SCIENCE CHINA
Information Sciences

• LETTER •

July 2023, Vol. 66 179204:1–179204:2
https://doi.org/10.1007/s11432-021-3413-x

Consensus of hybrid linear multi-agent systems with periodic jumps

Ying ZHANG[1,3] & Youfeng SU[2*]

[1]*Center for Discrete Mathematics, Fuzhou University, Fuzhou 350116, China;*
[2]*College of Computer and Data Science, Fuzhou University, Fuzhou 350116, China;*
[3]*School of Mathematics and Statistics, Fuzhou University, Fuzhou 350116, China*

Received 19 July 2021/Revised 16 October 2021/Accepted 23 December 2021/Published online 7 December 2022

Citation Zhang Y, Su Y F. Consensus of hybrid linear multi-agent systems with periodic jumps. Sci China Inf Sci, 2023, 66(7): 179204, https://doi.org/10.1007/s11432-021-3413-x

Dear editor,

Multi-agent systems have attracted extensive attention in control communities due to their wide range of applications, such as multiple spacecraft systems [1] and wireless sensors [2]. The consensus problem, which aims to drive the states of all agents to a common trajectory asymptotically, is one of the most fundamental cooperative control problems. So far, most results on consensus problem mainly focused on either continuous-time or discrete-time multi-agent systems individually [3–6]. In modeling real-world phenomena, it is more usual to consider a type of system that exhibits characteristics of both continuous-time (flow) dynamics and discrete-time (jump) dynamics, called the hybrid system [7]. A detailed review is given in Appendix B.

This study aims to investigate the consensus problem for hybrid linear multi-agent systems with periodic jumps over a directed communication graph. The distinguishing features for studying this class of multi-agent systems are of two aspects. On one hand, many interesting mechanical systems, such as multiple spinning and bouncing disks [8] and multiple RC circuits [9], can neither be modeled by the continuous-time model nor the discrete-time model individually, but have to be described in the hybrid sense. Hence, our study provides a systematic method for consensus problems of these practical multi-agent models. On the other hand, several control areas can be embraced by a hybrid system [7], such as sampled-data control systems and periodic impulsive systems. Hence, our study also gives a unified synthesis process for these consensus scenarios.

Technically, we establish the consensus protocol in the hybrid sense, containing both flow and jump dynamics. The technical novelties are of two aspects. Firstly, both the hybrid distributed state feedback and output feedback control laws are developed so as to deal with the hybrid structure of the plant, which is stabilizable and detectable in the hybrid sense, while neither its flow dynamics nor jump dynamics need to be stabilizable and detectable. In particular, a novel hybrid distributed observer combining both continuous output and discrete output is developed without requiring either of them to be detectable. Secondly, novel feedback and observer gain assignment algorithms that involve the modified H_∞ type Riccati inequality are proposed, where the obligatory elementary transformation is included, and properties of controllable and observable subspaces are utilized. The main contributions are detailed in Appendix D.

Problem formulation. Consider a class of hybrid linear multi-agent systems governed by the flow dynamics

$$\dot{\tau} = 1,\ \dot{x}_i = Ax_i + Bu_{Fi},\ i = 1, \ldots, N, \tag{1a}$$

whether $(\tau, x_i) \in [0, \tau_d] \times \mathbb{R}^n$ and the jump dynamics

$$\tau^+ = 0,\ x_i^+ = Ex_i + Fu_{Ji},\ i = 1, \ldots, N, \tag{1b}$$

whether $(\tau, x_i) \in \{\tau_d\} \times \mathbb{R}^n$, where $\tau \in \mathbb{R}$, $x_i \in \mathbb{R}^n$, $u_{Fi} \in \mathbb{R}^{m_1}$, and $u_{Ji} \in \mathbb{R}^{m_2}$ are the clock variable, state, flow input, and jump input of the i-th subsystem, respectively. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m_1}$, $E \in \mathbb{R}^{n\times n}$, and $F \in \mathbb{R}^{n\times m_2}$ are all constant matrices. $\tau_d > 0$ is a known constant that represents the dwell-time between two consecutive jumps. All solutions to system (1a) and (1b) are defined on the common hybrid time domain $\mathcal{T} := \{(t, k) : t \in [t_k, t_{k+1}], k \in \mathbb{N}, t_k := k\tau_d\}$. The measurable outputs of system (1a) and (1b) are defined as

$$y_{Fi}(t, k) := C_F x_i(t, k), \tag{1c}$$
$$y_{Ji}(k) := C_J x_i(t_k, k - 1),\ i = 1, \ldots, N, \tag{1d}$$

where $y_{Fi}(t, k) \in \mathbb{R}^{q_1}$, $y_{Ji}(k) \in \mathbb{R}^{q_2}$, $C_F \in \mathbb{R}^{q_1\times n}$, and $C_J \in \mathbb{R}^{q_2\times n}$ are constant matrices. Four motivating examples are collected in Appendix C so as to illustrate the practical motivation of the hybrid system (1). The information exchange among agents is described by a directed communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the normalized weighted adjacency matrix $\Omega = [\omega_{ij}] \in \mathbb{R}^{N\times N}$, where their definitions can be found in Appendix E.

Problem 1 (Hybrid consensus)**.** Given the hybrid linear multi-agent system (1) with a directed communication graph $\mathcal{G}$, find a distributed dynamic feedback control pair

* Corresponding author (email: yfsu@fzu.edu.cn)

(u_{Fi}, u_{Ji}) such that, for any initial conditions, the closed-loop system has the property that $\lim_{t+k\to\infty}(x_i(t,k) - x_j(t,k)) = 0$, $i, j = 1, \ldots, N$.

Assumption 1. Each subsystem of the hybrid linear multi-agent system (1) is stabilizable and detectable.

Assumption 2. The communication graph $\mathcal{G}$ contains a directed spanning tree.

Both Assumptions 1 and 2 are very mild. In particular, Appendix F has shown PBH tests for the stabilizability and detectability of hybrid system (1), which indeed cover the standard stabilizability and detectability PBH tests for the linear time-invariant system (see Appendix J).

Distributed dynamic state feedback control. Assume that for each agent, the full states of itself and its neighbors are available for feedback. We consider the distributed dynamic state feedback control law with flow dynamics

$$\dot{\tau} = 1, \quad \dot{\xi}_i = -A^{\mathrm{T}}\xi_i, \tag{2a}$$

whether $(\tau, x_i, \xi_i) \in [0, \tau_d] \times \mathbb{R}^n \times \mathbb{R}^n$, jump dynamics

$$\tau^+ = 0, \quad \xi_i^+ = \mathrm{e}^{A^{\mathrm{T}}\tau_d}\bar{K}_F \sum_{j=1}^{N} \omega_{ij}(x_i - x_j), \tag{2b}$$

whether $(\tau, x_i, \xi_i) \in \{\tau_d\} \times \mathbb{R}^n \times \mathbb{R}^n$, and controller output

$$u_{Fi} = B^{\mathrm{T}}\xi_i, \quad u_{Ji} = K_J \sum_{j=1}^{N} \omega_{ij}(x_i - x_j), \tag{2c}$$

where $\bar{K}_F$ and K_J are determined by Algorithm H1 of Appendix H.

Distributed dynamic output feedback control. Assume that for each agent, only the outputs of itself and its neighbors are available for feedback. We consider the distributed dynamic output feedback control law with flow dynamics

$$\dot{\tau} = 1,\ \dot{\hat{x}}_i = A\hat{x}_i + Bu_{Fi},\ \dot{\zeta}_i = C_F^{\mathrm{T}}\varphi_{Fi} - A^{\mathrm{T}}\zeta_i,\ \dot{\xi}_i = -A^{\mathrm{T}}\xi_i, \tag{3a}$$

whether $(\tau, \hat{x}_i, \zeta_i, \xi_i) \in [0, \tau_d] \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, where $\varphi_{Fi} = \sum_{j=1}^{N} \omega_{ij}(C_F\hat{x}_i - y_{Fi} - C_F\hat{x}_j + y_{Fj})$, jump dynamics

$$\begin{aligned} &\tau^+ = 0,\ \hat{x}_i^+ = E\hat{x}_i + Fu_{Ji} + \bar{L}_F\mathrm{e}^{A^{\mathrm{T}}\tau_d}\zeta_i + L_J\varphi_{Ji}, \\ &\zeta_i^+ = 0,\ \xi_i^+ = \mathrm{e}^{A^{\mathrm{T}}\tau_d}\bar{K}_F \sum_{j=1}^{N} \omega_{ij}(\hat{x}_i - \hat{x}_j), \end{aligned} \tag{3b}$$

whether $(\tau, \hat{x}_i, \zeta_i, \xi_i) \in \{\tau_d\} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, where $\varphi_{Ji} = \sum_{j=1}^{N} \omega_{ij}(C_J\hat{x}_i - y_{Ji} - C_J\hat{x}_j + y_{Jj})$, and controller output

$$u_{Fi} = B^{\mathrm{T}}\xi_i, \quad u_{Ji} = K_J \sum_{j=1}^{N} \omega_{ij}(\hat{x}_i - \hat{x}_j), \tag{3c}$$

where $\bar{K}_F$ and K_J are the same as those in (2), and $\bar{L}_F$ and L_J can be determined by Algorithm H2 of Appendix H. Here observers with the states $\hat{x}_i$ in (3) are of the hybrid form containing both flow and jump dynamics.

With real matrices $\tilde{A}$, $\tilde{B}_v$, $\bar{A}$, $\bar{C}_v$, and complex numbers λ_i, $i = 2, \ldots, N$, defined in Algorithms H1 and H2, the main theorem is given as follows.

Theorem 1. Assume that Assumptions 1 and 2 hold. Let

$$\begin{aligned} \delta_c = \sup_{\delta>0}\{\delta | \exists P > 0 \text{ s.t.} \\ \tilde{A}^{\mathrm{T}}P\tilde{A} - P - (1-\delta^2)\tilde{A}^{\mathrm{T}}P\tilde{B}_v(\tilde{B}_v^{\mathrm{T}}P\tilde{B}_v)^{-1}\tilde{B}_v^{\mathrm{T}}P\tilde{A} < 0\}. \end{aligned}$$

If there exists $\alpha_c \in \mathbb{R}$ satisfying the inequality $|1 - \alpha_c\lambda_i| < \delta_c$, Problem 1 is solved by the distributed dynamic state feedback control law (2). In addition, let

$$\begin{aligned} \delta_o = \sup_{\delta>0}\{\delta | \exists Q > 0 \text{ s.t.} \\ \bar{A}Q\bar{A}^{\mathrm{T}} - Q - (1-\delta^2)\bar{A}Q\bar{C}_v^{\mathrm{T}}(\bar{C}_vQ\bar{C}_v^{\mathrm{T}})^{-1}\bar{C}_vQ\bar{A}^{\mathrm{T}} < 0\}. \end{aligned}$$

If there exists $\alpha_o \in \mathbb{R}$ satisfying the inequality $|1 - \alpha_o\lambda_i| < \delta_o$, Problem 1 is solved by the distributed dynamic output feedback control law (3).

The proof of Theorem 1 can be found in Appendix I. The validity of Algorithms H1 and H2 is discussed in Remarks 3 and 4 of Appendix H, respectively. The robustness of our design is discussed in Remark 8 of Appendix I. Two corollaries of Theorem 1 are given in Appendix J, which provide the solvability of the sampling consensus problem of continuous-time linear multi-agent systems and the consensus problem of discrete-time linear multi-agent systems, respectively. As a practical application, the consensus of four bouncing disks moving on a horizontal plane between parallel walls is given in Appendix K to illustrate the proposed consensus algorithms.

Conclusion. The consensus problem for a class of hybrid linear multi-agent systems has been studied. By utilizing the modified H_∞ type Riccati inequalities, both distributed dynamic state feedback and distributed dynamic output feedback control laws have been presented to solve this problem. Our results extend the consensus studies from purely continuous-time or discrete-time multi-agent systems to hybrid multi-agent systems. Future studies will focus on the hybrid linear multi-agent systems subject to jointly-connected switching communication graphs. The hybrid multi-agent systems with nonlinear dynamics and time-varying periods are the other two significant future extensions.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61773122, 62173092).

Supporting information Appendixes A–K. The supporting information is available online at **info.scichina.com** and **link.springer.com**. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Cai H, Huang J. Leader-following adaptive consensus of multiple uncertain rigid spacecraft systems. Sci China Inf Sci, 2016, 59: 010201
2 Zhang Z, Li J X, Liu L. Distributed state estimation and data fusion in wireless sensor networks using multi-level quantized innovation. Sci China Inf Sci, 2016, 59: 022316
3 Hengster-Movric K, You K, Lewis F L, et al. Synchronization of discrete-time multi-agent systems on graphs using Riccati design. Automatica, 2013, 49: 414–423
4 Ma C-Q, Zhang J-F. Necessary and sufficient conditions for consensusability of linear multi-agent systems. IEEE Trans Automat Contr, 2010, 55: 1263–1268
5 Zhang H W, Lewis F L, Das A. Optimal design for synchronization of cooperative systems: state feedback, observer and output feedback. IEEE Trans Automat Contr, 2011, 56: 1948–1952
6 Zhang Y, Su Y F. Cooperative output regulation for linear uncertain MIMO multi-agent systems by output feedback. Sci China Inf Sci, 2018, 61: 092206
7 Goebel R, Sanfelice R G, Teel A R. Hybrid Dynamical Systems: Modeling, Stability, and Robustness. Princeton: Princeton University, 2012
8 Possieri C, Teel A R. Structural properties of a class of linear hybrid systems and output feedback stabilization. IEEE Trans Automat Contr, 2017, 62: 2704–2719
9 Carnevale D, Galeani S, Menini L, et al. Hybrid output regulation for linear systems with periodic jumps: solvability conditions, structural implications and semi-classical solutions. IEEE Trans Automat Contr, 2016, 61: 2416–2431