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Supplementary Information: Experimental Entanglement Quantification for Unknown Quantum States in a Semi-Device-Independent Manner

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Appendix A On the quantity a_1

As shown in the main text, the quantity a_1 is fundamental in quantifying the entanglement of unknown quantum states from the probability distribution in a semi-DI manner. In our experiment, the lower bounds for a_1 are obtained by applying the concept of non-degenerate Bell inequality directly. For example, the results of a_1 for the qutrit-qutrit demonstration can be seen in Fig. C1, where when the observed Bell value approaches the maximal, a_1 becomes increasingly closer to 1.

Appendix B On the quantity $\hat{F}$

Suppose that the probability distribution $p(\vec{a}|\vec{x})$ is obtained by measuring the target quantum state ρ with local measurements $\{M_{\vec{x}}\}$. Let $|\phi\rangle$ be arbitrary n-partite pure product states, and $q^*(\vec{a}|\vec{x})$ be the correlation produced by measuring $|\phi\rangle$ with the same local measurements $\{M_{\vec{x}}\}$. Then, there exist probability distributions $q_i^*(a_i|x_i)$ such that $q^*(\vec{a}|\vec{x}) = \prod_{i=1}^n q_i^*(a_i|x_i)$, and for any $\vec{x}$ it holds that

$$F(|\phi\rangle\langle\phi|, \rho) \leqslant F(q_{\vec{x}}^*, p_{\vec{x}}) = \sum_{\vec{a}} \sqrt{q^*(\vec{a}|\vec{x}) p(\vec{a}|\vec{x})},$$

where $p_{\vec{x}} \equiv p(\cdot|\vec{x})$, $q_{\vec{x}}^* \equiv q^*(\cdot|\vec{x})$, and the inequality comes from the fact that any quantum measurement cannot make the fidelity between two quantum states smaller. This means $F(|\phi\rangle\langle\phi|, \rho) \leqslant \min_{\vec{x}} F(q_{\vec{x}}^*, p_{\vec{x}})$, and

$$F(|\phi\rangle\langle\phi|, \rho) \leqslant \max_{q} \min_{\vec{x}} F(q_{\vec{x}}, p_{\vec{x}}),$$

where the maximization is over product correlations q and $q_{\vec{x}} \equiv q(\cdot|\vec{x})$. Combining this with the max-min inequality

$$\max_{q} \min_{\vec{x}} F(q_{\vec{x}}, p_{\vec{x}}) \leqslant \min_{\vec{x}} \max_{q} F(q_{\vec{x}}, p_{\vec{x}}),$$

we have that

$$F(|\phi\rangle\langle\phi|, \rho) \leqslant \min_{\vec{x}} \max_{q} F(q_{\vec{x}}, p_{\vec{x}}).$$

Therefore, we eventually obtain an upper bound for the fidelity between the target state and a pure product state, denoted as $\hat{F}$, based on the probability distribution $p(\vec{a}|\vec{x})$ only. Indeed, once $\vec{x}$ is fixed, the inner maximization can be computed using symmetric embedding [1] and the shifted higher-order power method (SHOPM) algorithm [2], yielding a correct answer up to numerical precision with very high probability.

Appendix C Details of the Bell-type inequalities used in this work

In our experiment, we test three Bell-type inequalities to quantify the entanglement of high-dimensional and multi-partite states. We now give the details of these inequalities below.

The CGLMP inequality we used is from Ref. [3], which takes the form of

$$\mathrm{P}(A_2 \geqslant B_2) + \mathrm{P}(B_2 \geqslant A_1) + \mathrm{P}(A_1 \geqslant B_1) + \mathrm{P}(B_1 > A_2) \leqslant 3.$$

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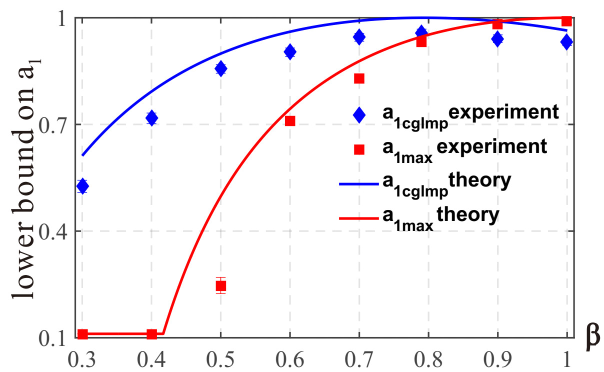

Figure C1 Results of lower bounding a_1 for the qutrit-qutrit states $|\Phi(\beta)\rangle$ with $\beta \in [0.3, 1]$. Experimental results are marked as blue and red dots for the two inequalities respectively, and the theoretical predictions (produce quantum correlations using perfect quantum states and measurements, then apply our method) are plotted in the blue and red lines. The error bars are calculated from 100 simulations of Poisson statistics.

The Bell inequality tailored to maximally entangled states is from Ref. [4], which reads

$$I_{d,m} = \sum_{k=0}^{\lfloor d/2 \rfloor -1} (\alpha_k \sum_{i=0}^{m} [P(A_i = B_i + k) + P(B_i = A_{i+1} + k)] - \beta_k \sum_{i=0}^{m} [P(A_i = B_i - k - 1) + P(B_i = A_{i+1} - k - 1)]),$$

where $\alpha_k = \frac{1}{2d} \tan(\frac{\pi}{2m})[g(k) - g(\lfloor \frac{d}{2} \rfloor)]$ and $\beta_k = \frac{1}{2d} \tan(\frac{\pi}{2m})[g(k+1-1/m) + g(\lfloor \frac{d}{2} \rfloor)]$ (with $g(x) = \cot(\pi(x + 1/2m)/d)$).

Last, the MABK inequality was defined in Refs. [5–7], and we are using its form for three-qubit quantum states, which reads

$$I_{MABK} = |\langle A_1 B_1 C_2 + A_1 B_2 C_1 + A_2 B_1 C_1 - A_2 B_2 C_2 \rangle| / 2 \leqslant 1.$$

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