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Exponential stability analysis of switched positive nonlinear systems with impulsive effects via multiple max-separable Lyapunov functions

Dadong TIAN[1], Jianwei XIA[2*], Yuangong SUN[3] & Mei LI[1]

[1]College of Information Science and Engineering, Shandong Agricultural University, Taian 271018, China;
[2]School of Mathematic Sciences, Liaocheng University, Liaocheng 252059, China;
[3]School of Mathematical Sciences, University of Jinan, Jinan 250022, China

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Dear editor,

Switched positive systems consist of a series of positive subsystems and a ruler that regulates the switching among the subsystems, and these systems have been investigated extensively in the past few decades [1]. Up to now, switched positive linear systems (SPLSs) have been well understood [2]. However, stability theory of switched positive nonlinear systems (SPNSs) is still less well-developed. Ref. [3] was devoted to the stability analysis of a class of SPNSs, but the systems were delay-free. Besides, an impulsive disturbance, an abrupt change of state, may occur at the moment of switching. And the impulse can also be delayed, that is, the amplitude of the impulse is determined by the previous state of the system. Either the delayed impulse or switching may cause system instability and oscillation, which is serious and worth studying. This article studies the exponential stability of SPNSs with delayed impulse, and the delay of impulse can be unbounded.

The SPNSs with delayed impulse are described by

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{f}_{\sigma(t)}(\boldsymbol{x}(t))+\boldsymbol{g}_{\sigma(t)}\left(\boldsymbol{x}\left(t-\tau(t)\right)\right),\\ t\geqslant 0,\ t\neq t_k,\\ \Delta\boldsymbol{x}(t_k)=C_{\sigma(t_k)}\boldsymbol{x}((t_k-d(t_k))^-),\\ \boldsymbol{x}(t)=\boldsymbol{\theta}(t),\quad t\in[-\tau_{\max},0],\end{cases}\tag{1}$$

where $\boldsymbol{x}(t)\in\mathbb{R}^n$ is the state variable and $\sigma(t):[0,+\infty)\to M=\{1,2,\ldots,m\}$ denotes the switching signal. m is the amount of all subsystems. $\boldsymbol{\theta}(t)\in C([-\tau_{\max},0],\mathbb{R}^n)$ is the initial condition. The continuous function $\tau(t)$ is time-delay with $\tau(t)\in[0,\tau_{\max}]$. The nonnegative diagonal matrix $C_p\in\mathbb{R}^{n\times n}=\mathrm{diag}\{C_{p11},C_{p22},\ldots,C_{pnn}\}$ is called an impulsive matrix. Let $\boldsymbol{x}(t^+)=\lim_{h\to0^+}\boldsymbol{x}(t+h)$ and $\boldsymbol{x}(t^-)=\lim_{h\to0^-}\boldsymbol{x}(t+h)$. $0=t_0<t_1<t_2<\cdots<t_k<\cdots$ is any given switching signal. $\Delta\boldsymbol{x}(t)=\boldsymbol{x}(t_k^+)-\boldsymbol{x}(t_k^-)$. In this article, let $\boldsymbol{x}(t_k)=\boldsymbol{x}(t_k^+)$ and $\sigma(t_k)=\sigma(t_k^+)$ for convenience.

Assumption 1. Continuous function $d(t)$ is nonnegative and satisfies $t-d(t)>-\tau_{\max}$ for any $t\geqslant0$.

Remark 1. Assumption 1 implies $d(t)$ can be unbounded. For example, let $d(t)=\frac{1}{2}t$ or $d(t)=\ln(t+1)$. Then $\lim_{t\to+\infty}t-d(t)=+\infty$. The bounded time-varying delay is a special case of Assumption 1. Moreover, there is not any restriction on the derivative of $d(t)$ in Assumption 1. That is, $d(t)$ can be non-differentiable.

Assumption 2. For any $p\in M$ and $\boldsymbol{x}\in\mathbb{R}^n$, there exists a nonnegative constant λ^* such that $\boldsymbol{f}_p:\mathbb{R}^n\to\mathbb{R}^n$ and $\boldsymbol{g}_p:\mathbb{R}^n\to\mathbb{R}^n$ satisfy

$$\boldsymbol{f}_p(\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{x}))\leqslant\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{f}_p(\boldsymbol{x})),\ \boldsymbol{g}_p(\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{x}))\leqslant\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{g}_p(\boldsymbol{x})),\ \forall\lambda>\lambda^*.$$

Remark 2. Obviously, the mapping $\boldsymbol{f}(\lambda\boldsymbol{x})=\lambda\boldsymbol{f}(\boldsymbol{x})$ satisfies Assumption 2 with respect to $\boldsymbol{f}(\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{x}))=\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{f}(\boldsymbol{x}))$ and $\boldsymbol{r}=\{1,1,\ldots,1\}$. That is, Assumption 2 can be applied to more general cases than the homogeneous vector fields with degree one given in [3]. For example, let

$$\boldsymbol{f}(\boldsymbol{x}(t))=\begin{bmatrix}x_1^{\frac{1}{2}}x_2^{\frac{1}{4}}+2x_1+k_1\\ x_1x_2^{\frac{1}{2}}+8x_2+k_2\end{bmatrix},$$

where $k_1\geqslant0,k_2\geqslant0$. It is easy to verify that $\boldsymbol{f}(\lambda\boldsymbol{x})\neq\lambda\boldsymbol{f}(\boldsymbol{x})$, which implies $\boldsymbol{f}$ is not a homogeneous vector field of degree one. However, for the dilation map $\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{x})$ with $\boldsymbol{r}=(1,2)$, if $k_1=k_2=0$, then $\boldsymbol{f}(\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{x}))\leqslant\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{f}(\boldsymbol{x}))$, $\forall\boldsymbol{x}\in\mathbb{R}^n,\lambda>\lambda^*=0$. If $k_1>0,k_2>0$, then $\boldsymbol{f}(\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{x}))<\delta_\lambda^{\boldsymbol{r}}(\boldsymbol{f}(\boldsymbol{x}))$, $\forall\boldsymbol{x}\in\mathbb{R}^n,\lambda>\lambda^*=1$. Therefore, $\boldsymbol{f}$ satisfies Assumption 2.

Assumption 3. (i) $\boldsymbol{f}_p$ is cooperative and $\boldsymbol{g}_p$ is order-preserving on $\mathbb{R}_+^n\backslash\{\boldsymbol{0}\}$. (ii) $\boldsymbol{f}_p$, $\boldsymbol{g}_p$ are continuous and continuously differentiable on $\mathbb{R}_+^n\backslash\{\boldsymbol{0}\}$.

Remark 3. It follows from (i) of Assumption 3 that system (1) is positive under arbitrary switching, that is,

* Corresponding author (email: njustxjw@126.com)

$\boldsymbol{x}(t) \in \mathbb{R}^n_+$ for all $t \geqslant 0$ with the initial condition $\boldsymbol{\theta}(t) \in \mathbb{R}^n_+$. This is called SPNSs. The condition (ii) of Assumption 3 implies the existence and uniqueness of the solution of system (1).

Let $r_{\max} = \max_{1\leqslant j\leqslant n} r_j$, $r_{\min} = \min_{1\leqslant j\leqslant n} r_j$, where $\boldsymbol{r} = (r_1, r_2, \ldots, r_n)$, $r_j > 0$ $(j = 1, 2, \ldots, n)$. For a fixed vector $\boldsymbol{\upsilon}_p \gg \boldsymbol{0}$, and $\boldsymbol{x} \in \mathbb{R}^n$, the multiple max-separable Lyapunov function is defined by $V_{\boldsymbol{\upsilon}_p}(\boldsymbol{x}) = \max_{1\leqslant j\leqslant n}(\frac{|x_j|}{\upsilon_{pj}})^{\frac{r_{\max}}{r_j}}$.

Lemma 1. Considering the system (1), for any $p, q \in M$, assume that $\boldsymbol{\upsilon}_p \gg \boldsymbol{0}, \boldsymbol{\upsilon}_q \gg \boldsymbol{0}$, and the state variable $\boldsymbol{x}(t) \in \mathbb{R}^n_+$. Then

$$V_{\overline{\boldsymbol{\upsilon}}}(\boldsymbol{x}(t)) \leqslant V_{\boldsymbol{\upsilon_p}}(\boldsymbol{x}(t)) \leqslant \beta V_{\boldsymbol{\upsilon_q}}(\boldsymbol{x}(t)) \leqslant \beta V_{\underline{\boldsymbol{\upsilon}}}(\boldsymbol{x}(t)), \qquad (2)$$

where $\beta = \alpha^l$, $l = \frac{r_{\max}}{r_{\min}}$, $\alpha = \max_{1\leqslant j\leqslant n} \frac{\overline{\upsilon}_j}{\underline{\upsilon}_j}$ with $\overline{\upsilon}_j = \max_{p\in M} \upsilon_{pj}$, $\underline{\upsilon}_j = \min_{p\in M} \upsilon_{pj}$, $\overline{\boldsymbol{\upsilon}} = [\overline{\upsilon}_1, \overline{\upsilon}_2, \ldots, \overline{\upsilon}_n]$, and $\underline{\boldsymbol{\upsilon}} = [\underline{\upsilon}_1, \underline{\upsilon}_2, \ldots, \underline{\upsilon}_n]$.

Theorem 1. Suppose Assumptions 1–3 hold. If for any $p \in M$, there exists $\boldsymbol{\upsilon}_p \gg \boldsymbol{0}$ such that $\boldsymbol{f}_p(\boldsymbol{\upsilon}_p) + \boldsymbol{g}_p(\boldsymbol{\upsilon}_p) \ll \boldsymbol{0}$ and for any $p \in M$, there exists a constant $h \geqslant 1$ satisfying $(I + C_p)\boldsymbol{\upsilon}_p \leqslant h\boldsymbol{\upsilon}_p$, then system (1) is exponentially stable (ES) under any average-dwell-time (ADT) switching signal satisfying $\tau_a > \frac{\ln(\beta\eta)}{\lambda_0}$, where $\eta = h^l$ and $0 < \lambda_0 < \min_{p\in M} \min_{1\leqslant j\leqslant n} \lambda_{pj}$ with λ_{pj} satisfying

$$\lambda_{pj} + \frac{f_{pj}(\boldsymbol{\upsilon}_p)}{\upsilon_{pj}} + \mathrm{e}^{\lambda_{pj}\tau_{\max}} \frac{g_{pj}(\boldsymbol{\upsilon}_p)}{\upsilon_{pj}} = 0. \qquad (3)$$

Remark 4. Compared with the results presented in [4], the impulsive and switching events are considered in this study. If for any $p \in M$, let the impulsive matrix $C_p \equiv 0$ and $\sigma(t) \equiv p$, then system (1) of this study degenerates to system (1) in [4]. The author of [3] only studied the switched positive systems without delays and impulse. If let impulsive matrix $C_p \equiv 0$ and $\tau(t) \equiv 0$, then system (1) degenerates to system (3.1) in [3]. Therefore, the main results in [3, 4] can be regarded as the special cases of this study. Moreover, the systems considered in this study satisfying Assumption 2 are not constrained to be degree one. As stated in Remark 2, Assumption 2 can be applied to more general cases than the homogeneous vector fields with degree one given in [3, 4]. In order to grapple with the difficulties resulting from state delays and impulsive delays, a model transformation is introduced. Then by employing comparison principle and designing appropriate switching signals, Theorem 1 is proved.

Remark 5. For fixed $\boldsymbol{\upsilon}_p \gg \boldsymbol{0}$ and $\tau_{\max}$, the expression of left-hand side of (3) can be regarded as a function with respect to λ_{pj} and it is strictly monotonically increasing in λ_{pj}. Furthermore, it follows from $(\boldsymbol{f}_p + \boldsymbol{g}_p)\boldsymbol{\upsilon}_p \ll \boldsymbol{0}$ that Eq. (3) has a unique positive solution λ_{pj}. Note that λ_{pj} is monotonically decreasing in $\tau_{\max}$, and this implies that the exponential decay rate presented in (3) deteriorates with an increasing upper bound of the system delay $\tau_{\max}$.

Remark 6. From Theorem 1, for all $p \in M$, if there exists a positive vector $\boldsymbol{\upsilon} \gg \boldsymbol{0}$ such that $\boldsymbol{f}_p(\boldsymbol{\upsilon}) + \boldsymbol{g}_p(\boldsymbol{\upsilon}) \ll \boldsymbol{0}$, and let impulsive matrix $C_p = 0$, then $\beta = \alpha^l = 1$ and $\eta = h^l = 1$ are obtained. Furthermore, it follows from $\tau_a = \frac{\ln(\beta\eta)}{\lambda_0} = 0$ that Eq. (1) is ES under arbitrary switching. So for the SPNSs without impulsive effects, the stability condition under arbitrary switching can be obtained by reducing the multiple max-separable Lyapunov function a common max-separable Lyapunov function.

Next, as a special case, the impulsive SPLS is considered.

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{\sigma(t)}(\boldsymbol{x}(t)) + \boldsymbol{B}_{\sigma(t)}\left(\boldsymbol{x}\left(t - \tau(t)\right)\right), \\ t \geqslant 0, \ t \neq t_k, \\ \Delta\boldsymbol{x}(t_k) = C_{\sigma(t_k)}\boldsymbol{x}((t_k - d(t_k))^-), \\ \boldsymbol{x}(t) = \boldsymbol{\theta}(t), \quad t \in [-\tau_{\max}, 0]. \end{cases} \qquad (4)$$

Corollary 1. Assume A_p is a Metzler matrix, B_p and C_p are nonnegative, where $A_p = [a^p_{ij}]_{n\times n}$ and $B_p = [b^p_{ij}]_{n\times n}$. If for any $p \in M$, there exists $\boldsymbol{\upsilon}_p \gg \boldsymbol{0}$ such that $(A_p + B_p)\boldsymbol{\upsilon}_p \ll \boldsymbol{0}$, and for any $p \in M$, there exists a constant $h \geqslant 1$ satisfying $(I + C_p)\boldsymbol{\upsilon}_p \leqslant h\boldsymbol{\upsilon}_p$, then system (4) is ES under any ADT switching signal satisfying $\tau_a > \frac{\ln(\beta\eta)}{\lambda_0}$, where $0 < \lambda_0 < \min_{p\in M} \min_{1\leqslant j\leqslant n} \lambda_{pj}$ with λ_{pj} being the solution of the following equation:

$$\lambda_{pj} + \sum_{i=1}^{i=n} \frac{1}{\upsilon_{pj}} a^p_{ji}\upsilon_{pi} + \mathrm{e}^{\lambda_{pj}\tau_{\max}} \sum_{i=1}^{i=n} \frac{1}{\upsilon_{pj}} b^p_{ji}\upsilon_{pi} = 0.$$

In the following, an extension of Theorem 1 to general impulsive switched linear systems is presented, in which the systems are not necessary positive. Let $\tilde{A}_p = [\tilde{a}^p_{ij}]_{n\times n}$, $|B_p| = [|b^p_{ij}|]_{n\times n}$ and $|C_p| = [|c^p_{ij}|]_{n\times n}$, where $\tilde{a}^p_{jj} = a^p_{jj}$, $\tilde{a}^p_{ij} = |a^p_{ij}|$ for $i \neq j$. Then we have Theorem 2.

Theorem 2. For system (4), if for any $p \in M$, there exists $\boldsymbol{\upsilon}_p \gg \boldsymbol{0}$ such that $(\tilde{A}_p + |B_p|)\boldsymbol{\upsilon}_p \ll \boldsymbol{0}$, and for any $p \in M$, there exists a constant $h \geqslant 1$ satisfying $(I + |C_p|)\boldsymbol{\upsilon}_p \leqslant h\boldsymbol{\upsilon}_p$, then system (4) is ES under ADT switching signal satisfying $\tau_a > \frac{\ln(\beta\eta)}{\lambda_0}$, where $0 < \lambda_0 < \min_{p\in M} \min_{1\leqslant j\leqslant n} \lambda_{pj}$ and λ_{pj} is the unique solution of the following equation:

$$\lambda_{pj} + \left(a^p_{jj} + \sum_{i=1, i\neq j}^{i=n} \frac{1}{\upsilon_{pj}} \tilde{a}^p_{ji}\upsilon_{pi} \right) + \mathrm{e}^{\lambda_{pj}\tau_{\max}} \sum_{i=1}^{i=n} \frac{1}{\upsilon_{pj}} |b^p_{ji}|\upsilon_{pi} = 0. \qquad (5)$$

Preliminaries and necessary definitions can be found in Appendix A. The proofs of Lemma 1, Theorems 1 and 2 can be found in Appendixes B–D, respectively.

Conclusion. This article attempted to analyze stability for delayed switched positive nonlinear systems with impulsive effects via multiple max-separable Lyapunov functions, and the SPLSs are the special case of our study.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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