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A model predictive control strategy with switching cost functions for cooperative operation of trains

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Abstract The cooperative control of trains is proposed as an innovative method for further improving operation efficiency. Model predictive control (MPC) has been widely discussed for multiple trains because it can handle the challenges posed by the cooperative control problem, such as complex constraints. In real situations, multiple objectives, such as comfort and safety, must be considered when controlling multiple trains with MPC, and the total objective may change during operation, affecting control performance. In this paper, a distributed structure based on switching cost function model predictive control (ScMPC) for multiple trains in a switching situation is given, where the cost functions of the train control problem change with the variable demand of cooperative operation. Furthermore, the feasibility of the proposed method and stability of the closed-loop system are proved to guarantee the stable operation of the controlled trains. Finally, the control method's effectiveness is verified. Three kinds of cost functions are given, and their control performance is compared to show the effect of different weights and the advantage of ScMPC.

Keywords cooperative control, switching cost functions, model predictive control, automatic train operation, railway train

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1 Introduction

Railway has become one of the most popular forms of transportation worldwide due to its rapidity, economy, and punctuality. Because of the growing demand for higher efficiency and better service, the automation level of train operation attracts much attention, and it has recently been improved, leading to more exact and efficient train operation [1,2]. As a typical example, the Jing-Zhang high-speed railway in China was equipped with an automatic train control system. An important achievement is that trains can operate automatically at 350 km/h on this line. Therefore, research on exact and stable control methods is essential theoretically and practically. Much work focusing on tracking control methods for single-train operation control is given [3, 4]. The designed controllers guarantee that trains can track specific speed profiles under disturbance without considering real-time constraints from other trains.

However, trains are not isolated when they operate on the line, and their competition for resources occurs not only on the line but also in the station. Further reducing the tracking distance between trains and strengthening the connection between trains during operation have become a worldwide and irreversible trend. For example, work on the moving block system and even virtual coupling is included in the Shift2rail project of the European Union. To achieve this target, innovation in communication and advanced control methods is needed to enhance the relationship between different trains in management, information transmission, and cooperative control [5–8]. In these research fields, the cooperative control of trains has recently become a popular topic because it directly relates to operation performance and safety.

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An essential aim of cooperative control methods for multiple trains is improving operation efficiency. Different from a single-train scenario, multiple trains are considered a convoy with the same desired velocity in the cooperative operation mode. In addition, on the basis of the advanced control method and real-time communication technology, a train in the convoy can track the one ahead at a shorter distance after achieving a cooperative state. A series of control methods have been researched for cooperative train control, such as fuzzy control [9], robust control [10], and model predictive control (MPC). The main task of the controller is to guarantee the stable operation of the trains to achieve the cooperative desired states and satisfy various constraints. In many studies, the constraints resulting from line conditions and train motion are transformed into input-constrained and state-constrained problems, and only one type of constraint is considered [11, 12]. Therefore, MPC has been widely discussed recently for its ability to deal with multiple constraints. In [13], an event-triggered MPC strategy is given to control multiple high-speed trains under random switching communication topologies. In [14], an efficient velocity control method for multiple trains under communication constraints is investigated, and an MPC scheme is used to solve the control problem. When applying MPC in the train control field, some researchers pay more attention to the model description. In other words, the proof of the stability and feasibility of the system needs to be completed.

Since train operation control in an MPC structure is described as a multi-objective optimal control problem, particularly in a cooperative scenario, another issue discussed is the choice of cost functions. Several factors, such as energy saving and tracking errors, should be considered at the same time, while some conflicts could occur during the optimal process. Determining an appropriate cost function for the predictor in an MPC structure is vital in controller design, for it is associated with control performance, as shown in [15]. Therefore, many studies have been conducted on the choice of cost functions and weights for trains. Most of these studies place a priority on one kind of cost function, such as mainly focusing on the energy consumption in [16, 17] or the tracking error in [13]. However, the environment of trains and the objectives could change during the operation. In this situation, a single cost function and weight may not satisfy the higher requirement. Particularly in the cooperative control mode, the states of the convoy are more complex and variable. How to guarantee the changing demand and states without the loss of control performance is a challenging task. Therefore, to further improve the control performance of multiple trains in complex and variable situations, an MPC method with switching cost functions for cooperative train control facing changing cost functions is given in this paper.

The main task of designing a controller in an MPC structure with switching cost functions is to prove the feasibility of the solution and the stability of the closed-loop system. In [18], a switched MPC algorithm for nonlinear continuous-time systems is proposed, and essential assumptions need to be satisfied to guarantee the stability of the problem. A time-varying and state-dependent cost function is used in the MPC structure in [19], and the final control action is chosen among the calculated Pareto optimal solutions under the cost function. In [20], a lexicographic multi-objective model predictive control scheme is proposed for nonlinear systems that have to switch objective prioritization. The MPC method with switching cost functions is also used in industrial fields, such as chemical processes [21–23]. However, the mentioned methods cannot be applied directly to train operation control because of the characteristics of train operation, including the unique operation objective and constraints. A suitable MPC structure for multiple trains facing switching cost functions is designed considering the constraints and train dynamic motion.

On the basis of the discussion mentioned above, model predictive controllers with switching cost functions are designed for multiple trains, which should achieve cooperative operation. Compared with the existing literature, the main contribution of this paper is as follows:

- A switching cost function MPC (ScMPC) method is proposed to adapt to the varying cooperative demand better. At the same time, the control performance is guaranteed. Furthermore, trains calculate their control inputs in a distributed way, reducing the computation burden of applications.

- Under switching cost functions, the feasibility and stability of the closed-loop system are verified. In addition, terminal cost, terminal region, and local input are given in the switching condition.

Notation. First, notations are introduced. Let $\|x\|_2$ denote the 2-norm for any vector $x \in \mathbb{R}^n$, where $\mathbb{R}$ is the field of real numbers. Additionally, the weighted norm $\|x\|_p$ is defined as $\|x\|_p := \sqrt{x^{\mathrm{T}} P x}$, where P is any symmetric positive definite matrix and satisfies $P \in \mathbb{R}^{n \times n}$. For any symmetric matrix M, $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote the largest and smallest real parts of its eigenvalue, respectively.

2 Train motion and the distributed ScMPC problem

In this section, the longitudinal dynamic motion for a multiple-train convoy is given. Then the distributed MPC problem with switching cost functions is introduced for the cooperative control of the convoy.

2.1 Longitudinal dynamic motion of trains

Consider a convoy formation that comprises N trains running on a railway line. According to the modeling guidelines given in [24, 25], the longitudinal dynamic motion of the ith train, where $i \in [1, N]$, is defined as follows:

$$\frac{\mathrm{d}p_i(t)}{\mathrm{d}t} = v_i(t), \tag{1a}$$

$$m_i \frac{\mathrm{d}v_i(t)}{\mathrm{d}t} = f_i(t) - m_i R(v_i(t)), \tag{1b}$$

$$\frac{\mathrm{d}f_i(t)}{\mathrm{d}t} = \frac{1}{\iota}\left(F_i(t) - f_i(t)\right), \tag{1c}$$

where $p_i(t)$ and $v_i(t)$ represent the position and velocity of the train i, m_i is the mass of the ith train, $F_i(t)$ and $f_i(t)$ are the controlled force of the servo motor and the integrated driving/braking force, respectively. Moreover, the inertial lag is also considered and defined as ι. The other major external force for train operation is the air resistance $m_i R(v_i(t))$, where $R(v_i(t)) = C_a + C_b v_i(t) + C_c v_i^2(t)$, and C_a, C_b, and C_c are the Davis coefficients that can be calculated beforehand on the basis of historical operation data.

Let $x_{i,1}(t) = p_i(t)$, $x_{i,2}(t) = v_i(t)$ and $x_{i,3}(t) = \frac{1}{m_i} f_i(t) - R(v_i(t))$. Then, the function (1) is reduced to $\dot{x}_{i,1}(t) = x_{i,2}(t)$, $\dot{x}_{i,2}(t) = x_{i,3}(t)$, and $\dot{x}_{i,3}(t) = \frac{1}{m_i \iota}\left(F_i(t) - (1 + \iota(C_b + 2C_c v_i(t)))\, f_i(t)\right) + (C_b + 2C_c v_i(t))\, R(v_i(t))$. Let $F_i(t) = m_i \iota u_i(t) + (1 + \iota(C_b + 2C_c v_i(t)))\, f_i(t) - m_i \iota (C_b + 2C_c v_i(t)) R(v_i(t))$ to compensate for the resistance and inertial lag, and $u_i(t)$ is part of the control input that needs to be designed. In addition, let $\boldsymbol{x}_i(t) = [x_{i,1}(t), x_{i,1}(t), x_{i,3}(t)]^{\mathrm{T}}$ for simplification. When trains operate cooperatively, they should reach the same velocity and maintain the desired distance. For that purpose, the tracking errors of train i in a convoy are given as $\tilde{\boldsymbol{x}}_i(t) = [\tilde{\boldsymbol{x}}_{i,1}(t), \tilde{\boldsymbol{x}}_{i,2}(t), \tilde{\boldsymbol{x}}_{i,3}(t)]^{\mathrm{T}} = [x_{i,1}(t) - x_{i-1,1}(t) + p_{\mathrm{des}}^i, x_{i,2}(t) - x_{i-1,2}(t), x_{i,3}(t) - x_{i-1,3}(t)]^{\mathrm{T}}$ and $\tilde{u}_i(t) = u_i(t) - u_{i-1}(t)$, where p_{des}^i is a constant representing the desired tracking distance between train $i-1$ and train i. In addition, define train 0 as a virtual leading train. In particular, a predefined speed profile $[p_0(t), v_0(t), u_0(t)]$ is given to train 0, and thus $\boldsymbol{x}_0(t)$ and $u_0(t)$ are known. When trains operate cooperatively, the desired state of train 1 is $\boldsymbol{x}_0(t)$. Other trains need to reach $v_0(t)$ and maintain the desired distance from adjacent trains.

Combining tracking errors with the dynamic motion of trains shows the following equation:

$$\dot{\tilde{\boldsymbol{x}}}_i(t) = A\tilde{\boldsymbol{x}}_i(t) + B\tilde{u}_i(t), \quad i = 1, 2, \ldots, N, \tag{2}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

It can be seen that (A, B) is controllable. In addition, on the basis of model (1), a function that can represent the energy consumption of train i in time interval $[0, T]$ is usually defined as

$$W_i = \int_0^T |F_i(t) v_i(t)| \mathrm{d}t, \tag{3}$$

when $F_i(t) > 0$. The braking process of trains is neglected when calculating the energy consumption because of the wide use of regenerative braking systems that can transmit kinetic energy to electricity [26].

To introduce the cooperative operation and ScMPC method for trains, some basic techniques, such as reliable real-time communication, should be provided to meet the information need. In this paper an assumption is made.

Assumption 1. Suppose that all the trains are instrumented with a real-time communication module. In this case, trains in the convoy can instantly send and receive operation states and control commands, as well as real-time changes of the desired speed profile, which also means the prediction problem of each train is solved instantly if the communication is available.

2.2 Distributed MPC with switching cost functions

In real scenarios, the cooperative objective could be affected by the environment and upper demand in real time. Hence, a situation in which the desired objective changes is considered. For example, the desired velocity needs to be adjusted due to external constraints. In this situation, the main task of trains is to track the changed objective as soon as possible. In contrast, if the desired velocity remains constant, the weight of velocity tracking could decrease slightly, which means admitting velocity deviation while increasing the weight of comfort. Therefore, an ScMPC method is chosen to meet variable requirements. Because of limited computational resources, a distributed MPC structure is employed in this paper, where trains calculate their optimal control input in a subsequent order. In contrast to a centralized MPC scheme, the proposed distributed structure reduces the model complexity by dividing the whole control system problem into a series of subsystems control problems while considering the correlation between subsystems. In other words, the constraints between trains in a convoy are guaranteed, and every train has an independent controller.

In Figure 1, the whole structure of the proposed control method for multiple trains is given. This structure comprises three main parts, upper demand, cost switching, and cooperative calculation. The upper demand determines the whole objective of the cooperative trains. Since the requirements and line conditions are complex and variable, the objective may change and switch during operation. Therefore, it is further divided into stage objectives based on the switching time. Specifically, when the cost switching part receives a sudden change signal from the upper demand part, it will adjust the prediction cost and then trigger switches between stage objectives to improve control performance. The switched real-time stage objective is sent to the cooperative calculation part. Finally, on the basis of that objective, the predictor and controller in an MPC structure calculate the control input for every train in the convoy subsequently.

The main procedure of the control progress can be described as follows. Dynamic function (2) is used as a prediction model in the MPC structure during the operation. After receiving a switching signal, the cost function used in the prediction part, which is determined by the real-time requirements, is changed and transferred into the cooperative calculation part. Furthermore, trains in the convoy will operate according to the switched cost functions. By minimizing the cost function given in Section 2 under constraints, the control input $u_i(t)$ is optimized, and then the control force $F_i(t)$ is calculated according to $u_i(t)$ and applied to the ith train. Specifically, in a sampling period, the first train transmits its states and control input to the second train, which calculates its predictive states and input based on that information. The processor will repeat until the last train in the convoy obtains its control input. Define a set of $j \in \Lambda := \{1, 2, \ldots, M\}$ different cost functions. At each sampling instant, one cost function is chosen to be minimized, which means that switches occur at instants t_k. Let $\sigma(\cdot)$ denote the switching signal, which is a piecewise constant function. Its value changes when the switching command is detected. Following the definition in [27, 28], let $N_{\sigma(t_1,t_2)}$ denote the total number of switchings in time interval $(t_1, t_2]$. In time interval $(0, T]$, the switching times can be written as $\tau_1, \tau_2, \ldots, \tau_{N_{\sigma(0,T)}}$. Furthermore, the value of time units that lies in two switches on average is defined as τ_a. The index of the active cost function in a time interval $[\tau_j, \tau_{j+1})$ is defined as j. Notably, the switching time τ_j in which different cost functions are switched is determined according to the operation scenarios, and it should coincide with some of the sampling time t_k, which means that $\tau_j = t_k$ should hold for $j = 0, 1, \ldots$ and some $k = 0, 1, \ldots$.

On the basis of the above analysis, the optimal control problem of trains in an ScMPC framework is described as (4) at time t_k, where T_p is the prediction horizon, and $t_{k+1} - t_k := \delta \leqslant T_p$ is the sampling time. T_c is the control horizon, which can be equal to or less than T_p. Here $T_c = T_p := T$ with $T = h\delta, h = 1, 2, \ldots$ is chosen in the theoretical proof. In this work, a continuous MPC structure is used to achieve the desired control performance. In addition, the train dynamics motion (1) and the state equation (2) show that the train control system can be mathematically described as a continuous-time

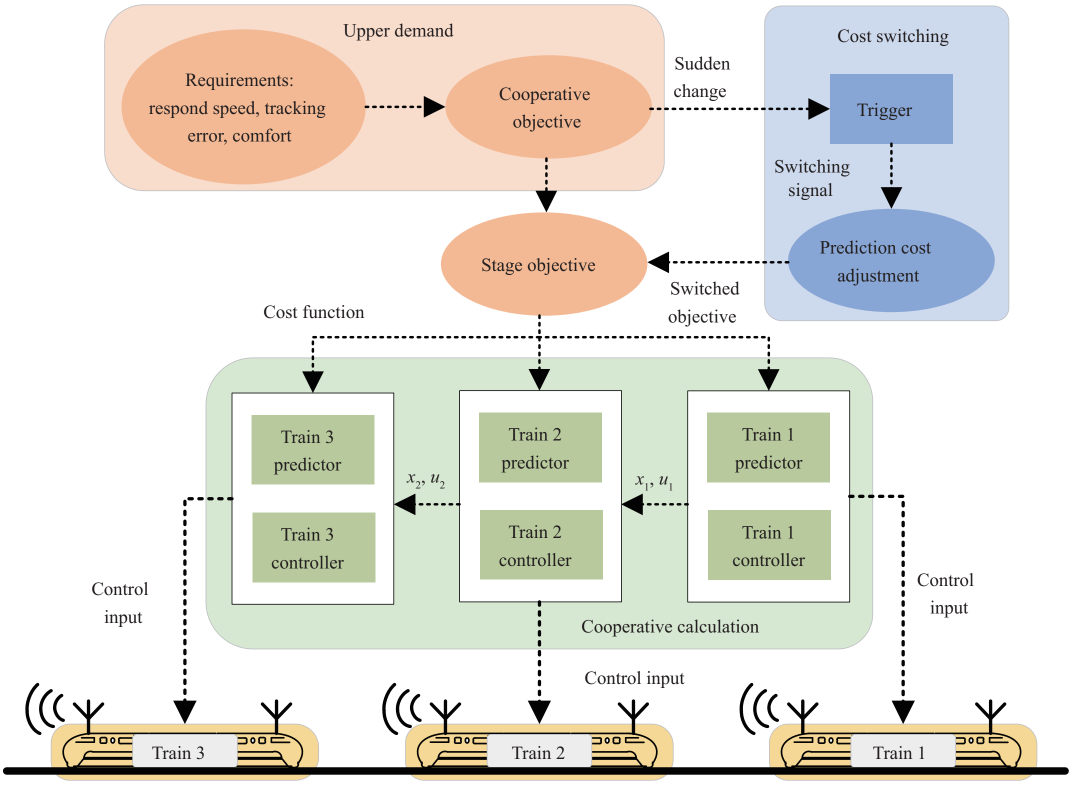

Figure 1 (Color online) ScMPC structure for multiple trains.

form,

$$
\begin{aligned}
&\min_{u_i(\cdot|t_k)} J_{i,j}\left(\boldsymbol{x}_i(\cdot|t_k), u_i(\cdot|t_k)\right) = \int_{t_k}^{t_k+T} L_{i,j}\left(\tilde{\boldsymbol{x}}_i(s|t_k), \bar{u}_i(s|t_k)\right) \mathrm{d}s + F_{i,j}^f\left(\tilde{\boldsymbol{x}}_i(t_k+T|t_k)\right),\\
&\dot{\tilde{\boldsymbol{x}}}_i(s|t_k) = A\tilde{\boldsymbol{x}}_i(s|t_k) + B\tilde{u}_i(s|t_k),\\
&\tilde{\boldsymbol{x}}_i(t_k|t_k) = \tilde{\boldsymbol{x}}_i(t_k),\\
&(\tilde{\boldsymbol{x}}_i(s|t_k), \bar{u}_i(s|t_k)) \in \mathcal{Z}_{i,j},\ \forall s \in [t_k, t_k+T],\\
&\tilde{\boldsymbol{x}}_i(t_k+T|t_k) \in \Omega_{i,j}^f,
\end{aligned}
\tag{4}
$$

where $\tilde{\boldsymbol{x}}_i(\cdot|t_k)$ and $\bar{u}_i(\cdot)$ represent the predicted states and input of train i over the prediction horizon, respectively, $J_{i,j}\left(\boldsymbol{x}_i(\cdot|t_k), u_i(\cdot|t_k)\right)$ is the cost function for each train, $L_{i,j}\left(\tilde{\boldsymbol{x}}_i(s|t_k), \bar{u}_i(s|t_k)\right)$ is a continuous and positive definite stage cost at any non-switching time, and $\mathcal{Z}_{i,j} = \mathcal{X}_{i,j} \times \mathcal{U}_{i,j}$ is the state and input constraint sets. Moreover, state region $\mathcal{X}_{i,j} \subseteq \mathbb{R}^3$ and input region $\mathcal{U}_{i,j} \subseteq \mathbb{R}^1$ contain the origin. The positive definite terminal cost $F_{i,j}^f\left(\tilde{\boldsymbol{x}}_i(t_k+T|t_k)\right)$ is related to the terminal region $\Omega_{i,j}^f$, which is set to guarantee the stability of the system with $\tilde{\boldsymbol{x}}_i(t_k+T|t_k) \in \Omega_{i,j}^f$ required. By solving problem (4), an optimal input and state trajectories for train i can be obtained and defined as $U_i^*(t_k) = [u_i^*(t_k), u_i^*(t_k+\delta), \ldots, u_i^*(t_k+T-\delta)]$ and $X_i^*(t_k) = [\boldsymbol{x}_i^*(t_k+\delta), \ldots, \boldsymbol{x}_i^*(t_k+T)]$, respectively, in which the subscript j means the problem is solved with the cost function J_j. The optimal input $U_0^*(t_k) = [u_0(t_k), u_0(t_k+\delta), \ldots, u_0(t_k+T-\delta)]$ and state $X_0^*(t_k) = [\boldsymbol{x}_0(t_k+\delta), \ldots, \boldsymbol{x}_0(t_k+T)]$ of train 0 are also known variables.

In most situations, trains are expected to track the desired speed profile and keep a predefined distance from adjacent trains, so a major objective is to reduce the tracking error. In addition, considering traveler satisfaction, the change rate of acceleration and deceleration should also be included. Since $u_i(t)$ is a part of the derivation of $\boldsymbol{x}_{i,3}(t)$, which represents the acceleration/deceleration value, it should be included as an objective in the MPC structure. The cost functions and constraint sets are given according to the above characteristics of train operation. Define $L_{i,j}\left(\tilde{\boldsymbol{x}}_i(t), \bar{u}_i(t)\right)$ and $F_{i,j}^f(\boldsymbol{x}_i(t))$ as follows:

$$
L_{i,j}(\tilde{\boldsymbol{x}}_i(t), \bar{u}_i(t)) = \|\tilde{\boldsymbol{x}}_i(t)\|_{Q_{i,j}}^2 + \|\bar{u}_i(t)\|_{R_{i,j}}^2, \tag{5}
$$

$$F^f_{i,j}(\tilde{\boldsymbol{x}}_i(t)) = \|\tilde{\boldsymbol{x}}_i(t)\|^2_{S_{i,j}}, \tag{6}$$

where $Q_{i,j}$, $R_{i,j}$ and $S_{i,j}$ are weight matrices. For different trains, the choices of weights can differ. Without loss of generality, the same matrices can also be chosen for the trains in the convoy if these trains are usually supposed to achieve the desired states synchronously.

On the basis of the requirement of safety and comfort, constraints (7)–(9) are made for all the trains, and the total state constraint set is $\mathcal{X}_{i,j} = \mathcal{X}^1_{i,j} \cup \mathcal{X}^2_{i,j} \cup \mathcal{X}^3_{i,j}$. The minimum distance between train i and train $i-1$ is defined as $p_{i,\min}$, and this value is chosen based on train parameters and safety conditions. Define $v_{\max}$ as the maximum velocity of the trains which is determined according to the line conditions and the type of trains. The acceleration and deceleration are also limited because of the force bounds. Additionally, comfort is one of the most important factors influencing traveler satisfaction.

$$\boldsymbol{x}_{i,1}(t) \in \mathcal{X}^1_{i,j} = \{\boldsymbol{x}_{i,1}(t) \leqslant \boldsymbol{x}_{i-1,1}(t) - p_{i,\min}\}, \tag{7}$$

$$\boldsymbol{x}_{i,2}(t) \in \mathcal{X}^2_{i,j} = \{0 \leqslant \boldsymbol{x}_{i,2}(t) \leqslant v_{\max}\}, \tag{8}$$

$$\boldsymbol{x}_{i,3}(t) \in \mathcal{X}^3_{i,j} = \{a_{\min} \leqslant \boldsymbol{x}_{i,3}(t) \leqslant a_{\max}\}. \tag{9}$$

Remark 1. The maximum velocity of trains $v_{\max}$ is limited by the automatic train protection (ATP) system in real scenarios. In this method, an active speed limit is set in the proposed MPC method to guarantee the operation safety. Therefore, under the MPC structure, control inputs are adjusted to avoid the appearance of an emergency brake.

As mentioned above, $u_i(t)$ represents the change rate of acceleration and deceleration. Therefore, it is also limited to guarantee the comfort of travelers, and $u_{\min}$ and $u_{\max}$ should be chosen based on line conditions and experience. The constraint is defined as $u_i(t) \in \mathcal{U}_i = \{u_{\min} \leqslant u_i(t) \leqslant u_{\max}\}$.

3 Feasibility and stability analysis

Based on the dynamic motion and control structure given before, the feasibility of control solutions and stability of the closed-loop system is discussed in this section. A terminal controller and constraint sets are given, which can guarantee the iterative feasibility and stability of the control problem (4). Furthermore, the asymptotical stability of the system in the switching cost functions condition is proved.

3.1 Feasible solution and terminal constraints

The closed-loop stability of the traditional MPC scheme has been illustrated in many studies; see [29]. One helpful method to guarantee the asymptotical stability of the system is applying terminal constraints and costs to the optimal control problem in each prediction horizon. The definition of terminal constraints and costs can be found in [30]. A local control law is used with terminal constraints and costs, and the following Lemma is usually given.

Lemma 1 ([29])**.** For the system $\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), u^*(t)), \boldsymbol{x}(t) = \boldsymbol{x}_0$, suppose there is an auxiliary local control law $u^f(t) \in \mathcal{U}, \forall \boldsymbol{x} \in \Omega^f$ satisfying $\frac{\mathrm{d}}{\mathrm{d}t}F^f(\boldsymbol{x}, u^f(t)) + L(\boldsymbol{x}, u^f(t)) \leqslant 0, \forall \boldsymbol{x} \in \Omega^f$. Then the closed-loop system is nominally asymptotically stable in some invariant set Ω^f.

The proof of Lemma 1 can be found in [31]. In a time interval $[t_k, t_{k+1}]$, suppose the active cost function of train i is $J_{i,j}$ for some $j \in \Lambda$, and there is a feasible solution $u_i(t) = u^*_i(t)$ in this time for system (2). Based on the proof in [29, 32, 33], a feasible solution for a time interval $[t_{k+1}, t_{k+1} + T]$ can be written as

$$\bar{u}_i(\cdot) = \begin{cases} u^*_i(s), & s \in [t_{k+1}, t_k + T], \\ u^f_{i,j}(s), & s \in [t_k + T, t_{k+1} + T], \end{cases} \tag{10}$$

where $u^f_{i,j}(t)$ is the designed terminal control input. Here a traditional feedback form $u^f_{i,j}(t) = k^f_{i,j}\tilde{\boldsymbol{x}}_i(t) - u_{i-1}(t)$ is chosen to guarantee the asymptotic stability of the system. It is easy to obtain that $\dot{\tilde{\boldsymbol{x}}}_i(t) = (A + Bk^f_{i,j})u^f_{i,j}(t)$. In addition, following the method given in [18], a same terminal function $F^f_{i,j}(\tilde{\boldsymbol{x}}_i(t)) = F^f_i(\tilde{\boldsymbol{x}}_i(t))$ and a local control law $u^f_{i,j}(t) = u^f_i(t), \forall j \in \Lambda$ are chosen because it might be difficult for the controller to satisfy the terminal region and the condition in Lemma 1 for different cost functions and local controllers at the same time. In this case, the derivation of $F^f_{i,j}(\tilde{\boldsymbol{x}}_i(t)) = F^f_i(\tilde{\boldsymbol{x}}_i(t))$ is calculated based on (6), which is $\frac{\mathrm{d}}{\mathrm{d}t}F_i^f(\tilde{\boldsymbol{x}}_i(t)) = \tilde{\boldsymbol{x}}_i^{\mathrm{T}}(t)((A+Bk_i^f)^{\mathrm{T}}S_i + S_i(A+Bk_i^f))\tilde{\boldsymbol{x}}_i(t)$. To satisfy the condition given in Lemma 1, the terminal $F_i^f(\tilde{\boldsymbol{x}}_i(t))$ and the terminal region Ω_i^f are calculated such that $\frac{\mathrm{d}}{\mathrm{d}t}F_i^f(\tilde{\boldsymbol{x}}_i(t)) + \max_{j\in\Lambda}L_{i,j}(\tilde{\boldsymbol{x}}_i(t), u_i^f(t)) \leqslant 0, \forall \tilde{\boldsymbol{x}}_i(t) \in \Omega_i^f$. The detailed procedures on how to calculate the gain k_i^f, the terminal cost function $F_i^f(\tilde{\boldsymbol{x}}_i(t))$, and the terminal region Ω_i^f for problem (4) after the transformation can be found in [31, 34].

Remark 2. It is seen that the stable region of the closed-loop system is affected by the values of weights because the choice of weights is directly related to the "three ingredients", i.e., terminal cost, terminal constraints, and local control input [29]. As a result, choosing different cost functions for trains with different cooperative demands can influence the control performance and the stability of the system, for example, a lower weight on position tracking error which has a relatively larger initial value.

When calculating the terminal region and terminal cost, the constraints defined in (7)–(9) are transformed to $\tilde{x}_{i,1}(t) \in \{\tilde{x}_{i,1}(t) \leqslant -p_{\min}^i\}$, $\tilde{x}_{i,2}(t) \in \{-x_{2,i-1}(t) \leqslant \tilde{x}_{i,2}(t) \leqslant v_{\max} - x_{2,i-1}(t)\}$, and $\tilde{x}_{i,3}(t) \in \{a_{\min} - x_{i-1,3}(t) \leqslant \tilde{x}_{i,3}(t) \leqslant a_{\max} - x_{i-1,3}(t)\}$. However, the constraints cannot be used directly for the time-varying bounds. When calculating the terminal set, notice that the desired velocity is always lower than the maximum velocity due to the consideration of safety. Furthermore, the desired states for trains in the convoy are known since the speed profile of train 0 and desired distances are predefined. Therefore, the following transformation is made when calculating the terminal set: $\{\tilde{x}_{i,2}(t) \leqslant \min\{v_{\max} - v_0(t)\}\}$, $\{\tilde{x}_{i,3}(t) \geqslant \min\{\dot{v}_0(t) - a_{\min}\}\}$, and $\{\tilde{x}_{i,3}(t) \leqslant \min\{a_{\max} - \dot{v}_0(t)\}\}$.

Remark 3. For a sufficiently small $\delta > 0$, the feasibility of the open-loop optimal control problem for trains under constraints for all $t > 0$ can be implied by its feasibility at $t = 0$. The designed controller for each train will find a feasible solution with an initial prediction horizon after the operation start. If the initial solution for the problem cannot be found, the prediction horizon will increase until a feasible solution is solved.

3.2 Stability of distributed ScMPC

It is noticed that the change of cost functions influences the offline calculation values, including the terminal region and the local solution, which is shown in Algorithm 1. However, the feasibility of the local control input $u_i^f(t)$ at any time instant does not depend on the switching signals. It also means that the switching MPC problem's recursive feasibility is guaranteed with an arbitrary switching signal. To ensure the asymptotical stability of the closed-loop system, the average dwell-time of the switching signal δ should satisfy some conditions in Lemma 2, while none priori knowledge of δ has to be known before switching.

Algorithm 1 Distributed ScMPC

1: Set $j = k = 0$; initialize states of trains at $t_0 = 0$; obtain the speed profile and control input of train 0; for each train i, offline compute S_i and terminal set Ω_i^f; define the operation ending time T_f; set $\tau_i = 0$;
2: Calculate an initial feasible solution for problem (4) with prediction horizon T_p; increase T_p until the solution is found;
3: **for** each sampling instant $t_k < T_f$ **do**
4:   Set $i = 1$;
5:   **for** $i \leqslant N-1$ **do**
6:     **if** $\sigma(t_k) \neq \sigma(t_{k-1})$ **then**
7:       $j := j+1$, $\tau_i = t_k$;
8:     **end if**
9:     Measure the states of train i; solve problem (4) using stage cost function $J_{i,j}$; obtain the optimal control sequence $U_i^*(t_k) = [u_i^*(t_k), u_i^*(t_k+\delta), \ldots, u_i^*(t_k+T-\delta)]$ and the predicted trajectory $X_{i,j}^*(t_k) = [\boldsymbol{x}_i^*(t_k+\delta), \ldots, \boldsymbol{x}_i^*(t_k+T)]$ based on $U_{i-1}^*(t_k)$ and $X_{i-1}^*(t_k)$; calculate a feasible solution $\bar{u}_i(t_{k+1})$ for the next time instant;
10:     **if** $i < N-1$ **then**
11:       Transmit $U_i^*(t_k)$ and $X_i^*(t_k)$ to train $i+1$;
12:       Set $i = i+1$;
13:     **end if**
14:     Calculate the control force $F_i(t_k)$ using the real-time states and the first part of the computed optimal control input $U_i^*(t_k)$; apply the force to the train;
15:   **end for**
16:   Set $k = k+1$;
17: **end for**

Lemma 2 ([18])**.** For the switching objectives MPC problem under state and input constraints, suppose it is feasible for all $t \geqslant 0$. Define a stage Lyapunov function $V(\boldsymbol{x})$ and a positive constant λ_0. If the following holds: a constant $\kappa > 1$ can always be found such that $V_{\eta_m}(\boldsymbol{x}) \leqslant \kappa V_{\eta_n}(\boldsymbol{x}), \forall \eta_{m,n} \in \Lambda$,

$V_j(\boldsymbol{x}(t_2)) - V_j(\boldsymbol{x}(t_1)) \leqslant -\lambda_0 \int_{t_1}^{t_2} V_j(x(s))\mathrm{d}s$ for any time interval $[t_1, t_2]$ between two switches and $j \in \Lambda$, and the average dwell-time τ satisfies $\tau > \frac{\ln \kappa}{\lambda_0}$, then the closed-loop system is asymptotically stable.

The following result shows how the lemma can be applied to the cooperative train control problem for multiple trains to guarantee the feasibility and stability of the system, and a control algorithm is also given.

Proposition 1. Suppose there is an initial feasible solution for the multiple control problem (4). For all stage cost functions, there always exist a terminal region and a terminal cost satisfying lemma 1. Then under the cost function given in (5) and (6), the optimal problem for the trains in the convoy is feasible for all $t \geqslant 0$. Furthermore, all trains can achieve cooperative operation by using Algorithm 1.

Proof. Firstly, it is shown how the second condition is satisfied. In Algorithm 1, suppose the control problem is feasible at the initial time. At time t_k, define $V_{i,j}(\boldsymbol{x}_i(t_k), u_i(t_k)) = J_{i,j}\,(\boldsymbol{x}_i(t_k), u_i(t_k))$ as a control Lyapunov function for each train in a single time stage. For simplification, define $V_{i,j}(\boldsymbol{x}_i(t_k), u_i(t_k))$ and $J_{i,j}\,(\boldsymbol{x}_i(t_k), u_i(t_k))$ as $V_{i,j}(\boldsymbol{x}_i(t_k))$ and $J_{i,j}\,(\boldsymbol{x}_i(t_k))$, respectively. By applying (10) and the optimality of $J^*_{i,j}(\boldsymbol{x}_i(t))$, inequality $J^*_{i,j}(\boldsymbol{x}_i(t_{k+1})) - J^*_{i,j}(\boldsymbol{x}_i(t_k)) \leqslant -\int_{t_k}^{t_{k+1}}(\|\tilde{\boldsymbol{x}}_i(s)\|^2_{Q_{i,j}} - \|\bar{u}_i(s)\|^2_{R_{i,j}})\mathrm{d}s$ holds.

In addition, it can be concluded that $V_{i,j}(\boldsymbol{x}_i(t_{k+1})) - V_{i,j}(\boldsymbol{x}_i(t_k)) \leqslant -\int_{t_k}^{t_{k+1}} V_{i,j}(\boldsymbol{x}_i(s))\mathrm{d}s$ holds based on the definition of $V_{i,j}(\boldsymbol{x}_i(t_k))$, which further indicates the first condition in Lemma 2, and the detailed proof can be found in [31]. Then the second condition of Lemma 2 is discussed. When applying a stage cost function as (6), the stage control Lyapunov function for train i satisfies

$$V_{i,j}(\boldsymbol{x}_i(t_k)) \leqslant \xi_{j,1} T\omega_{j,1}(\boldsymbol{x}_i(t_k)) + \xi_{j,2}\omega_{j,2}(\boldsymbol{x}_i(t_k)) = \frac{1}{\lambda_{0,j}} L_{i,j}(\tilde{\boldsymbol{x}}_i(t_k), \bar{u}_i(t_k)), \tag{11}$$

where $\beta_{j,1}$, $\beta_{j,2}$, $\xi_{j,1}$ and $\xi_{j,2}$ are positive constants, $\omega_{j,1}(\boldsymbol{x}_i(t_k))$ and $\omega_{j,2}(\boldsymbol{x}_i(t_k))$ are designed functions such that $L_{i,j}(\boldsymbol{x}_i(t_k), \bar{u}_i(t_k)) \geqslant \beta_{j,1} T\omega_{j,1}(\boldsymbol{x}_i(t_k))$ and $F^f_{i,j}(\tilde{\boldsymbol{x}}_i(t_k), \bar{u}_i(t_k)) \geqslant \beta_{j,2}\omega_{j,2}(\tilde{\boldsymbol{x}}_i(t_k))$ hold for all $\tilde{\boldsymbol{x}}_i(t) \in \Omega^f_i$ and $u_i(t) \in \mathcal{U}_{i,j}$, and $\frac{1}{\lambda_{0,j}} = (\frac{\xi_{j,1}T}{\beta_{j,1}} + \frac{\xi_{j,2}}{\beta_{j,2}})$. Besides, for $\tilde{\boldsymbol{x}}_i(t_k) \in \Omega^f_i$, a feasible solution $\hat{u}_i(t_k)$ and its corresponding state $\hat{\boldsymbol{x}}_i(t_k)$ exist in stage j and guarantee that the inequalities $L_{i,j}(\hat{\boldsymbol{x}}_i(s), \hat{u}_i(s)) \leqslant \xi_{j,1} T\omega_{j,1}(\boldsymbol{x}_i(t_k)), t_k \leqslant s \leqslant t_k + T$ and $F^f_{i,j}(\hat{\boldsymbol{x}}_i(t_k + T)) \leqslant \xi_{j,2}\omega_{j,2}(\boldsymbol{x}_i(t_k))$ hold.

In addition, for all $j, p \in \Lambda$ and $j \neq p$, inequality $\Gamma_{\min}\Xi_J \leqslant J_{i,j}(\boldsymbol{x}_i(t)) \leqslant \Gamma_{\max}\Xi_J$ is obtained for any feasible input and resulting states at time t_k based on the stage cost functions defined in (6), where $\Gamma_{\min} = [\lambda_{\min}(Q_{i,j}), \lambda_{\min}(R_{i,j}), \lambda_{\min}(S_{i,j})]$ and $\Gamma_{\max} = [\lambda_{\max}(Q_{i,j}), \lambda_{\max}(R_{i,j}), \lambda_{\max}(S_{i,j})]$ are parameter vectors, and $\Xi_J = [\int_{t_k}^{t_k+T} \|\tilde{\boldsymbol{x}}_i(s|t_k)\|^2\mathrm{d}s, \int_{t_k}^{t_k+T} \|\bar{u}_i(s|t_k)\|^2\mathrm{d}s, \int_{t_k}^{t_k+T} \|\tilde{\boldsymbol{x}}_i(t_k + T|t_k)\|^2\mathrm{d}s]^{\mathrm{T}}$. Then the stage cost function satisfies $V_{i,j}(\boldsymbol{x}_i(t_k)) = J_{i,j}(\boldsymbol{x}_i(t_k), u^*_i(t)) \leqslant \mu_{i,\max} J_{i,p}(\boldsymbol{x}_i, u^*_i(t)) = \mu_{i,\max} V_{i,p}(\boldsymbol{x}_i(t_k))$, and a constant $\mu_{i,\max}$ can always be found based on the optimality of the solution $u^*_i(t)$. It is noticed that the first condition in Lemma 2 is satisfied when $\mu_{i,\max} = \max\{\mu_{i,jp}\}, \forall j, p \in \Lambda, j \neq p$, where $\mu_{i,jp} = \max\{\frac{\lambda_{\max}(Q_{i,j})}{\lambda_{\min}(Q_{i,p})}, \frac{\lambda_{\max}(R_{i,j})}{\lambda_{\min}(R_{i,p})}, \frac{\lambda_{\max}(S_{i,j})}{\lambda_{\min}(S_{i,p})}\}$.

Based on the above discussion, if the switching time satisfies the third condition in Lemma 2, the closed-loop system (2) is asymptotically stable. Since the distance between two stations is long, the constraint of average dwell-time can be satisfied most time. In addition, the total number of switches is finite during the operation because trains have definite departure and arrival stations. Therefore, after the last switch has occurred, asymptotical stability can be established by considering the last active cost function, which is given in standard MPC arguments.

Remark 4. In the proposed ScMPC structure for multiple trains, the cost functions are considered in a quadratic form, which can be extended to a more general case as long as the stage cost functions for train i satisfy $L_{i,\ell_1}(\tilde{\boldsymbol{x}}_i(t), \bar{u}_i(t)) \leqslant \zeta_{i,\ell} L_{i,\ell_2}(\tilde{\boldsymbol{x}}_i(t), u_i(t))$ for all $\ell_1, \ell_2 \in \Lambda$ and $(\boldsymbol{x}_i(t), u_i(t)) \in \mathcal{Z}_i$ with terminal constraints $F_{i,\ell_1}(\tilde{\boldsymbol{x}}_i(t)) \leqslant \zeta_{i,\ell} F_{i,\ell_2}(\tilde{\boldsymbol{x}}_i(t)), \forall \tilde{\boldsymbol{x}}_i(t) \in \Omega^f_i$, where $\zeta_{i,\ell}$ and $\zeta_{i,\ell}$ are positive constants. Additionally, cost functions can be lower- and upper-bounded by exponential functions of $\|\boldsymbol{x}_i(t)\|$ and $\|u_i(t)\|$.

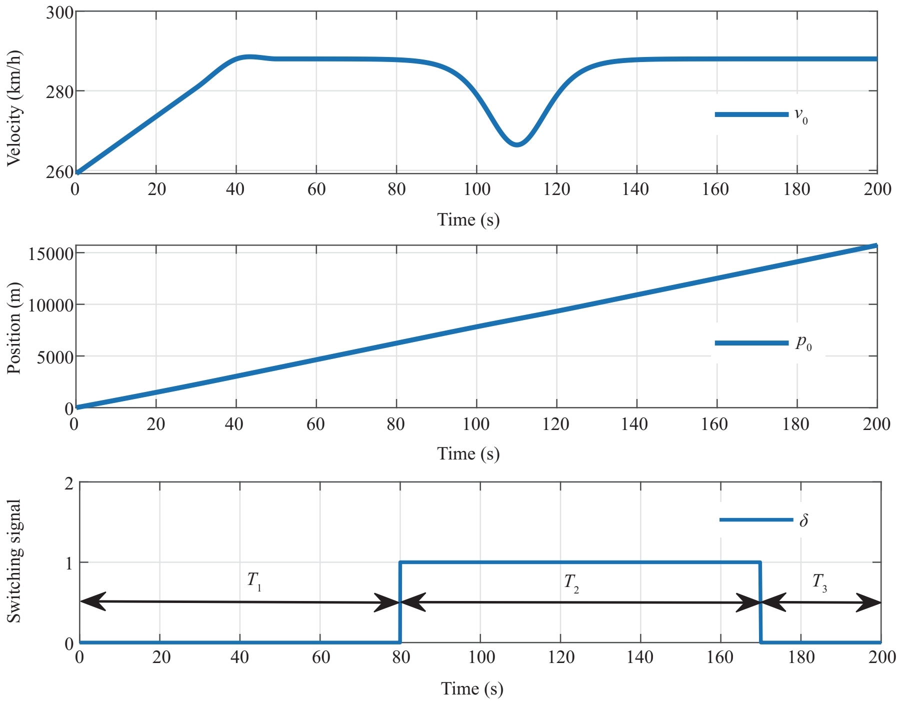

Figure 2 (Color online) Desired speed profile and switching signal.

4 Simulation

4.1 Test environment

A scenario is developed where multiple trains are required to achieve a cooperative state. And it is also used to compare the influence of different weights. Using MATLAB R2021b, the simulation is conducted on a PC (3.2-GHz Intel i5 CPU, 8-GB RAM, and 64-bit Windows 10). At each time step, the optimal control input of each train can be obtained within an average value of 0.031 s. Since a train only calculates its own control input, the complexity of calculation will not increase even more trains are considered. Therefore, the proposed method can guarantee real-time calculation and is applicable in practical applications.

A convoy formed by three trains with a mass of 800 tons for each train is chosen, while inertial lag with $\iota = 0.2$ is considered. Davis coefficients of three trains are defined as $C_a = 0.01176$, $C_b = 0.00077616$, and $C_c = 0.000016$. The position, velocity, and control input of train 0 are predefined and transmitted to Train 1 as its desired tracking speed profile, which is shown in Figure 2. To show the performance of the proposed ScMPC method, the tracking targets are changed with a switching signal during the operation. It is noticed that $p_0(t)$, $v_0(t)$, and $u_0(t)$ still need to satisfy the basic properties proposed in Section 2.

The simulation starts with an initial condition where three trains operate at velocities of 263, 256, and 252 km/h. The initial positions are 0, -4005, and -8005 m, respectively. Set the desired distance between two adjacent trains at 4000 m. The simulation is assumed to be in an ideal condition, without uncertainties in state measures and delay in communication.

4.2 Different cooperative objectives

The situation in which desired velocities of trains change is considered to certify the effectiveness of the proposed method in complex conditions. Higher weights on $\|u_i(t)\|$ can lead to a better performance in comfort, while those on tracking errors can guarantee a rapid response on target changes and an exact operation. Two kinds of operation objectives are given in the following simulation. Firstly, a comfort-first objective (CFO) based MPC is used in a normal situation. In this mode, higher weights are applied to the norm and the change of acceleration/deceleration when choosing a cost function. All the trains in the convoy can achieve the cooperative state in a relatively gentle way. Then, to better adapt to the sudden change of cooperative objective, an adjusting speed-first objective (SFO) based MPC is chosen, where

Table 1 Parameters of ScMPC

Parameter	Value	Parameter	Value
$u_{\min}$	-0.98	$u_{\max}$	0.98
$v_{\max}$	300 km/h	$p^1_{\min}$, $p^2_{\min}$, $p^3_{\min}$	3800 m
p^1_{des}, p^2_{des}, p^3_{des}	0 m, 4000 m, 4000 m	$a_{\min}$	-1.2 m/s^2
$a_{\max}$	1 m/s^2	δ	0.1 s
T_c	1 s	T_p	3 s

weights of tracking error are high in the cost function. By using these objectives, trains are controlled to achieve the changed desired states cooperatively with a higher speed. In other words, the errors of states are more important in this situation. Combining the advantages of SFO- and CFO-based MPC, the proposed ScMPC is verified in different situations, which can apply adaptive weights on comfort, tracking error and energy consumption.

The main parameters of ScMPC are given in Table 1. For simplification, consider trains in the convoy have the same parameters and cost functions with a subscript i omitted. Then the cost functions J_{η_1} and J_{η_2} are defined for all the trains in the convoy by the following groups of weights:

$$Q_{\eta_1} = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad Q_{\eta_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.05 \end{pmatrix}, \quad R_{\eta_1} = 1, \quad R_{\eta_2} = 0.05. \tag{12}$$

The auxiliary local control law using weights Q_{η_1} and R_{η_1} is calculated as $u_i^f(t) = 4.4721x_{i,1}(t) + 9.9989x_{i,2}(t) + 10.9544x_{i,3}(t) - u_0(t)$, and then system (2) with the terminal control input $u_i^f(t) = k_i^f \tilde{\boldsymbol{x}}_i(t) - u_{i-1}(t)$ is locally asymptotically stable with

$$S_{i,\eta_1} = \begin{pmatrix} 2.8704 & 3.1754 & 0.2878 \\ 3.1754 & 6.7784 & 0.6403 \\ 0.2878 & 0.6403 & 0.5657 \end{pmatrix},$$

as well as the terminal set $\Omega^f_{i,\eta_1} = \{\|\tilde{\boldsymbol{x}}_i(t)\|_{S_i} \leqslant 3.5\}$. Furthermore, to guarantee the condition in Proposition 1, the parameter that $\lambda_{i,0} = 0.1$ is calculated. Choose $\mu = 20$. Then it finally leads to $\tau_a > \frac{\ln \mu}{\lambda_{i,0}} = 30$.

Remark 5. In this simulation, assuming that trains track a fixed objective after $t = 170$ s. Therefore, the feasibility and stability of the proposed method can be calculated with a fixed weight problem with Q_{η_2} and R_{η_2}. In fact, the switching can be more flexible, and the simulation end time is not limited as long as the average dwell-time is satisfied.

Suppose the switching happens at $t = 80$ s and $t = 170$ s, and the switching signal σ is also given in Figure 2. Furthermore, to better compare the difference between the three cost objectives, the whole operating time is divided into three parts according to the switching time and also shown in Figure 2, which are defined as T_1, T_2, and T_3. It is noticed that the cost function in time stage T_3 is as same as that in T_1.

4.3 Results

Three groups of simulations with different cost functions have been developed. The results corresponding to the ScMPC method, which is applied on Train 1, appear in Figure 3(a). Different strategies are applied to train 2 and train 3, and the control performance in terms of comfort, adjustment speed, and energy consumption is shown in Figures 3(b)–(f). These figures not only illustrate that trains can achieve the desired cooperative state but also show a clear distinctive behavior between the two non-switching strategies, demonstrating that the SFO strategy maintains a faster response when facing a sudden desired tracking velocity change. In contrast, the CFO strategy has a better performance in comfort in normal operation at first. Another important result is that the ScMPC method combines the advantages of the above two strategies.

Figure 3 (Color online) States of trains under three kinds of cost functions. (a) Velocity, acceleration, and control input of train 1; (b) velocity of train 2 and train 3; (c) tracking distance of train 2 and train 3; (d) control input of train 2 and train 3; (e) acceleration of train 2 and train 3; (f) traction/braking force of train 2 and train 3.

To better show the difference between the performance of controllers with different cost functions, several results are given in Tables 2–4, where $W_{2,T_{1,2,3}}$ and $W_{3,T_{1,2,3}}$ represent the energy consumption of train 2 and train 3 in three time stages, respectively. W_2 and W_3 represent the total energy consumption during the operation. All the energy consumption values are calculated based on (3), and the units of them are 10^6 kJ. It can be seen that ScMPC has better performance than the other two methods in energy saving.

In Table 3, distance tracking errors $\mathrm{ED}_{2,3}$ and velocity tracking errors $\mathrm{EV}_{2,3}$ are given, which are

Table 2 Control performance comparison in terms of energy consumption

	W_{2,T_1}	W_{2,T_2}	W_{2,T_3}	W_2	W_{3,T_1}	W_{3,T_2}	W_{3,T_3}	W_3
CFO+MPC	1.3904	1.1599	0.03857	2.8889	1.4472	1.1689	0.33855	2.9547
SFO+MPC	1.3908	1.1303	0.33858	2.8597	1.447	1.1307	0.03871	2.9164
ScMPC	1.3904	1.1285	0.03857	2.8575	1.4472	1.1294	0.03873	2.9153

Table 3 Control performance comparison in terms of tracking errors

	ED_2 (km)	ED_3 (km)	EV_2 (m/s)	EV_3 (m/s)
CFO+MPC	2.9476	3.6188	7.5241	5.2768
SFO+MPC	4.1779	4.1984	8.2805	6.2191
ScMPC	2.9776	3.5674	7.6789	5.1776

Table 4 Control performance comparison in terms of comfort

	U_{2,T_1}	U_{2,T_2}	U_{2,T_3}	U_2	U_{3,T_1}	U_{3,T_2}	U_{3,T_3}	U_3
CFO+MPC	2.6485	1.2939	0.0005	2.9476	0.0014	3.3381	1.3974	3.6188
SFO+MPC	4.0482	1.0332	0.0069	4.1779	0.0110	4.0662	1.0452	4.1984
ScMPC	2.6485	1.3609	0.0033	2.9776	0.0049	3.3381	1.2582	3.5674

calculated by adding the norm of tracking distance error and velocity error, respectively. A larger error range is allowed during the adjustment in the CFO strategy, which is also shown in Figure 3(b). Therefore, trains need more time to achieve the new cooperative state. It is seen that the tracking error in CFO is twice as much as that in the SFO strategy. The control performance comparison in terms of comfort is given in Table 4, where $U_{2,T_{1,2,3}}$ and $U_{3,T_{1,2,3}}$ is calculated by summing the norm of control inputs of train 2 and train 3 in three time stages, respectively. In addition, define $U_2 = U_{2,T_1} + U_{2,T_2} + U_{2,T_3}$ and $U_3 = U_{3,T_1} + U_{3,T_2} + U_{3,T_3}$. It can be seen that the controller using the CFO strategy has better performance, whose acceleration change rate value is lower than that using the SFO strategy. Specifically, it can be seen in the time interval T_1, which also represents the normal operation scenario, that the rate of acceleration change when using CFO-based MPC is only half of that when using SFO-based MPC. In contrast, in the time interval T_2, the higher weight on comfort will cause a lower changing rate of acceleration. Since the desired velocity and position keep increasing, trains will choose a bigger force to achieve the latest cooperative state finally. In the switching situation, trains with the SFO strategy perform better in terms of comfort and energy saving. Based on the above analysis, it is clear that in a normal operation scenario, using the CFO strategy is more suitable for the requirement of comfort, while SFO is better in the switching scenario. Therefore, the proposed ScMPC method combines the advantages of the above two strategies, leading to more comfortable, exact and energy-saving controller performance, which is also shown in the simulation results.

5 Conclusion

A cooperative control method for multiple trains is proposed based on the distributed ScMPC method. Since the control performance difference caused by the choice of cost functions and weights has been discussed in many studies, this paper provides a solution that can adapt to several cooperative operation requirements in different conditions by changing weights. Additionally, the feasibility of the proposed method and the stability of the system are proved, which complements some proposed MPC methods for train control. In addition to the theoretical proof, the proposed control method is evaluated through a numerical simulation. Comfort-first and response-first cost functions, as well as one with switching costs, are chosen and compared in terms of control performance. The results indicate that trains can achieve and maintain cooperative operation using the ScMPC method. Moreover, better performance is obtained by applying the proposed method when facing a change in environment and cooperative requirements because a more appropriate weight is chosen for different situations.

Although this paper provides an ScMPC control for multiple trains, the difficulties associated with an actual implementation still need to be overcome. For example, the communication constraints such as time delays are not considered in this paper. In addition, future research will investigate how to deal with external measurement errors and improve controller robustness.

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