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Special Topic: Entanglement of Control and Optimization

Optimal output regulation for PMSM speed servo system using approximate dynamic programming

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In recent decades, the output regulation problem has received considerable attention from researchers in the control community because it simultaneously addresses reference signal tracking, disturbance rejection, and robustness [1]. Furthermore, when transient performance is considered by solving an optimization problem with a prescribed cost function, this output regulation problem is called an optimal output regulation (OOR) problem. Recently, the research on the OOR problem has mainly focused on unknown or partially unknown system dynamics [2]. In particular, because approximate dynamic programming (ADP) techniques can be used to obtain an approximated optimal controller from input and/or partial state data in the case of unknown system dynamics and unmeasurable disturbances [3], Gao and Jiang [2] first integrated ADP and output regulation theory to solve the OOR problem of linear systems.

Herein, motivated by [2], we formulate the speed tracking problem of a permanent magnet synchronous motor (PMSM) servo system as a linear optimal output regulation problem (LOORP) and propose a model-free optimal controller to solve this LOORP. The main contributions of our study are summarized as follows. On the one hand, an ADP-based model-free adaptive optimal state feedback controller (AOSFC) is proposed, where the feedback and feedforward gains are calculated using input and state data rather than using a system model. Compared with other model-based advanced control methods, such as internal model control [4] and sliding-mode control [5], the proposed model-free controller achieves superior speed tracking and transient performance as well as exhibits strong disturbance rejection ability for unmeasurable load torque disturbance. On the other hand, to the best of our knowledge, our study is the first attempt at applying OOR theory to a PMSM servo system, and the efficacy of our proposed model-free control law is experimentally verified.

Modeling and problem formulation. The dynamic model of surface-mounted PMSM under dq rotating reference frame is as follows [4]:

$$\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = \frac{3p\Phi_v}{2J} i_q - \frac{F_v}{J}\omega_r - \frac{1}{J}T_L, \tag{1}$$

$$\frac{\mathrm{d}i_d}{\mathrm{d}t} = -\frac{R_s}{L} i_d + p i_q \omega_r + \frac{1}{L} u_d, \tag{2}$$

$$\frac{\mathrm{d}i_q}{\mathrm{d}t} = -\frac{R_s}{L} i_q - p i_d \omega_r - \frac{p\Phi_v}{L}\omega_r + \frac{1}{L} u_q, \tag{3}$$

where ω_r is the rotor speed, i_d and i_q are dq axes stator currents, u_d and u_q are dq axes stator voltages, p is the pole pairs, T_L is the load torque, Φ_v is the flux linkage, J is the rotational inertia, L is the dq axes inductance, F_v is the viscous friction coefficient, and R_s is the stator resistance.

Herein, we aim to design a model-free optimal control law such that the rotor speed ω_r precisely and rapidly tracks the reference speed ω_d under parameter uncertainties as well as unmeasurable load torque disturbance. Like [5], the cascaded control structure of speed-current loops is employed. Taking i_q as the control input u, which will be used as reference current for q axis current loop, we can obtain a linear plant by motion equation (1) as follows:

$$\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = \frac{3p\Phi_v}{2J} u - \frac{F_v}{J}\omega_r - \frac{1}{J}T_L. \tag{4}$$

Letting $x = \omega_r$ and $d = T_L$, the state-space representation is obtained as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu + Ed, \\ e &= x - w_d, \end{aligned} \tag{5}$$

where $A = -\frac{F_v}{J}$, $B = \frac{3p\Phi_v}{2J}$, $E = -\frac{1}{J}$, and e denotes the speed tracking error.

The reference speed and load torque disturbance are assumed to be bounded and can be generated by a so-called exosystem as follows:

$$\dot{\nu} = W\nu, \quad \nu(0) = \nu_0, \quad \omega_d = M\nu, \quad d = N\nu, \tag{6}$$

where W, M, and N are appropriate matrices, but N is unknown.

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Combining (5) and (6), the plant is redescribed by

$$\begin{aligned} \dot{x} &= Ax + Bu + \hat{E}\nu, \\ \dot{\nu} &= W\nu, \\ e &= x + D\nu, \end{aligned} \tag{7}$$

with $D = -M$ and $\hat{E} = -\frac{N}{J}$ unknown.

Some assumptions of system (7) are listed as follows:

(1) The pair (A, B) is stabilizable.

(2) For all $\lambda \in \sigma(W)$, $\text{rank}\left[\begin{smallmatrix} A-\lambda & B \\ 1 & 0 \end{smallmatrix}\right] = 2$.

Now the speed tracking problem of system (4) is formulated as a LOORP of system (7) that aims to design a model-free optimal state feedback control law such that the closed-loop system is exponentially stable, $\lim_{t\to\infty} e(t) = 0$, and a predefined cost function is minimized.

To solve the LOORP of system (7), we solve a static optimization problem called Problem 1 and a dynamic optimization problem called Problem 2 (see [2]). Note that Problem 1 aims to find the optimal solution $(X_{\text{opt}}, U_{\text{opt}})$ for regulator equations associated with system (7), and Problem 2 aims to find the optimal feedback controller $\bar{u} = -K_{\text{opt}}\bar{x}$, where $\bar{x} = x - X_{\text{opt}}\nu$ and $\bar{u} = u - U_{\text{opt}}\nu$. Some notations and statements of Problems 1 and 2 are provided in Appendix A.

Design of AOSFC. Motivated by [2], we will propose an ADP-based model-free AOSFC to solve the LOORP of system (7). First, we develop a data-based iteration algorithm to solve Problem 2 with unknown A, B, and $\hat{E}$ of system (7) to obtain K_{opt}. Second, the solutions for Problem 2 are used to solve Problem 1 to obtain a solution pair $(X_{\text{opt}}, U_{\text{opt}})$ satisfying regulator equations. Thus, the optimal feedback and feedforward gains K_{opt} and $L_{\text{opt}} = U_{\text{opt}} + K_{\text{opt}}X_{\text{opt}}$ of the controller (11) can be obtained. Finally, a concrete algorithm for designing an AOSFC is summarized as Algorithm 1. Further details on the design of AOSFC are included in Appendix B.

Remark 1. First, the proposed AOSFC provides a reference current i_q^* for the q axis current loop. Second, the convergence of P_j and K_j obtained by (9) as well as the stability of the closed-loop system has been proven based on Theorems 2 and 3 in [2], respectively. Finally, ρ is added to control input in the learning phase $[t_0, t_r]$ to satisfy (8). ρ can be random noise and/or sinusoidal.

The simulation and experimental results are shown in Appendix C. Based on our study, we made the following observations. First, irrespective of whether the reference speed is sinusoidal, constant, or abrupt constant, the proposed AOSFC has the smallest maximal steady-state speed error despite a sinusoidal load torque disturbance. Second, the proposed AOSFC has better robustness to motor parameter variations than that of the three compared methods. Third, the proposed AOSFC can lead to considerably better transient performance than that of the three compared methods. Finally, in the case of suddenly added/removed load torque or sinusoidal load torque, the proposed AOSFC demonstrates considerably better disturbance rejection performance than that demonstrated by the three compared methods.

Algorithm 1 ADP-based algorithm for designing an AOSFC

1: Calculate X_0, X_1.

2: Employ control input $u = -K_0x + \rho$ during $[t_0, t_r]$. Keep computing $\Delta_{\overline{x}_i\overline{x}_i}, \Gamma_{\overline{x}_i\overline{x}_i}, \Gamma_{\overline{x}_i u}$, and $\Gamma_{\overline{x}_i\nu}$ for $i = 0, 1$, until rank condition

$$\text{rank}([\Gamma_{\overline{x}_i\overline{x}_i}, \Gamma_{\overline{x}_i u}, \Gamma_{\overline{x}_i\nu}]) = \frac{n(n+1)}{2} + (m+q)n \tag{8}$$

is met. Let $i = 0, j = 0$.

3: Compute P_j, K_{j+1} by

$$\begin{bmatrix} \text{vecs}(P_j) \\ \text{vec}(K_{j+1}) \\ \text{vec}((\hat{E} - S(X_i))^{\text{T}}P_j) \end{bmatrix} = (\Xi_{ij}^{\text{T}}\Xi_{ij})^{-1}\Xi_{ij}^{\text{T}}\Psi_{ij}. \tag{9}$$

4: Letting $j \leftarrow j+1$, repeat Step 3 until $\|P_j - P_{j-1}\| \leqslant \epsilon$ where ϵ is any small positive number.

5: Letting $i = 0, j* \leftarrow j$, solve $\hat{E}$ by (9) and compute $B = P_j^{-1}K_{j+1}^{\text{T}}\bar{R}$.

6: Letting $i = 1$, compute $S(X_1)$ by (9).

7: Compute $(X_{\text{opt}}, U_{\text{opt}})$ by

$$\begin{bmatrix} 0 & -I_q \otimes B \\ -I_q & 0 \end{bmatrix} \begin{bmatrix} \text{vec}(X) \\ \text{vec}(U) \end{bmatrix} = \begin{bmatrix} \text{vec}\Big(\hat{E} - S(X_1)\Big) \\ -\text{vec}(X_1) \end{bmatrix}. \tag{10}$$

8: Letting $K_{\text{opt}} = K_{j*}$ and $L_{\text{opt}} = U_{\text{opt}} + K_{\text{opt}}X_{\text{opt}}$, the AOSFC

$$u = -K_{\text{opt}}x + L_{\text{opt}}\nu \tag{11}$$

is obtained.

Conclusion. Herein, we have solved the LOORP of a PMSM speed servo system with an unknown model and unmeasurable disturbance. An ADP-based model-free AOSFC has been proposed, which achieves excellent speed tracking and transient performance as well as demonstrates strong disturbance rejection properties for unmeasurable disturbance. Simulation and experimental results have demonstrated the advantages of our method in steady-state and transient performances compared to other methods.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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