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Fully distributed event-triggered affine formation maneuver control over directed graphs: theory and experiment

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Appendix A Proof of Claim 1

For convenience, we first clarify the assumptions made throughout this paper. For the given formation ($\mathcal{G}$,r), we assume that the nominal formation $\{r_i\}_{i=1}^{i=N}$ affinely span on $\mathbb{R}^d$, which means that the nominal configuration r is supposed to be generic. Then we assume that there are $d+1$ leaders in $\mathcal{V}_l$, i.e., $M = d+1$. In addition, every follower in $\mathcal{V}_f$ is $(d+1)$-reachable from the agents in $\mathcal{V}_l$. In addition, all Ω_{ij} in Ω_{ff} have the same sign when i is fixed and they are opposite to Ω_{ii}. Moreover, Ω_{ff} is a diagonally-dominant matrix.

The dynamics of the ith agent are characterized by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, 2, \cdots, N. \tag{A1}$$

Denote $\bar{d}_i$ as the virtual adaptive gain whose dynamics are described by

$$\dot{\bar{d}}_i = \xi_i^T \Gamma \xi_i \tag{A2}$$

with initial value $1 \leqslant \bar{d}_i(0) \leqslant d_i(0)$. Let $\tilde{d}_i = d_i - \bar{d}_i$, $\bar{\rho}_i = \xi_i^T Q \xi_i$, and $\tilde{\rho}_i = \rho_i - \bar{\rho}_i$.

Define $C_i = \operatorname{sgn}(\sum_{j=1, j\neq i}^{N} w_{ij})$, $C = \operatorname{diag}(C_{M+1}, \cdots, C_N)$, $D = \operatorname{diag}(d_{M+1}, \cdots, d_N)$ and $\rho = \operatorname{diag}(\rho_{M+1}, \cdots, \rho_N)$. The closed-loop dynamics of system (A1) in compact form can be described by

$$\begin{aligned} \dot{x}_f &= (I_{N-M} \otimes A)x_f + (I_{N-M} \otimes B)u_f \\ &= (I_{N-M} \otimes A)x_f + [C(D+\rho) \otimes BK]\hat{\xi}. \end{aligned} \tag{A3}$$

It then follows that the dynamics of tracking error ξ can be given by

$$\begin{aligned} \dot{\xi} =& (I_{N-M} \otimes A)\xi + [\tilde{\Omega}_{ff}(D+\rho) \otimes BK]\hat{\xi} \\ =& (I_{N-M} \otimes A)\xi + [\tilde{\Omega}_{ff}(\bar{D}+\bar{\rho}) \otimes BK]\xi \\ &+ [\tilde{\Omega}_{ff}(\tilde{D}+\tilde{\rho}) \otimes BK]\xi + [\tilde{\Omega}_{ff}(D+\rho) \otimes BK]\tilde{\xi}, \end{aligned} \tag{A4}$$

where $\tilde{\Omega}_{ff} = C\Omega_{ff}$ and $\bar{D}, \bar{\rho}, \tilde{D}, \tilde{\rho}$ have similar structures to D and ρ.

Construct the following Lyapunov function:

$$V_1 = \sum_{i=M+1}^{N} r_i[(\bar{d}_i + \frac{1}{2}\bar{\rho}_i)\bar{\rho}_i + \frac{(\bar{d}_i - \alpha)^2}{2} + \beta \bar{d}_i^2 \bar{V}_0], \tag{A5}$$

where $\bar{V}_0 = \sum_{j=1}^{N} \frac{\varepsilon_j}{k_j}$, r_i, α and β are positive constants. The time derivative of V_1 along the trajectories of (A1) is given by

$$\begin{aligned} \dot{V}_1 =& \sum_{i=M+1}^{N} r_i[(\bar{d}_i + \bar{\rho}_i)\dot{\bar{\rho}}_i + ((1+2\beta\bar{V}_0)\bar{d}_i - \alpha + \bar{\rho}_i)\dot{\bar{d}}_i] + \sum_{i=M+1}^{N} r_i \beta \bar{d}_i^2 \dot{\bar{V}}_0 \\ =& \xi^T[(\bar{D}+\bar{\rho})R \otimes (QA + A^TQ) - (\bar{D}+\bar{\rho})(R\tilde{\Omega}_{ff} + \tilde{\Omega}_{ff}^T R)(\bar{D}+\bar{\rho}) \otimes \Gamma]\xi \\ &- 2\xi^T[(\bar{D}+\bar{\rho})R\tilde{\Omega}_{ff}(D+\rho) \otimes \Gamma]\tilde{\xi} - 2\xi^T[(\bar{D}+\bar{\rho})R\tilde{\Omega}_{ff}(\tilde{D}+\tilde{\rho}) \otimes \Gamma]\xi \\ &+ \xi^T[((2\beta\bar{V}_0+1)\bar{D} - \alpha I_{N-M} + \bar{\rho})R \otimes \Gamma]\xi - \sum_{i=M+1}^{N} \beta r_i \bar{d}_i^2 \sum_{j=1}^{N} (\varepsilon_j + \frac{\sigma_j}{k_j}\tilde{x}_j^T \tilde{x}_j), \end{aligned} \tag{A6}$$

where $R = \operatorname{diag}(r_{M+1}, \cdots, r_N)$ is a positive diagonal matrix such that $R\tilde{\Omega}_{ff} + \tilde{\Omega}_{ff}^T R > 0$.

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Then we have

$$\begin{aligned}&-\xi^T[(\bar{D}+\bar{\rho})(R\tilde{\Omega}_{ff}+\tilde{\Omega}_{ff}^T R)(\bar{D}+\bar{\rho})\otimes\Gamma]\\&\leqslant-\xi^T[\lambda_{\min}(\hat{L})(\bar{D}+\bar{\rho})^2\otimes\Gamma]\xi,\end{aligned}\tag{A7}$$

where $\hat{L}=(R\tilde{\Omega}_{ff}+\tilde{\Omega}_{ff}^T R)$ and $\lambda_{\min}(\hat{L})$ denotes the minimum eigenvalue of matrix $\hat{L}$.

Based on the well-known Young's inequality, we can obtain that

$$\begin{aligned}|\tilde{\rho}_i|&=\left|\hat{\xi}_i^T Q\hat{\xi}_i-\xi_i^T Q\xi_i\right|\\&\leqslant\alpha_1\bar{\rho}_i+(1+\frac{1}{\alpha_1})\lambda_{\max}(Q)\tilde{\xi}_i^T\tilde{\xi}_i,\end{aligned}\tag{A8}$$

where $\alpha_1=\frac{\lambda_{\min}(\hat{L})}{4\sigma_{\max}(R\tilde{\Omega}_{ff})}$, $\lambda_{\max}(\cdot)$ denotes the greatest eigenvalue of the matrix and $\sigma_{\max}(\cdot)$ represents the maximum singular value of the matrix.

Note that

$$\begin{aligned}\tilde{\xi}_i^T\tilde{\xi}_i&=\sum_{j=1}^N w_{ij}(\tilde{x}_i-\tilde{x}_j)^T\sum_{j=1}^N w_{ij}(\tilde{x}_i-\tilde{x}_j)\\&\leqslant 2\Omega_{ii}^2\tilde{x}_i^T\tilde{x}_i+2\sum_{j=1}^N w_{ij}\tilde{x}_j^T\sum_{j=1}^N w_{ij}\tilde{x}_j\\&\leqslant 2\Omega_{ii}^2\tilde{x}_i^T\tilde{x}_i+2\eta_i\sum_{j=1}^N\tilde{x}_j^T\sum_{j=1}^N\tilde{x}_j\\&\leqslant 2\eta_i^2\sum_{j=1}^N\tilde{x}_j^T\tilde{x}_j\\&\leqslant 2\eta_i^2\sum_{j=1}^N\gamma_j\varepsilon_j(0),\end{aligned}\tag{A9}$$

where $\eta_i=\sum_{j=1,j\neq i}^N|w_{ij}|$ and the last inequality is obtained by the triggering condition.

Next, we can derive from (A8) and (A9) that

$$|\tilde{\rho}_i|\leqslant\alpha_1\bar{\rho}_i+2(1+\frac{1}{\alpha_1})\lambda_{\max}(Q)\eta_i^2\sum_{j=1}^N\gamma_j\varepsilon_j(0).\tag{A10}$$

By using Young's inequality again, we can also obtain:

$$\begin{aligned}\left|\tilde{d}_i(t)\right|&=\left|\tilde{d}_i(0)+\int_0^t[\tilde{\xi}_i^T(\tau)\Gamma\tilde{\xi}_i(\tau)+2\tilde{\xi}_i^T(\tau)\Gamma\xi_i(\tau)]\,d\tau\right|\\&\leqslant\tilde{d}_i(0)+\alpha_1\bar{d}_i-\alpha_1\bar{d}_i(0)+2\lambda_{\max}(\Gamma)(1+\frac{1}{\alpha_1})\eta_i^2\sum_{j=1}^N\frac{\gamma_j\varepsilon_j(0)}{k_j}.\end{aligned}\tag{A11}$$

Combining (A10) and (A11), we have

$$\left|\tilde{d}_i+\tilde{\rho}_i\right|\leqslant\alpha_1(\bar{d}_i+\bar{\rho}_i)+\beta_1,\tag{A12}$$

where $\beta_1=\max_i\{\tilde{d}_i(0)+\alpha_1\bar{d}_i(0)\}+2(1+\frac{1}{\alpha_1})\max_i\{\eta_i^2\}\sum_{j=1}^N(\lambda_{\max}(Q)+\frac{\lambda_{\max}(\Gamma)}{k_j})\gamma_j\varepsilon_j(0)$.

Next, we can obtain that

$$\begin{aligned}&-2\xi^T[(\bar{D}+\bar{\rho})R\tilde{\Omega}_{ff}(\tilde{D}+\tilde{\rho})\otimes\Gamma]\xi\\&\leqslant\frac{\lambda_{\min}(\hat{L})}{4}\xi^T[(\bar{D}+\bar{\rho})^2\otimes\Gamma]\xi+\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\xi^T[(\tilde{D}+\tilde{\rho})^2\otimes\Gamma]\xi,\end{aligned}\tag{A13}$$

where

$$\begin{aligned}&\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\xi^T[(\tilde{D}+\tilde{\rho})^2\otimes\Gamma]\xi\\&\leqslant\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^N[\alpha_1(\bar{d}_i+\bar{\rho}_i)+\beta_1]^2\xi_i^T\Gamma\xi_i\\&=\frac{\lambda_{\min}(\hat{L})}{4}\xi^T[(\bar{D}+\bar{\rho})^2\otimes\Gamma]\xi+2\beta_1\sigma_{\max}(R\tilde{\Omega}_{ff})\xi^T[(\bar{D}+\bar{\rho})\otimes\Gamma]\xi\\&\quad+\frac{\beta_1^2\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\xi^T[I_{N-M}\otimes\Gamma]\xi.\end{aligned}\tag{A14}$$

Substituting (A14) into (A13) yields the following inequality:

$$\begin{aligned}&-2\xi^T[(\bar{D}+\bar{\rho})R\tilde{\Omega}_{ff}(\tilde{D}+\tilde{\rho})\otimes\Gamma]\xi\\ \leqslant&\frac{\lambda_{\min}(\hat{L})}{2}\xi^T[(\bar{D}+\bar{\rho})^2\otimes\Gamma]\xi+2\beta_1\sigma_{\max}(R\tilde{\Omega}_{ff})\xi^T\\ &\times[(\bar{D}+\bar{\rho})\otimes\Gamma]\xi+\frac{\beta_1^2\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\xi^T[I_{N-M}\otimes\Gamma]\xi.\end{aligned}\tag{A15}$$

Similar to (A10) and (A11), it then follows that

$$\begin{aligned}\rho_i\leqslant&(1+\alpha_1)\xi_i^TQ\xi_i+(1+\frac{1}{\alpha_1})\tilde{\xi}_i^TQ\tilde{\xi}_i\\ \leqslant&(1+\alpha_1)\bar{\rho}_i+2\lambda_{\max}(Q)(1+\frac{1}{\alpha})\eta_i^2\sum_{j=1}^N\gamma_j\varepsilon_j(0)\end{aligned}\tag{A16}$$

and

$$\begin{aligned}d_i=&d_i(0)+\int_0^t[\tilde{\xi}_i^T\Gamma\tilde{\xi}_i+2\tilde{\xi}_i^T\Gamma\xi_i^T+\xi_i^T\Gamma\xi_i]\,d\tau\\ \leqslant&d_i(0)+(1+\alpha_1)\bar{d}_i-(1+\alpha_1)\bar{d}_i(0)+2\lambda_{\max}(\Gamma)(1+\frac{1}{\alpha_1})\eta_i^2\sum_{j=1}^N\frac{\gamma_j\varepsilon_j(0)}{k_j},\end{aligned}\tag{A17}$$

where we use $\tilde{\xi}_i$ and ξ_i to denote $\tilde{\xi}_i(\tau)$ and $\xi_i(\tau)$.

Combine (A16) and (A17) to get the following result:

$$d_i+\rho_i\leqslant(1+\alpha_1)(\bar{d}_i+\bar{\rho}_i)+\beta_2,\tag{A18}$$

where $\beta_2=\max_i\{d_i(0)+(1+\alpha_1)\bar{d}_i(0)\}+2(1+\frac{1}{\alpha_1})\max_i\{\eta_i^2\}\sum_{j=1}^N[\lambda_{\max}(Q)+\frac{\lambda_{\max}(\Gamma)}{k_j}]\gamma_j\varepsilon_j(0)$.

Next, the second term in (A6) can be scaled as follows:

$$\begin{aligned}&-2\xi^T[(\bar{D}+\bar{\rho})R\tilde{\Omega}_{ff}(D+\rho)\otimes\Gamma]\tilde{\xi}\\ \leqslant&\frac{\lambda_{\min}(\hat{L})}{4}\xi^T[(\bar{D}+\bar{\rho})^2\otimes\Gamma]\xi+\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\tilde{\xi}^T[(D+\rho)^2\otimes\Gamma]\tilde{\xi}.\end{aligned}\tag{A19}$$

Substituting (A18) into the second term in (A19) yields

$$\begin{aligned}&\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\tilde{\xi}^T[(D+\rho)^2\otimes\Gamma]\tilde{\xi}\\ \leqslant&\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^N[(1+\alpha_1)(\bar{d}_i+\bar{\rho}_i)+\beta_2]^2\tilde{\xi}_i^T\Gamma\tilde{\xi}_i\\ =&\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^N[(1+\alpha_1)^2\bar{d}_i^2+2(1+\alpha_1)\bar{d}_i\beta_2\\ &+\beta_2^2]\tilde{\xi}_i^T\Gamma\tilde{\xi}_i+\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^N[(1+\alpha_1)^2\bar{\rho}_i\\ &+2(1+\alpha_1)^2\bar{d}_i+2\beta_2(1+\alpha_1)]\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i.\end{aligned}\tag{A20}$$

Noticing that $\bar{d}_i\geqslant1$, we have

$$\begin{aligned}&\sum_{i=M+1}^N[(1+\alpha_1)^2\bar{d}_i^2+2(1+\alpha_1)\bar{d}_i\beta_2+\beta_2^2]\tilde{\xi}_i^T\Gamma\tilde{\xi}_i\\ \leqslant&\sum_{i=M+1}^N[(1+\alpha_1)^2+2(1+\alpha_1)\beta_2+\beta_2^2]\bar{d}_i^2\tilde{\xi}_i^T\Gamma\tilde{\xi}_i\\ \leqslant&2\sum_{i=M+1}^N(1+\alpha_1+\beta_2)^2\lambda_{\max}(\Gamma)\eta_i^2\bar{d}_i^2\sum_{j=1}^N\tilde{x}_j^T\tilde{x}_j\end{aligned}\tag{A21}$$

and

$$\begin{aligned}&[(1+\alpha_1)^2\bar{\rho}_i+2(1+\alpha_1)^2\bar{d}_i+2\beta_2(1+\alpha_1)]\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i\\ \leqslant&[(1+\alpha_1)^2\bar{\rho}_i+2(1+\alpha_1)(1+\alpha_1+\beta_2)\bar{d}_i]\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i\\ \leqslant&\max[(1+\alpha_1)^2,2(1+\alpha_1)(1+\alpha_1+\beta_2)](\bar{d}_i+\bar{\rho}_i)\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i\\ \leqslant&\alpha_2(\bar{d}_i+\bar{\rho}_i)\bar{\rho}_i\sum_{j=1}^N\gamma_j\varepsilon_j(t),\end{aligned}\tag{A22}$$

where $\alpha_2 = 2\max\{(1+\alpha_1)^2, 2(1+\alpha_1)(1+\alpha_1+\beta_2)\}\lambda_{\max}(\Gamma)\max_i\{\eta_i^2\}$.

Note that the triggering condition along with the dynamics of the internal state actually indicates that $-(k_i+\sigma_i\gamma_i)\varepsilon_i \leqslant \dot{\varepsilon}_i \leqslant -k_i\varepsilon_i$. It then follows that

$$\exp[-(k_i+\sigma_i\gamma_i)t]\varepsilon_i(0) \leqslant \varepsilon_i(t) \leqslant \exp(-k_i t)\varepsilon_i(0), \tag{A23}$$

which means that ε_i decreases at an exponential rate. Thus there definitely exists a time threshold T_0 such that $\alpha_2\sum_{j=1}^N \gamma_j\varepsilon_j(t) \leqslant \frac{\alpha_1\lambda_{\min}(R)\lambda_{\min}(X)}{2\sigma_{\max}[R\tilde{\Omega}_{ff}]\lambda_{\max}(Q)}$ for all $t > T_0$, where $X = -(QA + A^TQ - \Gamma) > 0$. Then substituting (A21) and (A22) into (A19), we have

$$\begin{aligned} &-2\xi^T[(\bar{D}+\bar{\rho})R\tilde{\Omega}_{ff}(D+\rho)\otimes\Gamma]\tilde{\xi} \\ \leqslant&\frac{\lambda_{\min}(\hat{L})}{4}\xi^T[(\bar{D}+\bar{\rho})^2\otimes\Gamma]\xi + \frac{1}{2}\xi^T[(\bar{D}+\bar{\rho})R\otimes X]\xi \\ &+\frac{2\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^N(1+\alpha_1+\beta_2)^2\lambda_{\max}(\Gamma)\eta_i^2\bar{d}_i^2\sum_{j=1}^N\tilde{x}_j^T\tilde{x}_j. \end{aligned} \tag{A24}$$

Let $\bar{\alpha}_0 > 0$. Select $\alpha = \frac{\beta_1^2\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1\lambda_{\min}(R)} + \bar{\alpha}_0$ and $\beta \geqslant \frac{2\sigma_{\max}(R\tilde{\Omega}_{ff})}{\min_i\{r_i\}\times\alpha_1}(1+\alpha_1+\beta_2)^2\lambda_{\max}(\Gamma)\max_i\{\eta_i^2\}\max_j\{\frac{k_j}{\sigma_j}\}$, then substituting(A7), (A15) and (A24) into (A6) yields the following inequality:

$$\begin{aligned} \dot{V}_1 \leqslant& \xi^T[(\bar{D}+\bar{\rho})R\otimes(QA+A^TQ)]\xi - \xi^T[\frac{\lambda_{\min}(\hat{L})}{4}(\bar{D}+\bar{\rho})^2\otimes\Gamma]\xi \\ &+2\beta_1\sigma_{\max}(R\tilde{\Omega}_{ff})\xi^T[(\bar{D}+\bar{\rho})\otimes\Gamma]\xi + \frac{1}{2}\xi^T[(\bar{D}+\bar{\rho})R\otimes X]\xi \\ &+\xi^T[(1+2\beta\bar{V}_0(0))R(\bar{D}+\bar{\rho})\otimes\Gamma]\xi - \xi^T(\bar{\alpha}_0R\otimes\Gamma)\xi. \end{aligned} \tag{A25}$$

Selecting $\bar{\beta}_0 = 1 + 2\beta\bar{V}_0(0) + \frac{2\beta_1\sigma_{\max}(R\tilde{\Omega}_{ff})}{\lambda_{\min}(R)}$, it then follows that

$$\begin{aligned} \dot{V}_1 \leqslant&\xi^T[(\bar{D}+\bar{\rho})R\otimes(QA+A^TQ+\frac{X}{2}+\bar{\beta}_0\Gamma)]\xi \\ &-\xi^T[(\frac{\lambda_{\min}(\hat{L})}{4}(\bar{D}+\bar{\rho})^2+\bar{\alpha}_0R)\otimes]\xi. \end{aligned} \tag{A26}$$

Based on Young's inequality and selecting $\bar{\alpha}_0 \geqslant \frac{(1+\beta_0)^2\lambda_{\max}(R)}{\lambda_{\min}(\hat{L})}$, we have

$$\begin{aligned} \dot{V}_1 \leqslant&\xi^T[(\bar{D}+\bar{\rho})R\otimes(QA+A^TQ+\frac{X}{2}+\beta_0\Gamma)]\xi \\ &-(1+\beta_0)\xi^T[(\bar{D}+\bar{\rho})R\otimes\Gamma]\xi \\ =&-\frac{1}{2}\xi^T[(\bar{D}+\bar{\rho})R\otimes X]\xi \\ \leqslant&0. \end{aligned} \tag{A27}$$

Therefore, V_1 is bounded, which implies that ξ_i, $\bar{\rho}_i$ and $\bar{d}_i$ are bounded. Use V^∞ to denote the bounded limit of V_1 when $t \to \infty$. It then follows that

$$\int_0^\infty \xi^T[(\bar{D}+\bar{\rho})R\otimes X]\xi \leqslant 2[V_1(0) - V^\infty].$$

It can be inferred from (A16) and (A17) that ρ_i and d_i are bounded, i.e., $\tilde{d}_i$ and $\tilde{\rho}_i$ are bounded. It then can be deduced from the triggering condition that $\tilde{x}_i$ is bounded, implying that $\tilde{\xi}_i$ is also bounded. Thus we can deduce from (A4) that $\dot{\xi}$ is bounded. Hence, $\xi^T[(\bar{D}+\bar{\rho})R\otimes X]\dot{\xi}$ is bounded. By using Barbalat's Lemma, we can conclude that $\xi^T[(\bar{D}+\bar{\rho})R\otimes X]\xi \to 0$ as $t \to \infty$, i.e., $\xi \to 0$ as $t \to \infty$. This completes the proof of the first part of the claim.

Next we will show that the system does not exist Zeno behaviour under the proposed triggering condition. From the triggering condition, we have

$$\tilde{x}_i^T\tilde{x}_i \leqslant \gamma_i\varepsilon_i(0)\exp(-k_i t). \tag{A28}$$

The time derivative of $\tilde{x}_i$ can be obtained as follows:

$$\begin{aligned} \dot{\tilde{x}}_i &= \dot{\hat{x}}_i - \dot{x}_i \\ &= A\tilde{x}_i - (d_i+\rho_i)BK\hat{\xi}_i \end{aligned} \tag{A29}$$

It then follows that:

$$D^+\|\tilde{x}_i(t)\| \leqslant \|A\|\|\tilde{x}_i\| + (d_i+\rho_i)\|BK\hat{\xi}_i\|, \tag{A30}$$

where $D^+\|\tilde{x}_i(t)\|$ is the right-hand Dini derivative of $\tilde{x}_i$. Note that d_i, ρ_i and $\hat{\xi}_i$ is bounded, we can get that

$$\|\tilde{x}_i(t)\| \leqslant \frac{\Xi_i}{\|A\|}(\exp(\|A\|(t-t_k^i))-1), \tag{A31}$$

where Ξ_i is the upper bound of $(d_i+\rho_i)\|BK\xi_i\|$. Thus we can infer from the triggering condition that the triggering time t^i_{k+1} of the ith agent satisfies:

$$\begin{aligned}&[\frac{\Xi_i}{\|A\|}(\exp(\|A\|(t^i_{k+1}-t^i_k))-1)]^2\\ \geqslant&\varepsilon_i(0)\exp[-(k_i+\sigma_i\gamma_i)t^i_{k+1}],\end{aligned} \tag{A32}$$

which means that

$$\begin{aligned}&t^i_{k+1}-t^i_k\geqslant\\ &\frac{1}{\|A\|}\ln\left(1+\frac{\|A\|}{\Xi_i}\sqrt{\varepsilon_i(0)\exp[-(k_i+\sigma_i\gamma_i)t^i_{k+1}]}\right).\end{aligned} \tag{A33}$$

If Zeno behaviour exists, we can find a $T>0$ such that $t^i_k<T$ as $k\to\infty$. However, $t^i_{k+1}-t^i_k$ is strictly positive according to (A33). Hence, $t^i_{k+1}\to\infty$ as $k\to\infty$, which means that Zeno behaviour does not exist under the proposed control law and triggering condition. This completes the proof of the second part of the claim.

Appendix B Proof of Claim 2

The agent dynamics with perturbations are reformulated as follows:

$$\dot{x}_i=Ax_i+Bu_i+\varpi_i, i=1,\cdots,N, \tag{B1}$$

To prove Claim 2, here we no longer select virtue gain $\bar{d}_i$ and construct a different Lyapunov function candidate V_2 as follows:

$$V_2=\sum_{i=M+1}^{N}r_i[(d_i+\frac{1}{2}\bar{\rho}_i)\bar{\rho}_i+\frac{(d_i-\alpha)^2}{2}+\beta d_i^2\bar{V}_0]. \tag{B2}$$

Let $\varpi_l=\mathrm{diag}(Bu_1+\varpi_1,\cdots,Bu_M+\varpi_M)$, $\varpi_f=\mathrm{diag}(\varpi_{M+1},\cdots,\varpi_N)$ and $\phi=\mathrm{diag}(\phi_{M+1},\cdots,\phi_N)$, then the time derivative of V_2 along the trajectories of the system (B1) under the control law and the triggering condition is rewritten as follows:

$$\begin{aligned}\dot{V}_2=&\sum_{i=M+1}^{N}r_i[(d_i+\bar{\rho}_i)\dot{\bar{\rho}}_i+((1+2\beta\bar{V}_0)d_i-\alpha+\bar{\rho}_i)\dot{d}_i]+\sum_{i=M+1}^{N}r_i\beta d_i^2\dot{\bar{V}}_0\\ =&\xi^T[(D+\bar{\rho})R\otimes(P^{-1}A+A^TP^{-1})-(D+\bar{\rho})(R\tilde{\Omega}_{ff}+\tilde{\Omega}_{ff}^TR)(D+\bar{\rho})\otimes\Gamma]\xi\\ &-2\xi^T[(D+\bar{\rho})R\tilde{\Omega}_{ff}(D+\rho)\otimes\Gamma]\tilde{\xi}-2\xi^T[(D+\bar{\rho})R\tilde{\Omega}_{ff}\tilde{\rho}\otimes\Gamma]\xi\\ &+2\xi^T[(D+\bar{\rho})R\Omega_{ff}\otimes P^{-1}]\varpi_f+2\xi^T[(D+\bar{\rho})R\Omega_{fl}\otimes P^{-1}]\varpi_l\\ &+\hat{\xi}^T[((1+2\beta\bar{V}_0)D-\alpha I_{N-M}+\bar{\rho})R\otimes\Gamma]\hat{\xi}-\xi^T[\phi(D-I_{N-M})^2R\otimes P^{-1}]\xi\\ &-\sum_{i=M+1}^{N}r_i[(1+2\beta\bar{V}_0)d_i-\alpha]\phi_i(d_i-1)^2-\sum_{i=M+1}^{N}\beta r_id_i^2\sum_{j=1}^{N}(\varepsilon_j+\frac{\sigma_j}{k_j}\tilde{x}_j^T\tilde{x}_j).\end{aligned} \tag{B3}$$

Similar to (A7), we can derive that

$$\begin{aligned}&-\xi^T[(D+\bar{\rho})(R\tilde{\Omega}_{ff}+\tilde{\Omega}_{ff}^TR)(D+\bar{\rho})\otimes\Gamma]\\ \leqslant&-\xi^T[\lambda_{\min}(\hat{L})(D+\bar{\rho})^2\otimes\Gamma]\xi,\end{aligned} \tag{B4}$$

Following the similar line of (A8), we have

$$|\tilde{\rho}_i|\leqslant\alpha_1\bar{\rho}_i+\hat{\beta}_1, \tag{B5}$$

where $\alpha_1=\frac{\lambda_{\min}(\hat{L})}{4\sigma_{\max}(R\tilde{\Omega}_{ff})}$, $\hat{\beta}_1=2(1+\frac{1}{\alpha_1})\lambda_{\max}(P^{-1})\max_i\{\eta_i^2\}\sum_{j=1}^N\gamma_j\varepsilon_j(0)$.

Considering the fact that $d_i\geqslant1$ and $\alpha_1>0$, it then follows from (B5) that

$$|\tilde{\rho}_i|\leqslant\alpha_1(d_i+\bar{\rho}_i)+\hat{\beta}_1. \tag{B6}$$

Subsequently, we can obtain the following inequality:

$$\begin{aligned}&-2\xi^T[(D+\bar{\rho})R\tilde{\Omega}_{ff}\tilde{\rho}\otimes\Gamma]\xi\\ \leqslant&\frac{\lambda_{\min}(\hat{L})}{2}\xi^T[(D+\bar{\rho})^2\otimes\ \Gamma]\xi+2\hat{\beta}_1\sigma_{\max}(R\tilde{\Omega}_{ff})\\ &\times\xi^T[(D+\bar{\rho})\otimes\Gamma]\xi+\frac{\hat{\beta}_1^2\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\xi^T[I_{N-M}\otimes\Gamma]\xi.\end{aligned} \tag{B7}$$

Similar to (A16), we can also obtain that

$$\begin{aligned}\rho_i=&\tilde{\xi}_i^TP^{-1}\tilde{\xi}_i+2\tilde{\xi}_i^TP^{-1}\xi_i^T+\xi_i^TP^{-1}\xi_i\\ \leqslant&(1+\alpha_1)\bar{\rho}_i+\hat{\beta}_2\end{aligned} \tag{B8}$$

where $\hat{\beta}_2 = \hat{\beta}_1 = 2(1+\frac{1}{\alpha_1})\lambda_{\max}(P^{-1})\max_i\{\eta_i^2\}\sum_{j=1}^{N}\gamma_j\varepsilon_j(0)$. Obviously it then follows that

$$|d_i+\rho_i| \leqslant (1+\alpha_1)(d_i+\bar{\rho}_i)+\hat{\beta}_1. \tag{B9}$$

Leveraging the inequality (B9), we have the following relation:

$$\begin{aligned}
&-2\xi^T[(D+\bar{\rho})R\tilde{\Omega}_{ff}(D+\rho)\otimes\Gamma]\tilde{\xi} \\
\leqslant&\frac{\lambda_{\min}(\hat{L})}{4}\xi^T[(D+\bar{\rho})^2\otimes\Gamma]\xi+\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^{N}[(1+\alpha_1)^2\bar{\rho}_i+2(1+\alpha_1)(1+\alpha_1 \\
&+\hat{\beta}_1)d_i]\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i+\frac{2\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^{N}(1+\alpha_1+\hat{\beta}_1)^2\lambda_{\max}(\Gamma)\eta_i^2d_i^2\sum_{j=1}^{N}\tilde{x}_j^T\tilde{x}_j
\end{aligned} \tag{B10}$$

In view of the fact that $\tilde{\xi}=\hat{\xi}-\xi$, it can be seen that

$$\begin{aligned}
&\hat{\xi}^T[((1+2\beta\bar{V}_0)D-\alpha I_{N-M}+\bar{\rho})R\otimes\Gamma]\hat{\xi} \\
=&\xi^T[((1+2\beta\bar{V}_0)D+\bar{\rho})R\otimes\Gamma]\xi+\tilde{\xi}^T[((1+2\beta\bar{V}_0)D+\bar{\rho})R\otimes\Gamma]\tilde{\xi} \\
&+2\tilde{\xi}^T[((1+2\beta\bar{V}_0)D+\bar{\rho})R\otimes\Gamma]\xi-\alpha(\tilde{\xi}+\xi)^T(R\otimes\Gamma)(\tilde{\xi}+\xi).
\end{aligned} \tag{B11}$$

Letting $\alpha>1$ and considering the fact that $\bar{V}_0$ decays exponentially fast, there definitely exists a time threshold T_2 such that $1+2\beta\bar{V}_0\leqslant 2$ when $t\geqslant T_2$, which implies that

$$\begin{aligned}
&\hat{\xi}^T[((1+2\beta\bar{V}_0)D-\alpha I_{N-M}+\bar{\rho})R\otimes\Gamma]\hat{\xi} \\
\leqslant&\xi^T[((1+2\beta\bar{V}_0)D+\bar{\rho})R\otimes\Gamma]\xi+2\tilde{\xi}^T[(D+\bar{\rho})R\otimes\Gamma]\tilde{\xi}+4\tilde{\xi}^T[\sqrt{(D+\bar{\rho})R}\sqrt{(D+\bar{\rho})R}\otimes\Gamma]\xi \\
&-(\alpha-1)\xi^T(R\otimes\Gamma)\xi+2(\alpha^2-\alpha)\sum_{i=M+1}^{N}r_i\lambda_{\max}(\Gamma)\eta_i^2\sum_{j=1}^{N}\tilde{x}_j^T\tilde{x}_j \\
\leqslant&\xi^T[((1+2\beta\bar{V}_0)D+\bar{\rho})R\otimes\Gamma]\xi+4\tilde{\xi}^T[(D+\bar{\rho})R\otimes\Gamma]\tilde{\xi}+2\xi^T[(D+\bar{\rho})R\otimes\Gamma]\xi \\
&-(\alpha-1)\xi^T(R\otimes\Gamma)\xi+2(\alpha^2-\alpha)\sum_{i=M+1}^{N}r_i\lambda_{\max}(\Gamma)\eta_i^2\sum_{j=1}^{N}\tilde{x}_j^T\tilde{x}_j.
\end{aligned} \tag{B12}$$

Note that

$$\begin{aligned}
&4\tilde{\xi}^T[(D+\bar{\rho})R\otimes\Gamma]\tilde{\xi} \\
=&4\tilde{\xi}^T[DR\otimes\Gamma]\tilde{\xi}+4\tilde{\xi}^T[\bar{\rho}R\otimes\Gamma]\tilde{\xi} \\
\leqslant&8\sum_{i=M+1}^{N}r_id_i\lambda_{\max}(\Gamma)\eta_i^2\sum_{j=1}^{N}\tilde{x}_j^T\tilde{x}_j+4\sum_{i=M+1}^{N}r_i(d_i+\bar{\rho}_i)\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i.
\end{aligned} \tag{B13}$$

Substituting (B13) into (B12) yields:

$$\begin{aligned}
&\hat{\xi}^T[((1+2\beta\bar{V}_0)D-\alpha I_{N-M}+\bar{\rho})R\otimes\Gamma]\hat{\xi} \\
\leqslant&\xi^T[((1+2\beta\bar{V}_0)D+\bar{\rho})R\otimes\Gamma]\xi+2\xi^T[(D+\bar{\rho})R\otimes\Gamma]\xi+8\sum_{i=M+1}^{N}r_id_i\lambda_{\max}(\Gamma)\eta_i^2\sum_{j=1}^{N}\tilde{x}_j^T\tilde{x}_j \\
&+4\sum_{i=M+1}^{N}r_i(d_i+\bar{\rho}_i)\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i-(\alpha-1)\xi^T(R\otimes\Gamma)\xi+2(\alpha^2-\alpha)\sum_{i=M+1}^{N}r_i\lambda_{\max}(\Gamma)\eta_i^2\sum_{j=1}^{N}\tilde{x}_j^T\tilde{x}_j.
\end{aligned} \tag{B14}$$

Considering the fact that $\alpha>1$ and $\beta\bar{V}_0>0$, we can derive the following relation:

$$\begin{aligned}
&-\sum_{i=M+1}^{N}r_i[(1+2\beta\bar{V}_0)d_i-\alpha]\phi_i(d_i-1)^2 \\
\leqslant&-\sum_{i=M+1}^{N}r_i\phi_i(d_i-\alpha)(d_i-1)^2 \\
=&\sum_{i=M+1}^{N}r_i\phi_i[-(d_i-1)^3+(\alpha-1)(d_i-1)^2] \\
\leqslant&\sum_{i=M+1}^{N}r_i\phi_i[-\frac{1}{3}(d_i-1)^3+\frac{1}{3}(\alpha-1)^3].
\end{aligned} \tag{B15}$$

For a positive constant χ which satisfies $\chi<\min_{j\in\mathcal{V}_f}k_j$, we have

$$\begin{aligned}
\frac{\chi}{2}(d_i-\alpha)^2\leqslant&\frac{\chi}{2}(d_i-1)^2+\frac{\chi}{2}(\alpha-1)^2 \\
=&(\frac{\phi_i}{2})^{\frac{2}{3}}(d_i-1)^2\cdot\frac{\chi}{2}(\frac{\phi_i}{2})^{-\frac{2}{3}}+\frac{\chi}{2}(\alpha-1)^2 \\
\leqslant&\frac{\phi_i}{3}(d_i-1)^3+\frac{\chi^3}{6\phi_i^2}+\frac{\chi}{2}(\alpha-1)^2.
\end{aligned} \tag{B16}$$

Based on Young's inequality, we can derive the following result:

$$\begin{aligned}&2\xi^T[(D+\bar{\rho})R\Omega_{ff}\otimes P^{-1}]\varpi_f\\ \leqslant&\frac{1}{2}\xi^T[(D-I)^2\phi R\otimes P^{-1}]\xi+2\|(\sqrt{\phi^{-1}R}\Omega_{ff}\otimes\sqrt{P^{-1}})\varpi_f\|^2+4\|(\sqrt{R}\Omega_{ff}\\ &\otimes\sqrt{P^{-1}})\varpi_f\|^2+16\|(R^{\frac{1}{4}}\Omega_{ff}\otimes\sqrt{P^{-1}})\varpi_f\|^4+\frac{1}{2}\xi^T[(\bar{\rho}+\frac{1}{2}I_{N-M})R\otimes P^{-1}]\xi.\end{aligned}\tag{B17}$$

Next, the following inequality can be obtained analogously:

$$\begin{aligned}&2\xi^T[(D+\bar{\rho})R\Omega_{fl}\otimes P^{-1}]\varpi_l\\ \leqslant&\frac{1}{2}\xi^T[(D-I)^2\phi R\otimes P^{-1}]\xi+2\|(\sqrt{\phi^{-1}R}\Omega_{fl}\otimes\sqrt{P^{-1}})\varpi_f\|^2+4\|(\sqrt{R}\Omega_{fl}\\ &\otimes\sqrt{P^{-1}})\varpi_l\|^2+16\|(R^{\frac{1}{4}}\Omega_{fl}\otimes\sqrt{P^{-1}})\varpi_l\|^4+\frac{1}{2}\xi^T[(\bar{\rho}+\frac{1}{2}I_{N-M})R\otimes P^{-1}]\xi.\end{aligned}\tag{B18}$$

For brevity's sake, we let $\zeta_1=2\|(\sqrt{\phi^{-1}R}\Omega_{ff}\otimes\sqrt{P^{-1}})\|^2+4\|(\sqrt{R}\Omega_{ff}\otimes\sqrt{P^{-1}})\|^2$, $\zeta_2=16\|(R^{\frac{1}{4}}\Omega_{ff}\otimes\sqrt{P^{-1}})\|^4$, $\zeta_3=2\|(\sqrt{\phi^{-1}R}\Omega_{fl}\otimes\sqrt{P^{-1}})\|^2+4\|(\sqrt{R}\Omega_{fl}\otimes\sqrt{P^{-1}})\|^2$, and $\zeta_4=16\|(R^{\frac{1}{4}}\Omega_{fl}\otimes\sqrt{P^{-1}})\|^4$.

In light of the fact that $d_i\geqslant 1$, let $\beta\geqslant[2(1+\alpha_1+\hat{\beta}_1)^2\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}+2(\alpha^2-\alpha+4)\max_i\{r_i\}]\lambda_{\max}(\Gamma)\max_i\{\eta_i^2\}$ $\max_j\{\frac{k_j}{\sigma_j}\}$. Then substituting (B4), (B7), (B10), (B14), (B15), (B17) and (B18) into (B3) yields the following inequality:

$$\begin{aligned}\dot{V}_2\leqslant&-\chi V_2+\xi^T[(D+\bar{\rho})R\otimes(P^{-1}A+A^TP^{-1}+(\chi+1)P^{-1})]\xi-\frac{\lambda_{\min}(\hat{L})}{4}\xi^T[(D+\bar{\rho})^2\otimes\Gamma]\xi\\ &+2\hat{\beta}_1\sigma_{\max}(R\tilde{\Omega}_{ff})\xi^T[(D+\bar{\rho})\otimes\Gamma]\xi+\frac{\hat{\beta}_1^2\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\xi^T[I_{N-M}\otimes\Gamma]\xi\\ &+\xi^T[((1+2\beta\bar{V}_0)D+\bar{\rho}-(\alpha-1)I_{N-M})R\otimes\Gamma]\xi\\ &+\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^{N}[(1+\alpha_1)^2\bar{\rho}_i+2(1+\alpha_1)(1+\alpha_1+\hat{\beta}_1)d_i]\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i\\ &+4\sum_{i=M+1}^{N}r_i(d_i+\bar{\rho}_i)\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i+2\xi^T[(D+\bar{\rho})R\otimes\Gamma]\xi-\frac{1}{2}\xi^T(R\otimes P^{-1})\xi+\boldsymbol{\Upsilon},\end{aligned}\tag{B19}$$

where $\boldsymbol{\Upsilon}=\sum_{i=M+1}^{N}r_i[\frac{\chi^3}{6\phi_i^2}+\frac{\chi}{2}(\alpha-1)^2+\phi_i\frac{(\alpha-1)^3}{3}]+\zeta_1\iota^2+\zeta_2\iota^4+\zeta_3\kappa^2+\zeta_4\kappa^4$.

It is worth noting that

$$\begin{aligned}&\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^{N}[(1+\alpha_1)^2\bar{\rho}_i+2(1+\alpha_1)(1+\alpha_1\\ &+\beta_2)\bar{d}_i]\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i+4\sum_{i=M+1}^{N}r_i(d_i+\bar{\rho}_i)\bar{\rho}_i\tilde{\xi}_i^T\Gamma\tilde{\xi}_i\\ \leqslant&\frac{\sigma_{\max}(R\tilde{\Omega}_{ff})}{\alpha_1}\sum_{i=M+1}^{N}\hat{\alpha}_2(d_i+\bar{\rho}_i)\bar{\rho}_i\sum_{j=1}^{N}\gamma_j\varepsilon_j\\ \leqslant&\frac{1}{2}\xi^T[(D+\bar{\rho})R\otimes\hat{X}]\xi,\end{aligned}\tag{B20}$$

where $\hat{\alpha}_2=\{2\max[(1+\alpha_1)^2,2(1+\alpha_1)(1+\alpha_1+\hat{\beta}_1)]+\frac{8\alpha_1\max_i\{r_i\}}{\sigma_{\max}(R\tilde{\Omega}_{ff})}\}\lambda_{\max}(\Gamma)\max_i\{\eta_i^2\}$, $\hat{X}=-(P^{-1}A+A^TP^{-1}-2\Gamma)>0$, and the last inequality can be obtained similar to the analysis in (A22) and (A24).

Substituting (B20) into (B19), then similar to the proof in Appendix A, we can obtain the following inequality:

$$\begin{aligned}\dot{V}_2\leqslant&-\chi V_2+\xi^T[(D+\bar{\rho})R\otimes(AP^{-1}+P^{-1}A^T+(\chi\\ &+1)P^{-1}-2\Gamma+\frac{\hat{X}}{2})]-\frac{1}{2}\xi^T(R\otimes P^{-1})\xi+\boldsymbol{\Upsilon}.\end{aligned}\tag{B21}$$

By selecting $\varsigma>2(\chi+1)$, we can obtain that $\dot{V}_2\leqslant-\chi V_2$ if $\|\xi\|^2\geqslant\frac{2\boldsymbol{\Upsilon}}{\lambda_{\min}(P^{-1})\min_i\{r_i\}}$, which means that the tracking error δ and the adaptive gain d_i are ultimately bounded under the proposed fully distributed robust event-triggered control protocol. Then we complete the proof of the claim.

Appendix C Numerical Simulations

In this section, we use the following simulation results to verify the effectiveness of the proposed fully distributed event-triggered control algorithm. We consider 6 agents networked by the communication topology shown in Fig. C1 with dynamics described by (A1). Agents labeled 1, 2, and 3 are leaders, other agents are followers. Agents are assumed with double-integrator dynamics.

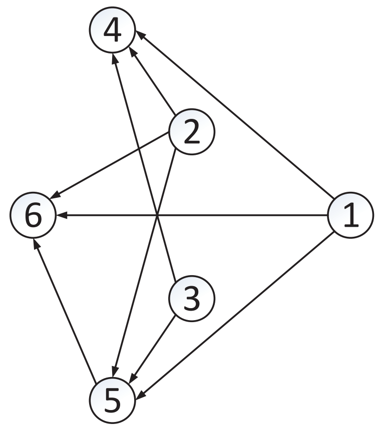

Figure C1 Communication topology for simulations.

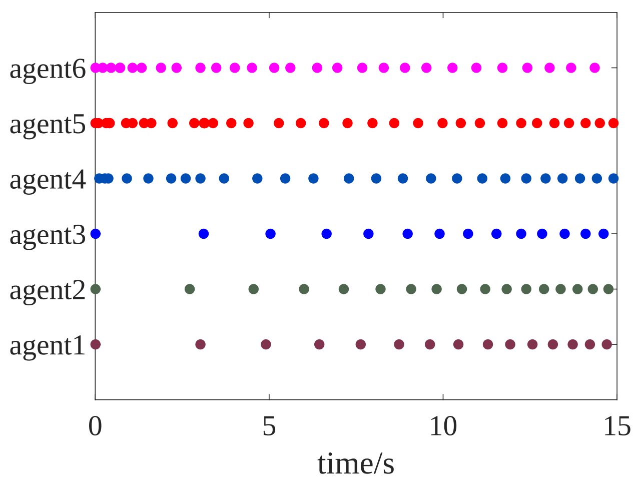

Figure C2 Triggering instants of each agent.

Then we construct the following stress matrix:

$$\Omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{2}{3} & \frac{7}{3} & -\frac{1}{3} & -\frac{4}{3} & 0 & 0 \\ -\frac{2}{3} & -\frac{1}{3} & \frac{7}{3} & 0 & -\frac{4}{3} & 0 \\ -1 & \frac{8}{5} & 0 & 0 & \frac{4}{5} & -\frac{7}{5} \end{bmatrix}$$

while the corresponding nominal configuration of the formation is set to

$$p^T = \begin{bmatrix} 3 & 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 2 & -2 & 0 \end{bmatrix}.$$

Triggering instants of the first 15 seconds of the formation are shown in Fig. C2. Agents only need to communicate with their neighbors and update their control protocols at certain instants. Other simulation results are displayed in the MOOP paper.

Appendix D Experimental Validations

The layout of our laboratory can be seen in Fig. D1. Our experimental equipment consists of the following three parts:

1) The crazyflie is a nano-quadcopter which only weighs 27g and is less than 10cm long. It is equipped with low-latency and long-range radio as well as Bluetooth LE, which allow us to steer it with python scripts and a remote controller. Due to its small size and light weight, we can use a lot of them to carry out indoor experiments in a small space.

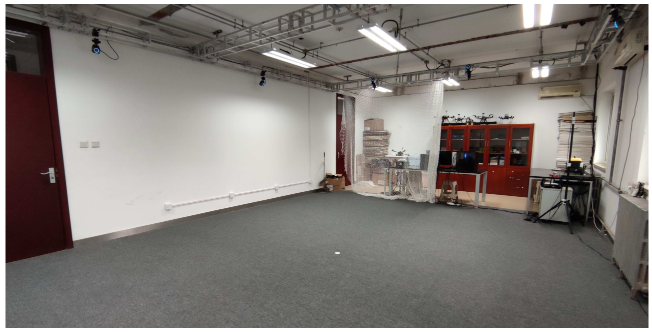

Figure D1 Layout of the laboratory.

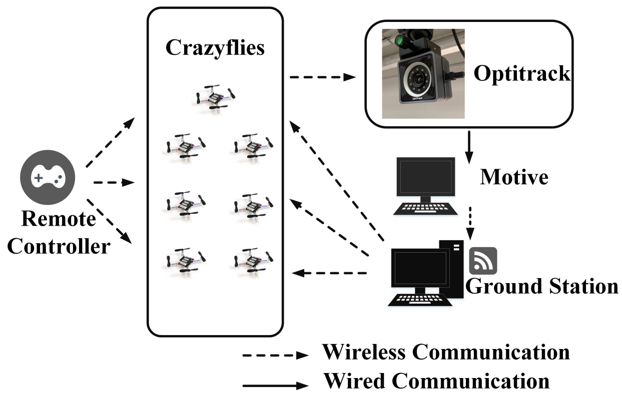

Figure D2 Architecture of the flight control system.

2) The optitrack system is a set of real-time local positioning system with precision of +/- 0.2mm even across large tracking areas. The optitrack system consists of ten high-precision cameras with native frame rate of 240FPS and a Motive software which is installed on a Windows 10 personal computer to provide data integration function. Through proper initialization and calibration, we can use the optitrack system to achieve high-accuracy real-time positioning of dynamic targets.

3) We use Robot Operating System (ROS) to construct the experimental platform. Wi-Fi 6 communication protocol is implemented for information interaction. Python scripts are run in a ground station to steer the crazyflies, while the optitrack system broadcasts the crazyflies' state information to the ground station.

The flight control experimental system is elucidated in Fig. D2. The optitrack motion capture system captures the crazyflies' position information and sends it to the host computer where Motive is installed. The Motive software integrates the data and broadcasts the information to the ground station in ROS through the Wi-Fi 6 communication protocol. Python scripts are run in the ground station utilizing position information of crazyflies broadcast by Motive. Finally, control commands for crazyflies are broadcast from the ground station accordingly via crazyradio, which is a long range open USB radio dongle and capable of 2.6-GHz radio communication. In addition, there is also a remote controller for crazyflies in case of emergencies. It is worth mentioning that the optitrack system can only provide position information, which means that we have to exploit appropriate Kalman filtering algorithm to obtain corresponding velocity information required by our control protocols.

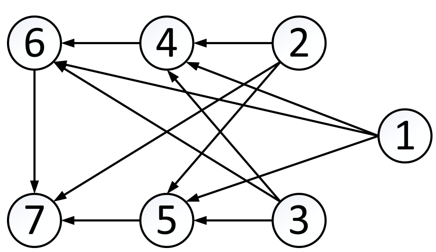

Figure D3 Communication topology for experiments.

In this experiment, we apply a different communication topology of 7 agents shown in Fig. D3. Agents labeled 1, 2 and 3 are leaders while the others are followers. The corresponding stress matrix is designed as follows:

$$\Omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1.5 & 0.5 & -1 & 0 & 0 & 0 \\ 1 & -0.5 & -1.5 & 0 & 1 & 0 & 0 \\ 3 & -2.5 & -1.5 & -1 & 0 & 2 & 0 \\ -3 & 0.5 & 3.5 & 0 & 1 & 1 & -3 \end{bmatrix}.$$

The corresponding nominal formation configuration is set to:

$$f^T = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}.$$

The experimental results and the corresponding analysis can be referred to the MOOP paper.