B_{i,t}^{j} \left(\frac{T_{j,t}}{D_{j,t}} \right) \right) \\ &- \frac{1}{|I_x|} \sum_{i \in I_x} \frac{\theta_{i,t}}{2} \left(B_{i,t}^{j} \left(\frac{T_{j,t}}{D_{j,t}} \right) \right)^2 - \frac{1}{|I_x|} \sum_{i \in I_x} \left(\lambda_{i,t} P_{j,t} B_{i,t}^{j} \left(\frac{T_{j,t}}{D_{j,t}} \right) \right). \end{aligned} \tag{15}$$

We define the replicator dynamics as

$$\frac{\partial y_t^{j,x}}{\partial t} = y_t^{j,x} \left(\frac{|I_x|}{L} f_t^{j,x} - \overline{f_t^x} \right), \tag{16}$$

and describe the dynamic process of power users, where t is time, and L is the total number of power users. $\overline{f_t^x} = \sum_{j \in J_x} y_t^{j,x} (|I_x|/L) f_t^{j,x}, x \in \chi$ is the average satisfaction function of power users in the population x.

3.2 Evolutionary equilibrium

The evolutionary equilibrium indicates that power users in a population do not change their strategies. Thereafter a stable state is reached in the evolutionary game. Furthermore, according to the replicator dynamics, the evolutionary equilibrium is achieved when $(|I_x|/L) f_t^{j,x} = \overline{f_t^x}$. We define the evolutionary equilibrium at time t as $\boldsymbol{y}_{i,t}^{*} = ({y_{i,t}^{1}}^{*}, \ldots, {y_{i,t}^{N}}^{*})$.

To describe the dynamic process of power users and achieve the evolutionary equilibrium, from (14)–(16), we define the iterative algorithm as

$$y_t^{j,x}(s+1) = y_t^{j,x}(s) + \delta_t y_t^{j,x}(s) \left(\frac{|I_x|}{L} f_t^{j,x}(s) - \overline{f_t^x}(s) \right), \tag{17}$$

where $y_t^{j,x} = y_{i,t}^{j}, i \in I_x, j \in J_x$, s denotes the iteration number and $\delta_t > 0$ is the step size. The terminal criterion is given by

$$\left| \frac{|I_x|}{L} f_t^{j,x}(s) - \overline{f_t^x}(s) \right| < \varepsilon,$$

where ε is an arbitrarily given positive constant.

The convergence of the evolutionary equilibrium is established by the Lyapunov method [35]. Therefore, we apply the following lemma and theorem.

Lemma 2 ([36])**.** The equilibrium is globally asymptotically stable if there exists a scalar function $V(Z)$ with continuous first-order derivative such that

- $V(Z)$ is positive definite;
- $\dot{V}(Z)$ is negative definite;
- $V(Z) \to \infty$ as $\|Z\| \to \infty$.

Theorem 1. The $y_t^{j,x}$ defined in the iterative algorithm (17) converges to an evolutionary equilibrium

$$\boldsymbol{y}_{i,t}^{*} = \left(y_{i,t}^{1\ *}, \ldots, y_{i,t}^{j\ *}, \ldots, y_{i,t}^{N\,*}\right),$$

where $y_{i,t}^{j\ *} = y_t^{j,x^*}, i \in I_x, j \in J_x$.

Proof. From the replicator dynamics $\partial y_t^{j,x}/\partial t = y_t^{j,x}((|I_x|/L)f_t^{j,x} - \overline{f_t^x})$, the evolutionary equilibrium is achieved when $(|I_x|/L)f_t^{j,x} = \overline{f_t^x}$. In the population x, the proportion of power users purchasing electricity from microgrid j is y_t^{j,x^*}. To analyze the stability of the evolutionary game model, we define the tracking error function $e_t^{j,x} = y_t^{j,x^*} - y_t^{j,x}$, and the Lyapunov function $V_t^{j,x} = (e_t^{j,x})^2/2$.

From the Lyapunov function, if $e_t^{j,x} = 0$, $V_t^{j,x} = (e_t^{j,x})^2/2 = 0$. If $e_t^{j,x} \neq 0$, $V_t^{j,x} = (e_t^{j,x})^2/2 > 0$. Therefore, $V_t^{j,x}$ is positive definite all the time. We define the derivative of $V_t^{j,x}$ with respect to t as

$$\begin{aligned}
\frac{\partial V_t^{j,x}}{\partial t} &= \frac{\partial (e_t^{j,x})^2/2}{\partial t} \\
&= -e_t^{j,x}\frac{\partial y_t^{j,x}}{\partial t} \\
&= -\left(y_t^{j,x^*} - y_t^{j,x}\right) y_t^{j,x}\left(\frac{|I_x|}{L}f_t^{j,x} - \overline{f_t^x}\right) \\
&= y_t^{j,x}\frac{|I_x|}{L}\left(y_t^{j,x} - y_t^{j,x^*}\right)\left(f_t^{j,x} - \sum_{j\in J_x} y_t^{j,x} f_t^{j,x}\right).
\end{aligned} \tag{18}$$

In order to determine whether $\partial V_t^{j,x}/\partial t$ is positive or negative, we should consider $y_t^{j,x} - y_t^{j,x^*}$ and $f_t^{j,x} - \sum_{j\in J_x} y_t^{j,x} f_t^{j,x}$. In the population x, considering the power users from the microgrid n with the maximum satisfaction function at time t, we have $f_t^{n,x} > f_t^{j,x}$, then

$$f_t^{n,x} - \sum_{j\in J_x} y_t^{j,x} f_t^{j,x} = \sum_{j\in J_x} y_t^{j,x} f_t^{n,x} - \sum_{j\in J_x} y_t^{j,x} f_t^{j,x} > 0.$$

According to (16), the proportion of power users purchasing electricity from microgrid n will increase, namely $y_t^{n,x} < y_t^{n,x*}$. Hence, the time derivative of $V_t^{j,x}$ is negative definite. The Lyapunov function $V_t^{j,x}$ is radially unbounded, as it tends to infinity as $\|e_t^{j,x}\| \to \infty$. According to Lemma 2, the equilibrium is globally asymptotically stable. The replicator dynamics of power users from the microgrid with the maximum satisfaction function will converge to equilibrium. Therefore, the iterative algorithm (17) will converge to an evolutionary equilibrium.

4 Stackelberg game between microgrids and power users

We obtain the Stackelberg equilibrium strategies between microgrids and power users through backward induction. As leaders, microgrids set real-time prices to maximize their profit functions. As followers, power users select the appropriate demands to maximize their satisfaction functions.

4.1 Non-cooperative game among microgrids

There is a non-cooperative static game among microgrids, wherein they jointly determine the appropriate real-time prices to maximize their profit functions.

Definition 1 ([37]). A non-cooperative game is $G = \{S_1, \ldots, S_n; U_1, \ldots, U_n\}$, where S_j is the player j, and U_j is the payoff function of player j. A strategy set $\boldsymbol{s}^* = ({s_1}^*, \ldots, {s_n}^*)$ is a Nash equilibrium if and only if

$$U_j({s_j}^*, {\boldsymbol{s}_{-j}}^*) \geqslant U_j(s_j, {\boldsymbol{s}_{-j}}^*), \quad s_j \in S_j.$$

We define the entire optimal strategy set excluding player j as

$${\boldsymbol{s}_{-j}}^* = ({s_1}^*, \ldots, {s_{j-1}}^*, {s_{j+1}}^*, \ldots, {s_n}^*).$$

Then, the optimization problem is given by the following equation:

$${s_j}^* \in \arg\max_{s_j \in S_j} U_j({s_1}^*, \ldots, {s_{j-1}}^*, s_j, {s_{j+1}}^*, \ldots, {s_n}^*).$$

There are partially overlapping sales areas among the microgrids, indicating that some power users are covered by more than one microgrid. Then, we can make the following assumption.

Assumption 2. For any microgrid j, there exist power users $i \in \boldsymbol{C}_{j,t}$ satisfying $|\boldsymbol{M}_{i,t}| \geqslant 2$.

As $T_{j,t}$ is usually different from $D_{j,t}$, the profit function of microgrid j at time t has two possibilities.

If $T_{j,t} \geqslant D_{j,t}$, from (11) and (12), we define the total demand of power users from microgrid j at time t as

$$\begin{aligned} D_{j,t} &= \sum_{i \in \boldsymbol{C}_{j,t}} \left(y_{i,t}^{j^*} B_{i,t}^{j^*} \right) \\ &= \sum_{i \in \boldsymbol{C}_{j,t}} \left(y_{i,t}^{j^*} \frac{(\lambda_{i,t}+2)\left(\sum_{l \in \boldsymbol{M}_{i,t}} P_l - |\boldsymbol{M}_{i,t}| P_{j,t} + \theta_{i,t} K_{i,t}\right)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}} \right). \end{aligned} \tag{19}$$

Considering the first-order partial derivatives of $D_{j,t}$ with respect to $P_{j,t}$, we have

$$\begin{aligned} \frac{\partial D_{j,t}}{\partial P_{j,t}} &= \frac{\partial \sum_{i \in \boldsymbol{C}_{j,t}} (y_{i,t}^{j^*} B_{i,t}^{j^*})}{\partial P_{j,t}} \\ &= \sum_{i \in \boldsymbol{C}_{j,t}} \left(y_{i,t}^{j^*} \frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}} \right). \end{aligned} \tag{20}$$

The profit function of microgrid j is given by

$$\begin{aligned} & F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t}) \\ &\quad = P_{j,t} D_{j,t} - \phi_{j,t}(T_{j,t}) \\ &\quad = P_{j,t} D_{j,t} - a_{j,t} T_{j,t}^2 - b_{j,t} T_{j,t} - c_{j,t}. \end{aligned} \tag{21}$$

Considering the first-order partial derivatives of (21) with respect to $P_{j,t}$, we have

$$\frac{\partial F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})}{\partial P_{j,t}} = D_{j,t} + P_{j,t} \frac{\partial D_{j,t}}{\partial P_{j,t}} - T_{j,t} - \frac{b_{j,t}}{2a_{j,t}}. \tag{22}$$

If Assumption 2 holds,

$$\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}} < 0. \tag{23}$$

From (20), we have $\partial D_{j,t}/\partial P_{j,t} < 0$. Then, Eq. (22) is less than zero. Therefore, if $T_{j,t} \geqslant D_{j,t}$, $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a monotonically decreasing function with respect to $P_{j,t}$.

If $T_{j,t} < D_{j,t}$, the profit function of microgrid j at time t is given by

$$\begin{aligned} & F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t}) \\ &\quad = P_{j,t} T_{j,t} - \phi_{j,t}(T_{j,t}) \\ &\quad = P_{j,t} T_{j,t} - a_{j,t} T_{j,t}^2 - b_{j,t} T_{j,t} - c_{j,t} \\ &\quad = (P_{j,t} - b_{j,t}) T_{j,t} - a_{j,t} T_{j,t}^2 - c_{j,t} \end{aligned}$$

$$= \frac{(P_{j,t} - b_{j,t})^2}{4a_{j,t}} - c_{j,t}. \quad (24)$$

Considering the first-order partial derivatives of (24) with respect to $P_{j,t}$, we have

$$\frac{\partial F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})}{\partial P_{j,t}} = \frac{P_{j,t} - b_{j,t}}{2a_{j,t}}. \quad (25)$$

As the profit function should be greater than 0, the real-time price satisfies $P_{j,t} > b_{j,t}$ and then Eq. (25) is greater than 0. Therefore, if $T_{j,t} < D_{j,t}$, $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a monotonically increasing function with respect to $P_{j,t}$.

Based on the abovementioned factors, if Assumptions 1 and 2 hold, when $T_{j,t} \geqslant D_{j,t}$, $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a monotonically decreasing function with respect to $P_{j,t}$. When $T_{j,t} < D_{j,t}$, $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a monotonically increasing function with respect to $P_{j,t}$.

Definition 2. Suppose $D \subset R^n$ is a nonempty convex set, and f is a real-valued function defined on D. For $\forall x, y \in D$ and $\forall a \in [0, 1]$, if

$$f(ax + (1 - a)y) \geqslant \min\{f(x), f(y)\},$$

then f is a quasi-concave function on D.

Theorem 2. If Assumptions 1 and 2 hold, then $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a quasi-concave function with respect to $P_{j,t}$.

Proof. For $\forall P_{j_1,t}, P_{j_2,t} \in \mathbf{\Gamma}_{j,t}$ and $\forall a \in [0, 1]$, if

$$F_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant \min\{F_{j,t}(P_{j_1,t}), F_{j,t}(P_{j_2,t})\},$$

then $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a quasi-concave function with respect to $P_{j,t}$. Without the loss of generality, we assume that $P_{j_1,t} \leqslant P_{j_2,t}$ and $P_{j_1,t} \leqslant aP_{j_1,t} + (1 - a)P_{j_2,t} \leqslant P_{j_2,t}$.

For the monotonicity of $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$, we consider the following four possibilities.

(1) If $T_{j,t}(P_{j_1,t}) < D_{j,t}(P_{j_1,t})$ and $T_{j,t}(P_{j_2,t}) < D_{j,t}(P_{j_2,t})$, then $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a monotonically increasing function with respect to $P_{j,t}$. Therefore,

$$F_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant F_{j,t}(P_{j_1,t}),$$

$$F_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant \min\{F_{j,t}(P_{j_1,t}), F_{j,t}(P_{j_2,t})\}.$$

From Definition 2, $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a quasi-concave function with respect to $P_{j,t}$.

(2) If $T_{j,t}(P_{j_1,t}) \geqslant D_{j,t}(P_{j_1,t})$ and $T_{j,t}(P_{j_2,t}) \geqslant D_{j,t}(P_{j_2,t})$, then $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a monotonically decreasing function with respect to $P_{j,t}$. Therefore,

$$F_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant F_{j,t}(P_{j_2,t}),$$

$$F_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant \min\{F_{j,t}(P_{j_1,t}), F_{j,t}(P_{j_2,t})\}.$$

From Definition 2, $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a quasi-concave function with respect to $P_{j,t}$.

(3) If $T_{j,t}(P_{j_1,t}) < D_{j,t}(P_{j_1,t})$ and $T_{j,t}(P_{j_2,t}) \geqslant D_{j,t}(P_{j_2,t})$, when

$$T_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) < D_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}),$$

then $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a monotonically increasing function with respect to $P_{j,t}$. Therefore,

$$F_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant F_{j,t}(P_{j_1,t}),$$

$$F_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant \min\{F_{j,t}(P_{j_1,t}), F_{j,t}(P_{j_2,t})\}.$$

When

$$T_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant D_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}),$$

then $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a monotonically decreasing function with respect to $P_{j,t}$. Therefore,

$$F_{j,t}(aP_{j_1,t} + (1 - a)P_{j_2,t}) \geqslant F_{j,t}(P_{j_2,t}),$$

$$F_{j,t}(aP_{j_1,t}+(1-a)P_{j_2,t}) \geqslant \min\{F_{j,t}(P_{j_1,t}), F_{j,t}(P_{j_2,t})\}.$$

From Definition 2, $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a quasi-concave function with respect to $P_{j,t}$.

(4) If $T_{j,t}(P_{j_1,t}) \geqslant D_{j,t}(P_{j_1,t})$ and $T_{j,t}(P_{j_2,t}) < D_{j,t}(P_{j_2,t})$, from (20), $D_{j,t}(P_{j,t})$ is a monotonically decreasing function with respect to $P_{j,t}$, and then

$$D_{j,t}(P_{j1}) \geqslant D_{j,t}(P_{j2}).$$

From (13), $T_{j,t}(P_{j,t})$ is a monotonically increasing function with respect to $P_{j,t}$, and then

$$T_{j,t}(P_{j2}) \geqslant T_{j,t}(P_{j1}).$$

Therefore, there is a conflict with the conditions.

In summary, if Assumptions 1 and 2 hold, then $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a quasi-concave function with respect to $P_{j,t}$.

Theorem 3. If Assumptions 1 and 2 hold, there exists a Nash equilibrium for the non-cooperative game among microgrids.

Proof. From Theorem 2, $F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})$ is a quasi-concave function with respect to $P_{j,t}$. The real-time price strategy set $\mathbf{\Gamma}_{\mathrm{j},t}$ of microgrid j is a closed, bounded, and convex subset of $\mathbb{R}$. Therefore, based on the existence theorem of the Nash equilibrium [38], we know that there is a Nash equilibrium for the non-cooperative game among microgrids.

To confirm the uniqueness of the Nash equilibrium strategy for the non-cooperative game among microgrids, we first proved the optimal price function of the microgrid j is standard [39]. Next, we established the following definition.

Definition 3 ([39])**.** A function $\boldsymbol{I}(\boldsymbol{P}) = [I_1(\boldsymbol{P}), I_2(\boldsymbol{P}), \ldots, I_N(\boldsymbol{P})]$ is standard if the following properties are satisfied for all $\boldsymbol{P} \geqslant \mathbf{0}$, where $\boldsymbol{P} = [P_1, P_2, \ldots, P_N]$.

- Positivity: $\boldsymbol{I}(\boldsymbol{P}) > \mathbf{0}$;
- Monotonicity: If $\boldsymbol{P} \geqslant \boldsymbol{P}'$, then $\boldsymbol{I}(\boldsymbol{P}) \geqslant \boldsymbol{I}(\boldsymbol{P}')$;
- Scalability: For all $\varpi > 1, \varpi\boldsymbol{I}(\boldsymbol{P}) > \boldsymbol{I}(\varpi\boldsymbol{P})$.

Theorem 4. If Assumptions 1 and 2 hold, there is a unique Nash equilibrium for the non-cooperative game among microgrids.

Proof. Since $T_{j,t}$ is usually different from $D_{j,t}$, the optimal price function of microgrid j has the following two possibilities:

If $T_{j,t} \geqslant D_{j,t}$, from (22), let

$$\frac{\partial F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})}{\partial P_{j,t}} = D_{j,t} + P_{j,t}\frac{\partial D_{j,t}}{\partial P_{j,t}} - T_{j,t} - \frac{b_{j,t}}{2a_{j,t}} = 0.$$

Then, from (13), (19) and (20), we obtain the optimal price function of microgrid j at time t,

$$\begin{aligned} I_j(\boldsymbol{P}_t) &= \frac{\sum_{i\in \boldsymbol{C}_{j,t}} (y_{i,t}^{j*}\frac{(\lambda_{i,t}+2)(\sum_{l\in \boldsymbol{M}_{i,t}\setminus\{j\}} P_l + \theta_{i,t}K_{i,t})}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}{\frac{1}{2a_{j,t}} - 2\sum_{i\in \boldsymbol{C}_{j,t}} (y_{i,t}^{j*}\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})} \\ &= \frac{2a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}} (y_{i,t}^{j*}\frac{(\lambda_{i,t}+2)(\sum_{l\in \boldsymbol{M}_{i,t}\setminus\{j\}} P_l + \theta_{i,t}K_{i,t})}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}{1 - 4a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}} (y_{i,t}^{j*}\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}, \end{aligned} \tag{26}$$

where $\boldsymbol{P}_t = [P_{1,t}, P_{2,t}, \ldots, P_{N,t}]$.

If $T_{j,t} < D_{j,t}$, from (25), let

$$\frac{\partial F_{j,t}(P_{j,t}, T_{j,t}, D_{j,t})}{\partial P_{j,t}} = \frac{P_{j,t} - b_{j,t}}{2a_{j,t}} = 0.$$

Then, we obtain the optimal price function of microgrid j at time t,

$$I_j(\boldsymbol{P}_t) = b_{j,t}. \tag{27}$$

Next, we have proven the optimal price function $I_j(\boldsymbol{P}_t)$ as standard.

• Positivity: If $T_{j,t} \geqslant D_{j,t}$ and Assumption 2 holds,

$$\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}} < 0.$$

Then, from (26), we have $I_j(\boldsymbol{P}_t) > 0$. If $T_{j,t} < D_{j,t}$, from (27), we have $I_j(\boldsymbol{P}_t) = b_{j,t} > 0$. Therefore, the optimal price function $I_j(\boldsymbol{P}_t)$ satisfies the positivity.

• Monotonicity: $\boldsymbol{P}_t \geqslant \boldsymbol{P}'_t$ means that $P_{j,t} \geqslant P'_{j,t}, j \in N$. If $\forall l \neq j,\ l \in N$,

$$\begin{aligned} &I_j([P_{1,t},\ldots,P_{j,t},\ldots,P_{N,t}]) \geqslant I_j([P_{1,t},\ldots,P'_{j,t},\ldots,P_{N,t}]),\\ &I_l([P_{1,t},\ldots,P_{j,t},\ldots,P_{N,t}]) \geqslant I_l([P_{1,t},\ldots,P'_{j,t},\ldots,P_{N,t}]), \end{aligned}$$

the monotonicity can be satisfied. Therefore, if $\partial I_j(\boldsymbol{P}_t)/\partial P_{j,t} \geqslant 0$ and $\partial I_l(\boldsymbol{P}_t)/\partial P_{j,t} \geqslant 0$, we can prove the optimal price function $I_j(\boldsymbol{P}_t)$ satisfies the monotonicity.

If $T_{j,t} \geqslant D_{j,t}$ and Assumption 2 holds, from (26),

$$\frac{\partial I_j(\boldsymbol{P}_t)}{\partial P_{j,t}} = 0.$$

$$\frac{\partial I_l(\boldsymbol{P}_t)}{\partial P_{j,t}} = \frac{2a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}{1-4a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})} > 0.$$

If $T_{j,t} < D_{j,t}$, from (27)

$$\frac{\partial I_j(\boldsymbol{P}_t)}{\partial P_{j,t}} = \frac{\partial I_l(\boldsymbol{P}_t)}{\partial P_{j,t}} = 0.$$

Therefore, the optimal price function $I_j(\boldsymbol{P}_t)$ satisfies the monotonicity.

• Scalability: For all $\varpi > 1$, if $T_{j,t} \geqslant D_{j,t}$ and Assumption 2 holds, from (26),

$$\begin{aligned} &\varpi\ I_j(\boldsymbol{P}_t) - I_j(\varpi \boldsymbol{P}_t)\\ &= \frac{2\varpi a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)(\sum_{l\in \boldsymbol{M}_{i,t}\setminus\{j\}}P_l+\theta_{i,t}K_{i,t})}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}{1-4a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}\\ &\quad - \frac{2a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)(\varpi\sum_{l\in \boldsymbol{M}_{i,t}\setminus\{j\}}P_l+\theta_{i,t}K_{i,t})}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}{1-4a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}\\ &= \frac{2\varpi a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)K_{i,t}}{|\boldsymbol{M}_{i,t}|})}{1-4a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})} - \frac{2a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)K_{i,t}}{|\boldsymbol{M}_{i,t}|})}{1-4a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})}\\ &= \frac{2(\varpi-1)a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)K_{i,t}}{|\boldsymbol{M}_{i,t}|})}{1-4a_{j,t}\sum_{i\in \boldsymbol{C}_{j,t}}(y_{i,t}^{j^*}\frac{(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}})} > 0; \end{aligned}$$

if $T_{j,t} < D_{j,t}$, from (27),

$$\begin{aligned} &\varpi\ I_j(\boldsymbol{P}_t) - I_j(\varpi \boldsymbol{P}_t)\\ &= \varpi b_{j,t} - b_{j,t}\\ &= (\varpi-1)b_{j,t} > 0. \end{aligned}$$

Therefore, the optimal price function $I_j(\boldsymbol{P}_t)$ satisfies the scalability. In summary, if Assumptions 1 and 2 hold, the optimal price function $I_j(\boldsymbol{P}_t)$ is a standard function.

Considering the real-time price strategy $\{P_{j,t}, j = 1,2,\ldots,N\}$, from Theorem 1, we obtain the evolutionary equilibrium $\boldsymbol{y}_{i,t}^*$. If Assumption 1 holds, from Lemma 1, we obtain the ${B_{i,t}^j}^*$. Let $\boldsymbol{I}(\boldsymbol{P}_t) = [I_1(\boldsymbol{P}_t), I_2(\boldsymbol{P}_t),\ldots,I_N(\boldsymbol{P}_t)]$. Then, from Theorem 3, there exists a Nash equilibrium $\boldsymbol{P}_t^*$ for the non-cooperative game among microgrids. The equilibrium strategy $\boldsymbol{P}_t^*$ satisfies $\boldsymbol{I}(\boldsymbol{P}_t^*) = \boldsymbol{P}_t^*$. We assume that $\dot{\boldsymbol{P}}_t^*$ and $\ddot{\boldsymbol{P}}_t^*$ are both Nash equilibrium strategies for the non-cooperative game among microgrids. Then, we have $\boldsymbol{I}(\dot{\boldsymbol{P}}_t^*) = \dot{\boldsymbol{P}}_t^*$ and $\boldsymbol{I}(\ddot{\boldsymbol{P}}_t^*) = \ddot{\boldsymbol{P}}_t^*$. From Definition 3, $\boldsymbol{I}(\cdot)$ is a standard function.

From the positivity of $\boldsymbol{I}(\cdot)$, we have $\dot{P}^*_{j,t} > 0$ and $\ddot{P}^*_{j,t} > 0$. Without the loss of generality, we assume that $\dot{P}^*_{j,t} < \ddot{P}^*_{j,t}$. Then, there is $\varpi > 1$ such that $\varpi\dot{P}^*_{j,t} = \ddot{P}^*_{j,t}$ and $\varpi\dot{\boldsymbol{P}}^*_t \geqslant \ddot{\boldsymbol{P}}^*_t$ hold.

From the monotonicity of $\boldsymbol{I}(\cdot)$ and $\varpi\dot{\boldsymbol{P}}^*_t \geqslant \ddot{\boldsymbol{P}}^*_t$, we have

$$\ddot{P}^*_{j,t} = I_j(\ddot{\boldsymbol{P}}^*_t) \leqslant I_j(\varpi\dot{\boldsymbol{P}}^*_t). \tag{28}$$

From the scalability of $\boldsymbol{I}(\cdot)$, we have

$$I_j(\varpi\dot{\boldsymbol{P}}^*_t) < \varpi I_j(\dot{\boldsymbol{P}}^*_t) = \varpi\dot{P}^*_{j,t}. \tag{29}$$

From (28) and (29), we have $\varpi\dot{P}^*_{j,t} > \ddot{P}^*_{j,t}$. There is a conflict with the condition $\varpi\dot{P}^*_{j,t} = \ddot{P}^*_{j,t}$. Thus, we have $\boldsymbol{P}^*_t = \dot{\boldsymbol{P}}^*_t = \ddot{\boldsymbol{P}}^*_t$. Therefore, we know that there is a unique Nash equilibrium for the non-cooperative game among microgrids.

4.2 Stackelberg game analysis

To maximize the profit functions, microgrids jointly determine the appropriate real-time prices to power users. Next, the power users are involved in the evolutionary game and finally obtain the evolutionary equilibrium. Then, the total demands of power users are transferred into the microgrids. We eventually obtain the Stackelberg equilibrium.

Definition 4 ([40]). If $\boldsymbol{P}^*_t = (P^*_{1,t}, P^*_{2,t}, \ldots, P^*_{N,t})$ is the equilibrium strategy of the real-time price among microgrids at time t. $\boldsymbol{y}^*_t = (\boldsymbol{y}^*_{1,t}, \boldsymbol{y}^*_{2,t}, \ldots, \boldsymbol{y}^*_{L,t})$ is an optimal response strategy among power users at time t. Then, a strategy pair $(\boldsymbol{P}^*_t, \boldsymbol{y}^*_t)$ is a Stackelberg equilibrium between microgrids and power users.

Theorem 5. If Assumptions 1 and 2 hold, there is a unique Stackelberg equilibrium between microgrids and power users.

Proof. From Theorem 4, there is a unique Nash equilibrium for the non-cooperative game among microgrids. Considering the real-time equilibrium price $\{P^*_{j,t}, j = 1, \ldots, N\}$, from Theorem 1, the proportion strategy of power user i purchasing electricity converges to the evolutionary equilibrium $\boldsymbol{y}^*_{i,t}$. $\boldsymbol{y}^*_{i,t}$ is an optimal response strategy of the power user i at time t. Therefore, from Definition 4, we know that there is a unique Stackelberg equilibrium between microgrids and power users.

4.3 An iterative algorithm based on Stackelberg-evolutionary joint game

In the real-time pricing scheme of multiple microgrids and multiple power users, since power users' feedback demands to microgrids instead of the optimal response demand curves, it is difficult for the microgrids to predict the optimal response demands of power users to certain real-time prices. Then, the microgrid j adjusts the real-time price according to the relationship between $D_{j,t}$ and $T_{j,t}$ to ensure a balance of power supply and demand and maximize profit functions. Therefore, from (12) and (13), we present an iterative algorithm to update the real-time price strategy $P_{j,t}$:

$$\begin{cases} P_{j,t}(s+1) = P_{j,t}(s) + \sigma_{j,t}(D_{j,t}(s) - T_{j,t}(s)), \\ D_{j,t}(s) = \sum\limits_{i \in \boldsymbol{C}_{j,t}} (y^j_{i,t}(s) B^j_{i,t}(s)), \\ T_{j,t}(s) = \frac{P_{j,t}(s) - b_{j,t}}{2a_{j,t}}, \end{cases} \tag{30}$$

where $\sigma_{j,t}$ is the generation demand ratio of microgrid j, and s denotes the iteration number. The terminal criterion is given by

$$|D_{j,t}(s) - T_{j,t}(s)| < \varepsilon, \tag{31}$$

where ε is an arbitrarily small positive constant.

Theorem 6. If Assumption 2 holds, and $\sigma_{j,t}$ satisfies

$$\sigma_{j,t} < \frac{4a_{j,t}}{1 - 2a_{j,t}\sum_{i \in \boldsymbol{C}_{j,t}} \frac{y^j_{i,t}(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}}}, \tag{32}$$

then the iterative algorithm (30) converges to the unique non-cooperative game equilibrium strategy among microgrids.

Proof. First, we define the mapping function $\Xi_{j,t}(\tau):[0,1]\to\mathbb{R}$, $j\in N$ as

$$\Xi_{j,t}(\tau)=\tau P_{j,t}(s)+(1-\tau)P_{j,t}^{*}+\sigma_{j,t}(D_{j,t}(\tau P_{j,t}(s)+(1-\tau)P_{j,t}^{*})-T_{j,t}(\tau P_{j,t}(s)+(1-\tau)P_{j,t}^{*})). \quad (33)$$

We have

$$\Delta=D_{j,t}(P_{j,t}(s))-T_{j,t}(P_{j,t}(s)). \quad (34)$$

Combining this with the iterative algorithm (30), we obtain

$$\begin{aligned}\left|P_{j,t}(s+1)-P_{j,t}^{*}\right| &= \left|P_{j,t}(s)+\sigma_{j,t}\Delta-P_{j,t}^{*}\right| \\ &= \left|\Xi_{j,t}(1)-\Xi_{j,t}(0)\right| \\ &= \left|\int_0^1 \frac{\mathrm{d}\Xi_{j,t}(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau\right| \\ &\leqslant \int_0^1 \left|\frac{\mathrm{d}\Xi_{j,t}(\tau)}{\mathrm{d}\tau}\right|\mathrm{d}\tau \\ &\leqslant \max_{\tau\in[0,1]}\left|\frac{\mathrm{d}\Xi_{j,t}(\tau)}{\mathrm{d}\tau}\right|, \quad j\in N. \end{aligned} \quad (35)$$

Next, we have

$$\Upsilon=\sum_{i\in \boldsymbol{C}_{j,t}}\frac{y_{i,t}^{j}(\lambda_{i,t}+2)\left(1-|\boldsymbol{M}_{i,t}|\right)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}}, \quad j\in N. \quad (36)$$

Then, $|\frac{\mathrm{d}\Xi_{j,t}(\tau)}{\mathrm{d}\tau}|$ can be calculated by

$$\begin{aligned}\left|\frac{\mathrm{d}\Xi_{j,t}(\tau)}{\mathrm{d}\tau}\right| &= \left|P_{j,t}(s)-P_{j,t}^{*}\right| \\ &\quad+\left|\sigma_{j,t}\left(\Upsilon\left(P_{j,t}(s)-P_{j,t}^{*}\right)-\frac{P_{j,t}(s)-P_{j,t}^{*}}{2a_{j,t}}\right)\right| \\ &= \left|\left(1+\sigma_{j,t}\left(\Upsilon-\frac{1}{2a_{j,t}}\right)\right)\left(P_{j,t}(s)-P_{j,t}^{*}\right)\right| \\ &= \left|1+\sigma_{j,t}\left(\Upsilon-\frac{1}{2a_{j,t}}\right)\right|\left|P_{j,t}(s)-P_{j,t}^{*}\right|, \end{aligned} \quad (37)$$

where $\boldsymbol{C}_{j,t}$ is the set of power users covered by the microgrid j at time t and $\boldsymbol{M}_{i,t}$ represents the set of microgrids covering power user i at time t.

Next, we have

$$\begin{aligned}\Theta &= \left|1+\sigma_{j,t}\left(\Upsilon-\frac{1}{2a_{j,t}}\right)\right| \\ &= \left|1+\sigma_{j,t}\left(\sum_{i\in \boldsymbol{C}_{j,t}}\frac{y_{i,t}^{j}(\lambda_{i,t}+2)\left(1-|\boldsymbol{M}_{i,t}|\right)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}}-\frac{1}{2a_{j,t}}\right)\right|. \end{aligned} \quad (38)$$

From (35) and (37), we can obtain

$$\begin{aligned}\left|P_{j,t}(s+1)-P_{j,t}^{*}\right| &\leqslant \max_{\tau\in[0,1]}\left|\frac{\mathrm{d}\Xi_{j,t}(\tau)}{\mathrm{d}\tau}\right| \\ &\leqslant \Theta\left|P_{j,t}(s)-P_{j,t}^{*}\right| \\ &\leqslant \Theta^{2}\left|P_{j,t}(s-1)-P_{j,t}^{*}\right| \\ &\leqslant \Theta^{s+1}\left|P_{j,t}(0)-P_{j,t}^{*}\right|. \end{aligned} \quad (39)$$

When $0<\Theta<1$ and $s\to\infty$,

$$\lim_{s\to\infty}\left|P_{j,t}(s+1)-P_{j,t}^{*}\right|=0, \quad (40)$$

namely, the iterative algorithm (30) converges to the unique non-cooperative game equilibrium strategy among microgrids.

To ensure $0 < \Theta < 1$, we have

$$0 < \left| 1 + \sigma_{j,t} \left(\sum_{i \in \boldsymbol{C}_{j,t}} \frac{y_{i,t}^{j}(\lambda_{i,t}+2)\left(1-|\boldsymbol{M}_{i,t}|\right)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}} - \frac{1}{2a_{j,t}} \right) \right| < 1.$$

If Assumption 2 holds, we can obtain

$$\sigma_{j,t} < \frac{4a_{j,t}}{1 - 2a_{j,t}\sum_{i \in \boldsymbol{C}_{j,t}} \frac{y_{i,t}^{j}(\lambda_{i,t}+2)(1-|\boldsymbol{M}_{i,t}|)}{|\boldsymbol{M}_{i,t}|\theta_{i,t}}}.$$

Remark 2. First, the microgrids jointly announce the initial real-time prices to power users. Then, the power users are involved in the evolutionary game. From (14) and (15), $f_t^{j,x}(s)$ and $\overline{f_t^{x}}(s)$ are calculated, respectively. From the iterative algorithm (17), the proportion strategy of power user i purchasing electricity converges to the evolutionary equilibrium $\boldsymbol{y}_{i,t}^{*}$. From (12) and (13), $D_{j,t}(s)$ and $T_{j,t}(s)$ are calculated, respectively. From the iterative algorithm (30), the real-time price of microgrid j converges to the Nash equilibrium strategy $P_{j,t}^{*}$ of the non-cooperative game among microgrids. Eventually, the Stackelberg equilibrium can be achieved.

5 Numerical examples

In this section, we have presented numerical results to verify the convergence of the iterative algorithm and analyze the equilibrium strategies of the game. We consider the scenario $N = 3$ and $L = 6$, with five populations in the system. The electricity consumption time is divided into 24 time slots representing the 24 h of a day ($H = 24$). The simulation conditions are as follows: in population 1, power user 2 is jointly covered by microgrids 1 and 2. In population 2, power user 4 is jointly covered by microgrids 2 and 3. In population 3, power user 1 is separately covered by microgrid 1. In population 4, power user 3 is separately covered by microgrid 2. In population 5, power users 5 and 6 are separately covered by microgrid 3. The cost parameters of the microgrid j are as follows, the sequence $\{a_{j,t}, j \in N, t \in H\}$ is independent and identically distributed and satisfies the uniformly distributed on $[0.01, 0.03]$. $b_{j,t} = 0.01$. $c_{j,t} = 0.1$. The initial real-time prices are $P_{j,t}(0) = 0.8$ \$/kWh. We define the power user parameters by $\theta_{i,t}$=0.1, $\lambda_{i,t}$=0.5, and $K_{i,t} = 30$ kWh. When the power user i is separately covered by the microgrid, the sequence $\{m_{i,t}, i \in L, t \in H\}$ is independent and identically distributed and satisfies the uniformly distributed on $[4, 6]$. Otherwise, the sequence $\{m_{i,t}, i \in L, t \in H\}$ satisfies the uniformly distributed on $[3, 5]$. The step sizes are $\delta_t = 0.06$ and $\sigma_{j,t} = 0.01$.

To verify the effectiveness and convergence of the iterative algorithm, we conduct a simulation and obtain the Stackelberg equilibrium strategy through backward induction. Figure 1 represents the convergence process of the real-time prices, where $P_1^* = P_2^* = 1.29$ \$/kWh, $P_3^* = 1.81$ \$/kWh. Since the microgrid covers several power users in a separate sales area, the power users are less sensitive to the real-time price owing to their fixed demand. Then, the real-time price of microgrid 3 is higher than those of microgrids 1 and 2. This finding means that increasing the number of power users in overlapping sales areas can help reduce the real-time price. Figure 2 indicates the convergence process of the proportions of users purchasing electricity. Populations 1 and 2 have a relatively higher proportion of purchasing electricity in microgrid 2, which is because microgrid 2 has higher sensitivity to the real-time price with several overlapping sales areas, resulting in increasing demands of power users. This finding suggests overlapping sales areas, signifying a higher proportion of power users purchasing electricity.

After several iterations, the proportion of purchasing electricity converges to the evolutionary equilibrium. Figure 3 represents the convergence process of the total demand of power users from each microgrid, where $D_1^* = 32.14$ kWh, $D_2^* = 64.28$ kWh, $D_3^* = 60$ kWh. The total demand of power users in microgrid 1 is the lowest because microgrid 1 covers fewer users. Although the number of users covered by microgrids 2 and 3 is the same, the total demand for power users in microgrid 2 is larger as microgrid 2 has more overlapping sales areas that lead to an increasing total demand for power users. Figure 4 depicts the convergence process of the production power of each microgrid, where $T_1^* = 32.14$ kWh, $T_2^* = 64.28$ kWh, $T_3^* = 60$ kWh. As the cost parameter of microgrid 2 is smaller than those of microgrids

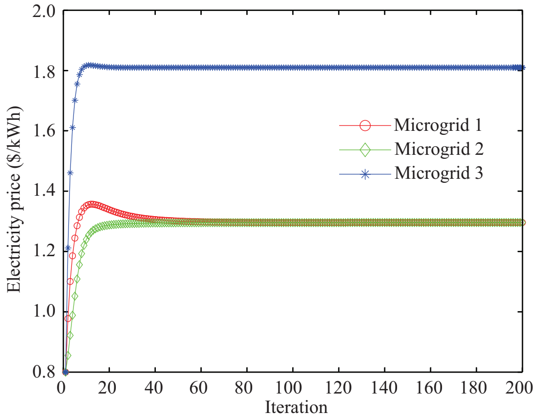

Figure 1 (Color online) Electricity prices of the microgrids.

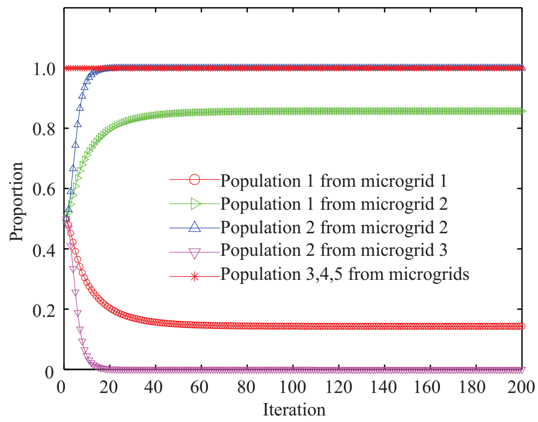

Figure 2 (Color online) Proportions of users purchasing electricity.

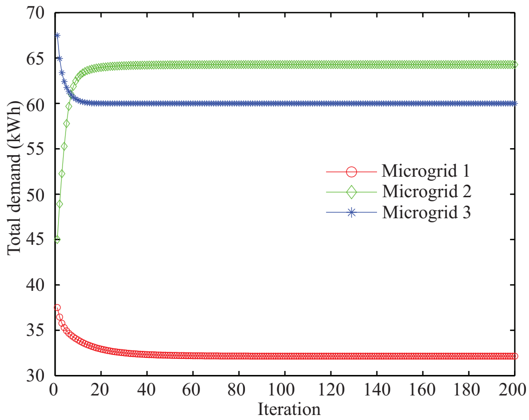

Figure 3 (Color online) Total demands of power users in the microgrids.

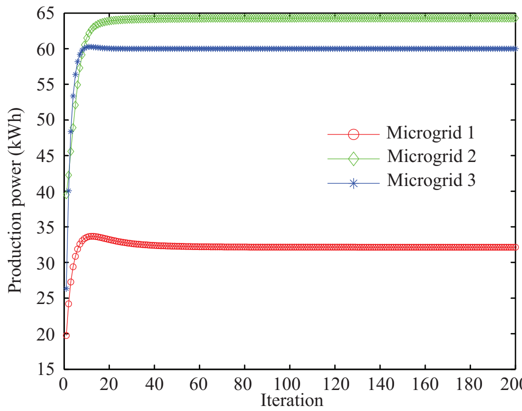

Figure 4 (Color online) Production powers of the microgrids.

1 and 3, the production power of microgrid 2 is higher than those of microgrids 1 and 3. According to the results in Figures 3 and 4, the total demand of power users and the production power of each microgrid converge to the same level, which depicts the convergence and effectiveness of the iterative algorithm.

Figure 5 depicts the convergence process of microgrids' profits. The profits of all microgrids achieve convergence, where $F_1 = 20.56\$$, $F_2 = 41.22\$$, and $F_3 = 53.9\$$. The profit of microgrid 3 is the highest because of its higher real-time price and more separate power users. Although the real-time prices of microgrids 1 and 2 are equal after several iterations, the profit of microgrid 2 is higher. This is because microgrid 2 has more overlapping sales areas and microgrid 1 covers fewer power users. Figure 6 shows the convergence process of the average satisfaction function of populations. Furthermore, the average satisfaction functions of populations 1 and 2 are higher than those of populations 3–5, indicating that power users in the overlapping sales areas obtain better satisfaction functions.

Figure 7 shows the real-time prices of the microgrids in a day. Figure 8 shows the total demands of power users in the microgrids in a day. Power users adjust their demands according to the real-time price signals at different time points. According to the results given in Figures 7 and 8, when the real-time prices of microgrids are higher, power users reduce electricity consumption, thereby reducing the demands during peak periods. When the real-time prices are lower, power users increase their electricity consumption, thereby increasing demands during the off-peak periods. The results exhibit the effectiveness of the iterative algorithm.

To evaluate the performance of the proposed algorithm, we considered a fixed pricing algorithm in [28] for comparison. In this algorithm, microgrids maintain a fixed electricity price at each time point. Then, power users have no incentive to change their demands owing to the lack of interaction. We select

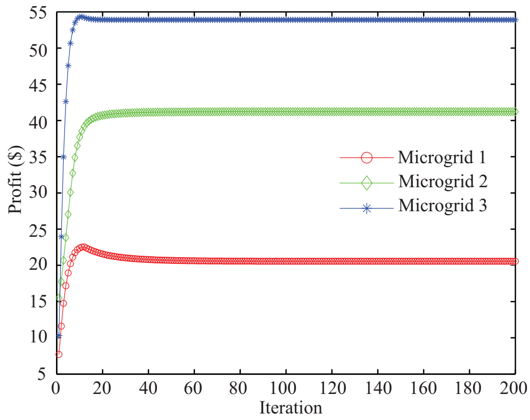

Figure 5 (Color online) The profits of the microgrids.

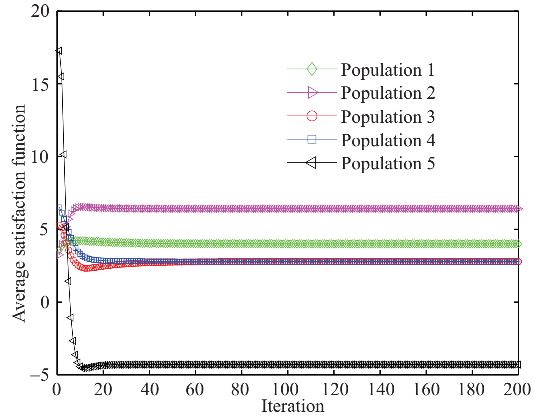

Figure 6 (Color online) The average satisfaction function of populations.

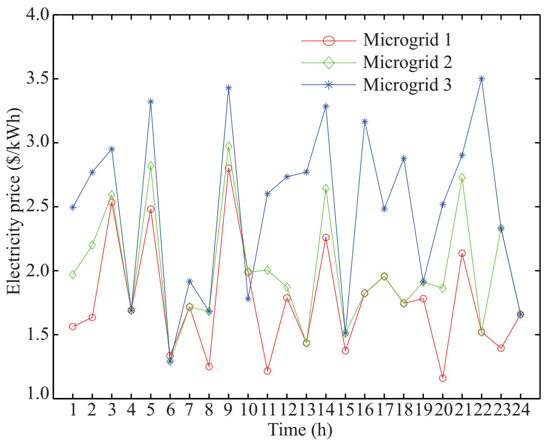

Figure 7 (Color online) The electricity prices of the microgrids in a day.

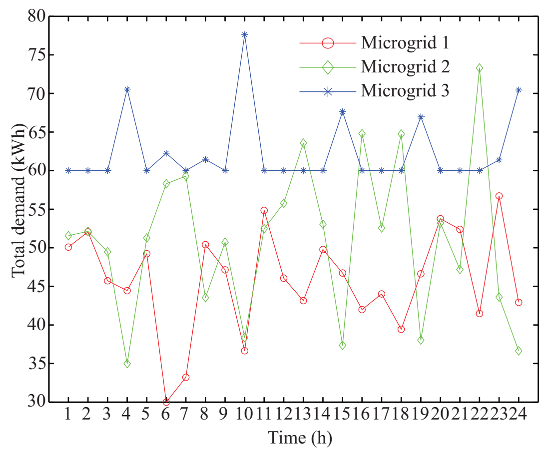

Figure 8 (Color online) The total demands of power users in the microgrids in a day.

microgrid 2 for comparison under different pricing algorithms.

Figure 9 exhibits the electricity price of microgrid 2 in a day under different pricing algorithms. According to the results in Figure 9, we calculate that the average electricity price of microgrid 2 in a day under the fixed pricing algorithm is 2.4974$. Meanwhile, we calculated the average electricity price of microgrid 2 in a day under the proposed algorithm to be 1.9967$. Obviously, the proposed algorithm could reduce the electricity price as the proposed algorithm could calculate the electricity price for each period in real time. The response of power users to the electricity price in the previous period influences the electricity price in the next period. Furthermore, microgrid 2 shows higher sensitivity to the demand with several overlapping sales areas. Figure 10 shows the profit of microgrid 2 in a day under different pricing algorithms. According to the results in Figure 10, we calculate that the average profit of microgrid 2 in a day under the fixed pricing algorithm is 42.7180$. Meanwhile, we calculate that the average profit of microgrid 2 in a day under the proposed algorithm is 50.3003$. Thus, it is evident that the proposed algorithm helps the microgrid obtain a greater profit because the cost parameters in the proposed algorithm are time-varying, while those in the fixed pricing algorithm are fixed. Thus, the proposed algorithm could reduce the cost of the microgrid and thereby obtain a greater profit.

6 Conclusion

We have presented a framework of the Stackelberg-evolutionary joint game to study the real-time pricing scheme of multiple microgrids and multiple power users. In this framework, the electricity consumption

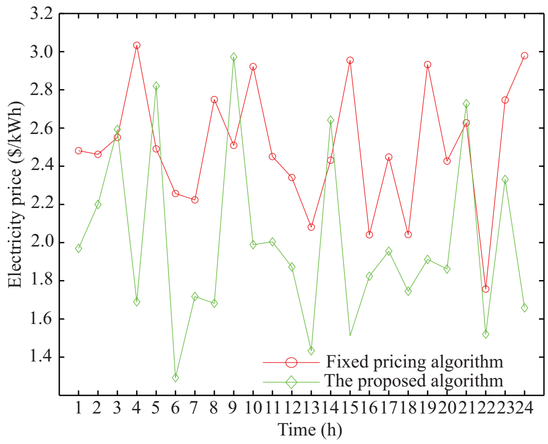

Figure 9 (Color online) The electricity prices of microgrid 2 in a day under different pricing algorithms.

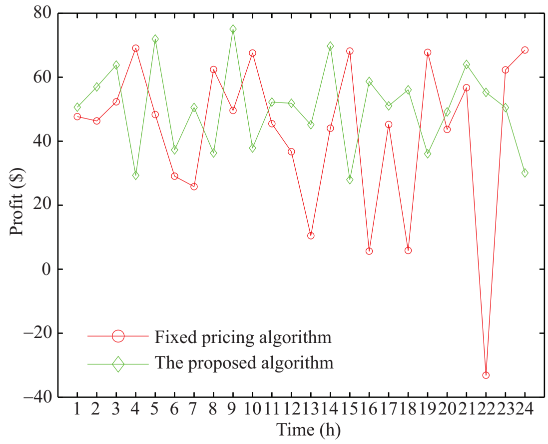

Figure 10 (Color online) The profits of microgrid 2 in a day under different pricing algorithms.

time is divided in a day into H periods. Then, the microgrids set the time-varying electricity prices and induce power users to shift their demands from the peak to off-peak periods. The convergence of the evolutionary equilibrium among power users was thus proven. Then, we next proved the existence and uniqueness of the Stackelberg equilibrium. Furthermore, we computed the equilibrium strategies by using an iterative algorithm. Numerical simulations demonstrate the effectiveness and the convergence of the iterative algorithm. Moreover, the algorithm reflected the changing relationship between the supply and demand at each time point and adjusted the demands of power users according to the real-time prices. When the number of power users is large, it is difficult to analyze the demand response management by the proposed algorithm. In the future, we plan to consider demand response management for multiple microgrids with a large number of power users involved.

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