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Fully distributed event-triggered affine formation maneuver control over directed graphs

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Affine formation maneuver control primarily aims to steer a group of agents to simultaneously execute various formation maneuvers such as translations, scales, and rotations. It has several potential applications in many fields, such as military, aerospace, and source-seeking. Ever since the pioneering work in [1], considerable attention has been devoted to affine formation maneuver control issues for systems with double-integrator, high-order, and general linear dynamics over undirected graphs [2,3]. In direct graphs, the corresponding control issues have been investigated in [4,5] for double-integrator and general linear multiagent systems. However, due to the directed-graph-induced asymmetry, the existing results usually require the knowledge of certain global information of the network and thus are not fully distributed. Moreover, in most existing studies, the inter-agent communication and control law updates have been executed in a continuous manner, which may not apply to scenarios with constrained communication bandwidth and limited energy resources. Therefore, it is reasonable to employ the so-called event-triggered methods to reduce unnecessary communication/updates [6–8].

This study aims to propose fully distributed event-triggered control protocols for agents with a leader-follower structure and generally linear dynamics to achieve affine formation maneuver control over directed graphs. Furthermore, to avoid the need for global information, the in-degree information is incorporated into the control law instead of a diagonal stabilizing matrix, and the proposed adaptive control protocol solely depends on local measurements. The dynamic event-triggered mechanism is exploited to avoid continuous interactions, and the Zeno behavior is excluded by showing a minimal time interval between any two triggering instants. In the presence of external disturbances, the σ-modification technique is used to guarantee that the formation tracking error can converge to a bounded set. The proposed control schemes are demonstrated to be useful via numerical simulations and hardware experiments.

Problem statement. Consider a group of agents with M leaders and $N-M$ followers. The dynamics of agent i are characterized by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^d$ and $u_i \in \mathbb{R}^p$ denote the state and the control input of agent i, respectively. Let $\mathcal{V}_f$ and $\mathcal{V}_l$ be the sets of followers and leaders, respectively.

Owing to the invariance to affine transformation, the stress matrix $\Omega = \left[\begin{smallmatrix} 0_{M\times M} & 0_{M\times(N-M)} \\ \Omega_{fl} & \Omega_{ff} \end{smallmatrix}\right]$ is chosen to describe the network topology.

Let $x_l = [x_1^{\mathrm{T}}, \ldots, x_M^{\mathrm{T}}]^{\mathrm{T}}$, $x_f = [x_{M+1}^{\mathrm{T}}, \ldots, x_N^{\mathrm{T}}]^{\mathrm{T}}$, and $p = [(x_l^*)^{\mathrm{T}}, (x_f^*)^{\mathrm{T}}]^{\mathrm{T}}$, where p is the target position. Using nonglobal assumptions about the directed graph, we can achieve affine localizability [2], which actually shows the nonsingularity of the matrix Ω_{ff} and implies the fact that $(\Omega \otimes I_d)p = 0$, i.e., $(\Omega_{fl} \otimes I_d)x_l^* + (\Omega_{ff} \otimes I_d)x_f^* = 0$. Then, the desired states of followers x_f^* can be uniquely determined as $x_f^* = -(\Omega_{ff}^{-1}\Omega_{fl} \otimes I_d)x_l^*$. We can derive the tracking error of followers as $\delta = x_f - x_f^* = x_f + (\Omega_{ff}^{-1}\Omega_{fl} \otimes I_d)x_l^*$. Without loss of generality, assume that leaders have already been steered properly, i.e., $x_l = x_l^*$. The primary objective of this study is then transferred to steering the followers such that $\delta \to 0$ as $t \to \infty$.

Fully distributed event-triggered protocols for directed graphs. A fully distributed event-triggered protocol will be proposed in this part. Assume that all agents in $\mathcal{V}_l$ have zero inputs. Partially motivated by [9], we propose the following control law for each follower:

$$\begin{aligned} u_i(t) &= \mathrm{sgn}\left(\sum_{j=1, j\neq i}^{N} w_{ij}\right)(d_i + \rho_i)K\hat{\xi}_i, \\ \dot{d}_i(t) &= \hat{\xi}_i^{\mathrm{T}}\Gamma\hat{\xi}_i, \quad i \in \mathcal{V}_f, \end{aligned} \tag{2}$$

where w_{ij} denotes the edge weight, $\hat{\xi}_i = \sum_{k=1}^{M} w_{ik}(\hat{x}_i - \hat{x}_k) + \sum_{j=M+1}^{N} w_{ij}(\hat{x}_i - \hat{x}_j)$, $\hat{x}_j(t) = \mathrm{e}^{A(t-t_k^j)}x_j(t_k^j)$, $\forall t \in [t_k^j, t_{k+1}^j)$ is the state estimation of agent j with t_k^j being the kth triggering instant, $\mathrm{sgn}(\cdot)$ denotes the sign function, $d_i(t)$ is the adaptive gain with initial value $d_i(0) \geqslant 1$, and

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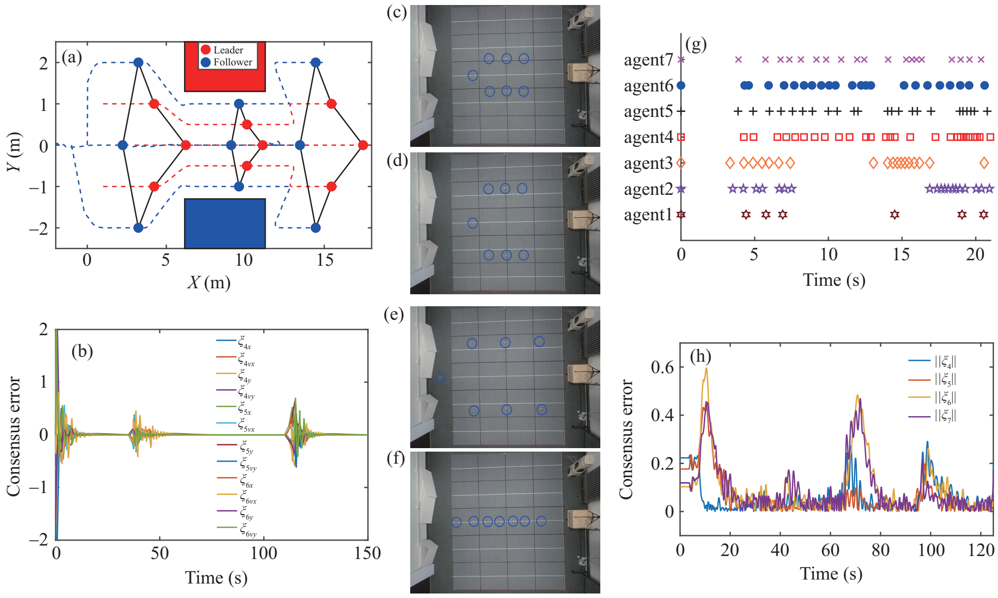

Figure 1 (Color online) (a) Trajectories of affine formation in simulation results; (b) consensus error of followers in simulation results; (c)–(f) snapshots of experiments; (g) triggering times of experiments; (h) consensus error of experiments.

$K=-E^{-1}B^{\mathrm{T}}Q$, $\Gamma=QBE^{-1}B^{\mathrm{T}}Q$ are feedback gain matrices with $Q>0$ being the solution to the following algebraic Riccati equation: $QA+A^{\mathrm{T}}Q-QBE^{-1}B^{\mathrm{T}}Q+L=0$, where $L>0$, $E>0$. In addition, $\rho_i=\hat{\xi}_i^{\mathrm{T}}Q\hat{\xi}_i$.

Define the estimation error as $\tilde{x}_i(t)=\hat{x}_i(t)-x_i(t)$. The triggering condition of agent i is devised as follows:

$$t_{k+1}^i=\inf\{t>t_k^i|\tilde{x}_i^{\mathrm{T}}(t)\tilde{x}_i(t)\geqslant\gamma_i\varepsilon_i(t)\},\quad i\in\mathcal{V},\tag{3}$$

where $t_k^0=0$, $\gamma_i\in\mathbb{R}^+$, and ε_i is an internal state with the following dynamics:

$$\dot{\varepsilon_i}(t)=-k_i\varepsilon_i(t)-\sigma_i\tilde{x}_i^{\mathrm{T}}(t)\tilde{x}_i(t),\tag{4}$$

where $\varepsilon_i(0)>0$, $k_i>0$ and $\sigma_i>0$.

Once the triggering condition (3) is satisfied, the corresponding agent broadcasts its sampled state information to its out-neighbors and updates its control law if it is a follower. Subsequently, when a follower receives updated information from its in-neighbors, its control law will also be updated.

We claim that the proposed event-triggered control protocol (2) and the triggering function (3) solve the affine formation maneuver control problem within the system (1). Moreover, the time interval between any two triggering instants is strictly positive.

To prove the claim, we first define $\xi_i=\sum_{k=1}^M w_{ik}(x_i-x_k)+\sum_{j=M+1}^N w_{ij}(x_i-x_j)$. Similar to the definition of x_f and x_l, let $\xi=(\Omega_{ff}\otimes I_d)x_f+(\Omega_{fl}\otimes I_d)x_l=(\Omega_{ff}\otimes I_d)\delta$. Noticing that Ω_{ff} is nonsingular, the objective $\delta\to 0$ can be transformed into $\xi\to 0$. Now, we construct a Lyapunov function as follows:

$$V_1=\sum_{i=M+1}^N r_i\left[\left(\bar{d}_i+\frac{1}{2}\bar{\rho}_i\right)\bar{\rho}_i+\frac{(\bar{d}_i-\alpha)^2}{2}+\beta\bar{d}_i^2\bar{V}_0\right],\tag{5}$$

where $\bar{V}_0=\sum_{j=1}^N\frac{\varepsilon_j}{k_j}$, $\bar{\rho}_i=\xi_i^{\mathrm{T}}Q\xi_i$. r_i, α, and β are positive constants, and $\bar{d}_i$ is the devised virtual adaptive gain whose dynamics are described by $\dot{\bar{d}}_i=\xi_i^{\mathrm{T}}\Gamma\xi_i$ with initial value $1\leqslant\bar{d}_i(0)\leqslant d_i(0)$.

Using Barbalat's lemma, we can conclude that $\xi\to 0$ as $t\to\infty$. Moreover, the minimal time interval between two triggering instants can be calculated as follows:

$$t_{k+1}^i-t_k^i \geqslant\frac{1}{\|A\|}\ln\left(1+\frac{\|A\|}{\Xi_i}\sqrt{\varepsilon_i(0)\exp[-(k_i+\sigma_i\gamma_i)t_{k+1}^i]}\right),\tag{6}$$

where Ξ_i is the upper bound of $(d_i+\rho_i)\|BK\xi_i\|$.

Fully distributed robust event-triggered protocols. In the previous section, the system (1) is free from external perturbations. However, in real-world implementations, the agent dynamics are often subject to external disturbances such as winds. Hence, it is imperative to design an event-triggered control algorithm with guaranteed robustness to external disturbances. In this study, the agent dynamics with perturbations are reformulated as follows:

$$\dot{x}_i=Ax_i+Bu_i+\varpi_i,\ i=1,\ldots,N,\tag{7}$$

where $\varpi_i\in\mathbb{R}^d$ denotes the external disturbance of agent i, satisfying the assumption that there exist positive constants ι and κ such that $\|\varpi_i\|\leqslant\iota$ for $i\in\mathcal{V}_f$ and $\|Bu_i+\varpi_i\|\leqslant\kappa$ for $i\in\mathcal{V}_l$.

The control law (2) needs to be modified to ensure that the consensus error ξ and the adaptive gain $d_i(t)$ will not diverge. Motivated by the well-known σ-modification technique, the proposed fully distributed robust event-triggered control protocol is proposed as

$$\begin{aligned}u_i(t)&=\mathrm{sgn}\left(\sum_{j=1,j\neq i}^N w_{ij}\right)(d_i+\rho_i)K\hat{\xi}_i,\\ \dot{d}_i(t)&=\hat{\xi}_i^{\mathrm{T}}\Gamma\hat{\xi}_i-\phi_i(d_i-1)^2,\quad i\in\mathcal{V}_f,\end{aligned}\tag{8}$$

where $\hat{\xi}_i$ and w_{ij} are defined in (2), ϕ_i is a positive constant, $\rho_i=\xi_i^{\mathrm{T}}P^{-1}\xi_i$, $K=-B^{\mathrm{T}}P^{-1}$, $\Gamma=P^{-1}BB^{\mathrm{T}}P^{-1}$, and $P>0$ is the solution to the following linear matrix inequality (LMI):

$$AP+PA^{\mathrm{T}}+\varsigma P-2BB^{\mathrm{T}}<0,\tag{9}$$

where $\varsigma > 1$ is a fixed coefficient.

We still choose (3) as the triggering condition of agent i. Then, the tracking error δ of followers and the adaptive gain d_i in (8) are ultimately bounded under the proposed robust protocol (8) and the triggering function (3).

To prove such a claim, a different Lyapunov function candidate V_2 is constructed as follows:

$$V_2 = \sum_{i=M+1}^{N} r_i \left[\left(d_i + \frac{1}{2}\bar{\rho}_i \right) \bar{\rho}_i + \frac{(d_i - \alpha)^2}{2} + \beta d_i^2 \bar{V}_0 \right]. \tag{10}$$

According to Barbalat's lemma, as the time moves toward infinity, the convergence error ξ will converge to a bounded set. Moreover, the Zeno behavior does not exist according to (6).

Simulation and experimental results. We performed simulations and experiments to verify the effectiveness of the proposed control algorithms. In numerical simulations, six agents with dynamics (1) are considered, and the trajectories of affine formation are illustrated in Figure 1(a), where the corresponding maneuvers are achieved according to different scenarios. The consensus error of three followers is shown in Figure 1(b), showing that the adaptive gain d_i tends to be fixed values according to the control law (3).

During the experimental validations, seven Crazyflie nano-quadcopters are considered to achieve affine formation maneuver control. We utilize Optitrack, a motion capture system, to provide the absolute coordinate system and the relative position information. Control commands for Crazyflies are broadcast from the ground station via crazyradio, which is a long-range open universal serial bus (USB) radio dongle capable of realizing 2.6 GHz radio communication.

The snapshots of the actual formation at 38, 58, 78, and 117 s are shown in Figures 1(c)–(f), where the translational, scaling, and shearing maneuvers are depicted in Figures 1(d)–(f), respectively. Figure 1(g) shows the triggering instants of seven drones in the first 23 s, which demonstrates the efficacy of the developed robust event-triggered control protocol (8) in reducing the number of communication interactions. Figure 1(h) shows the consensus error of follower drones. Despite the effect of wind disturbances, the final steady-state errors of followers still do not exceed 5 cm and the norms of the formation steady-state errors ξ_i do not exceed 0.1, demonstrating the good performance of the proposed event-triggered control protocol against external disturbances.

Conclusion. We have proposed fully distributed dynamic event-triggered control algorithms to solve the affine formation maneuver control problem of general linear multiagent systems over directed graphs, where the Zeno behavior has been theoretically excluded. In contrast to existing results, such as those reported by [4, 5], the control architecture has been derived in a fully distributed manner and is hence applicable to large-scale networks. Moreover, both simulations and experiments were performed to demonstrate the feasibility of the developed control laws.

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