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Fault estimator design based on an iterative-learning scheme according to the forgetting factor for nonlinear systems

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Dear editor,

With the rapid development of artificial intelligence and intelligent manufacturing, industrial systems have become more complex and costly. The occurrence of the fault will result in a decrease in system performance, equipment damage, or even safe accidents, resulting in severe property losses and casualties. Fault-tolerant control (FTC) is an effective technique to keep the system operating performance and improve the reliability and safety of the operating system when a fault occurs. Hence, more and more scholars pay attention to it, and many kinds of research have been published [1, 2].

Fault estimation is the most important step of FTC. It can reconstruct the fault signal and aid in fault accommodation or reconfiguration to compensate for the fault by designing a new controller to ensure the required stability and performance. Many scholars have drawn attention to fault estimation and have given massive approaches [3–8]. In these methods, the iterative-learning-based method has particular advantages. As the information of the previous iterations is employed in the current iteration, the iterative-learning-based method can obtain estimation results of high precision.

Many industrial systems operate in a complex environment and there will inevitably exist some noises and initial error which will effect the system's performance and the estimation results. For the iterative-learning-based method, these undesired factors will be accumulated into the final result with the iteration index increasing. To deal with this problem, the forgetting fatcor should be considered in fault estimation. The forgetting factor is a weight factor used in fault estimation. It describes the proportions of dwindling historical data and retaining new data in the iterative process.

In this study, the forgetting factor is considered in the estimator design for reducing the adverse effect brought by initial error and noises. Other work of this study is as follows. First, the state-space equation of nonlinear system is introduced. The actuator fault, noises, and initial error are considered in the system model. Then, a fault estimator based on the iterative-learning scheme is designed. Finally, the convergence condition of the proposed method is solved by Bellman-Gronwall lemma [9] and the λ-norm theory. The effect of forgetting factor is also verified.

Model and methodology. Based on the research results of [7, 8], we consider the following kind of system and the Luenberger observer of it:

$$\begin{aligned} \dot{x}_k(t) &= Ax_k(t) + g\left(x_k(t), t\right) + Bu_k(t) \\ &\quad + Ef_k(t) + w_k(t), \\ y_k(t) &= Cx_k(t) + v_k(t), \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{\hat{x}}_k(t) &= A\hat{x}_k(t) + g(\hat{x}_k(t), t) + Bu_k(t) \\ &\quad + E\hat{f}_k(t) + L(y_k(t) - \hat{y}_k(t)), \\ \hat{y}_k(t) &= C\hat{x}_k(t), \end{aligned} \tag{2}$$

where x denotes the state variable and $\hat{x}$ is the estimation of state variable. k is the number of iterations and x_k denotes the state variable in the kth interaction. u and y represent the control input and measurement output; $\hat{y}$ is the estimation of output. w and v denote the state noise and measurement noise, respectively. f is the fault signal, $\hat{f}$ denotes the estimation of fault. t denotes the time variable and $t \in [0, T]$. A, B, C, E and L are constants or constant matrices. For consistency of description, $g\left(x_k(t), t\right)$ is designed as Lipschitz nonlinear term and Assumption 1 is met.

Assumption 1. If function g is a Lipschitz function, the following inequality is satisfied:

$$\left\|g\left(x_i(t), t\right) - g\left(x_j(t), t\right)\right\| \leqslant \delta \left\|x_i(t) - x_j(t)\right\|, \tag{3}$$

where δ is the Lipschitz constant.

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Remark 1. Eqs. (1) and (2) describe a class of repetitive nonlinear system and its observer while $g(\hat{x}_k(t),t)\neq 0$. Linear systems can be regarded as a special case of (1) and (2) as $g(\hat{x}_k(t),t)=0$.

Assumption 2. The noises are bounded with $\|w\|\leqslant b_w$ and $\|v\|\leqslant b_v$.

The initial error is given as $e_k^0=x_k(0)-\hat{x}_k(0)$ and is bounded with $\|e_k^0\|\leqslant b_{e_0}$.

Definition 1. The λ norm of vector function $h:[0,T]\rightarrow R^m$ is defined as follows:

$$\|h\|_\lambda=\sup_{t\in[0,T]}\{\mathrm{e}^{-\lambda t}\|h(t)\|\},\quad \lambda>0. \tag{4}$$

For vector function $h:[0,T]\rightarrow\mathbb{R}^m$, the λ norm of it meets the following inequality:

$$\|h\|_\lambda\leqslant\sup_{t\in[0,T]}\|h(t)\|\leqslant\mathrm{e}^{\lambda T}\|h\|_\lambda. \tag{5}$$

Then the fault estimator is designed as

$$\begin{aligned}\hat{f}_{k+1}(t)&=(1-\theta)\,\hat{f}_k(t)+\theta\hat{f}_0(t)\\&\quad+\gamma\Delta\dot{y}_k(t)+\eta\Delta y_k(t),\end{aligned} \tag{6}$$

where $\Delta y_k(t)$ is defined as $\Delta y_k(t)=y_k(t)-\hat{y}_k(t)$, γ and η are the coefficients for the differential term and proportional term, respectively, and θ is the potential factor.

Theorem 1. Considering system (1), observer (2) and fault estimator (6), the error of fault estimation $r_k(t)=f(t)-\hat{f}_k(t)$ converges to a small range if the following conditions are satisfied:

(i) The number of iteration k is sufficiently large;

(ii) $\|(1-\theta)-\gamma EC\|<1$.

Then, the following equation holds:

$$\lim_{k\rightarrow\infty}\|r_k(t)\|_\lambda=0. \tag{7}$$

Proof. Based on (1) and the λ-norm theory, one can get that

$$\begin{aligned}\|\dot{e}_k(t)\|&\leqslant(\|(A-LC)\|+\delta)\,\|e_k(t)\|+\|L\|\,\|v(t)\|\\&\quad+\|E\|\,\|r_k(t)\|+\|w(t)\|\,,\\\|\Delta y_k(t)\|&\leqslant\|C\|\,\|e_k(t)\|+\|L\|\,\|v(t)\|\,.\end{aligned} \tag{8}$$

The error of fault estimation can be described as

$$\begin{aligned}\|r_{k+1}(t)\|&\leqslant\|1-\theta-\gamma CE\|\,\|r_k(t)\|+|\theta|\,\|r_0(t)\|\\&\quad+\|\gamma(A-LC)+\gamma\delta-\eta\|\,\|C\|\,\|e_k(t)\|\\&\quad+\|\gamma C\|\,\|w_k(t)\|+\|\gamma\|\,\|\dot{v}_k(t)\|\,.\end{aligned} \tag{9}$$

Based on Bellman-Gronwall lemma [9], substituting (1) into (2), we can obtain

$$\begin{aligned}&\|r_{k+1}(t)\|\\&\leqslant\rho\,\|r_k(t)\|+\theta\,\|r_0(t)\|+c_2b_w+c_3+c_4b_{e_0}\mathrm{e}^{c_1t}\\&\quad+c_4\int_0^t\mathrm{e}^{c_1(t-\tau)}(b_e\,\|r_k(\tau)\|+b_w+\|L\|\,b_v)\mathrm{d}\tau,\end{aligned} \tag{10}$$

where $\rho=\|1-\theta-\gamma CE\|$, $c_2=\|\gamma C\|$, $c_3=\sup_{t\in[0,T]}\|\gamma\|\|\dot{v}_k(t)\|$, $c_4=\|\gamma(A-LC)+\gamma\delta-\eta\|\,\|C\|$.

Multiplying the both sides of (10) with $\mathrm{e}^{-\lambda t}$, the following equation can be obtained based on Definition 1:

$$\begin{aligned}\|r_{k+1}(t)\|_\lambda&=\left(\rho+c_4b_e\frac{1-\mathrm{e}^{(c_1-\lambda)T}}{\lambda-c_1}\right)\|r_k(t)\|_\lambda\\&\quad+\theta\|r_0(t)\|_\lambda+(c_2b_w+c_3+c_4b_{e_0}\mathrm{e}^{c_1t})\mathrm{e}^{-\lambda t}\\&\quad+c_4(b_w+\|L\|\,b_v)\frac{1-\mathrm{e}^{(c_1-\lambda)T}}{\lambda-c_1}\|I\|_\lambda.\end{aligned} \tag{11}$$

To simplify (11), the parameter $\bar{\rho}$ is defined as $\bar{\rho}=\rho+c_4b_e\frac{1-\mathrm{e}^{(c_1-\lambda)T}}{\lambda-c_1}$ and the additional term is designed as $\varepsilon=\theta\|r_0(t)\|_\lambda+(c_2b_w+c_3+c_4b_{e_0}\mathrm{e}^{c_1t})\mathrm{e}^{-\lambda t}+c_4(b_w+\|L\|\,b_v)\frac{1-\mathrm{e}^{(c_1-\lambda)T}}{\lambda-c_1}\|I\|_\lambda$. Then, one can get that

$$\|r_k(t)\|_\lambda<\frac{\varepsilon}{1-\bar{\rho}}. \tag{12}$$

If the parameter $\lambda\rightarrow\infty$ and conditions (i) and (ii) are satisfied, the following conclusion can be obtained:

$$\lim_{k\rightarrow\infty}\|r_k(t)\|_\lambda=0. \tag{13}$$

Conclusion. In this study, an iterative-learning-based fault estimator with the forgetting factor is proposed in response to the requirement of fault estimation in a complex system. The iterative-learning scheme is verified to have high precision, and a forgetting factor is introduced to reduce the adverse effect of noises and initial error. Furthermore, in Theorem 1, the proposed method's convergence condition is solved and the effectiveness of the proposed fault estimator is demonstrated.

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References

1 Gao Z, Cecati C, Ding S X. A survey of fault diagnosis and fault-tolerant techniques-part I: fault diagnosis with model-based and signal-based approaches. IEEE Trans Ind Electron, 2015, 62: 3757–3767
2 Barrett J D. Diagnosis and fault-tolerant control. IEEE Trans Autom Control, 2004, 49: 493–494
3 Mao Z, Wang Y, Jiang B, et al. Fault diagnosis for a class of active suspension systems with dynamic actuators' faults. Int J Control Autom Syst, 2016, 14: 1160–1172
4 Hwang I, Kim S, Kim Y, et al. A survey of fault detection, isolation, and reconfiguration methods. IEEE Trans Contr Syst Technol, 2010, 18: 636–653
5 Jiang B, Chowdhury F N. Fault estimation and accommodation for linear MIMO discrete-time systems. IEEE Trans Contr Syst Technol, 2005, 13: 493–499
6 Yang H, Luo H, Kaynak O, et al. Adaptive SMO-based fault estimation for Markov jump systems with simultaneous additive and multiplicative actuator faults. IEEE Syst J, 2021, 15: 607–616
7 Jiang B, Staroswiecki M, Cocquempot V. Fault accommodation for nonlinear dynamic systems. IEEE Trans Automat Contr, 2006, 51: 1578–1583
8 Li F, Kenan D, Shuiqing X, et al. A nonrepetitive fault estimation design via iterative learning scheme for nonlinear systems with iteration-dependent references. Neural Comput Applic, 2022, 34: 5169–5179
9 Debeljkovic D L J, Kablar N A. Finite-time stability of linear singular systems: Bellman-Gronwall approach. In: Proceedings of the American Control Conference, 1999. 3: 1803–1806