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Adaptive dynamic surface control of high-order strict feedback nonlinear systems with parameter estimations

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Dear editor,

The dynamics of system components is usually governed by high-order differential equations in engineering practice. However, these high-order systems are mostly converted into first-order systems and treated within the framework of first-order systems. Recently, Duan [1] showed that more direct and effective control algorithms can be designed if the conversion process is saved. Based on this idea, he proposed an adaptive backstepping control approach for high-order strict feedback nonlinear systems (SFNSs) in [2]. Considering that the backstepping control approach in [2] still suffers from the "differential explosion" problem, a natural problem is how to extend the dynamic surface control (DSC) approach to high-order SFNSs. Furthermore, a few accurate parameter estimation approaches are proposed to improve the performance of the closed-loop adaptive control systems [3–5]. In these algorithms, there exist unbounded integral operations when computing the regression matrices, which may result in an unstable closed-loop system. To solve this problem, Na et al. [6] introduced a forgetting-factor-based approach. However, the system is assumed to satisfy the so-called persistent excitation (PE) condition.

In this study, an adaptive DSC method with accurate parameter estimation is proposed for high-order SFNSs. The main features of the proposed control method are summarized as follows. (1) Without converting the high-order SFNS into the first-order one, a more direct and concise adaptive DSC method is proposed by introducing multiple (if necessary) first-order low-pass filters at every step. Compared with [2], it avoids the "differential explosion" problem and repeated estimations of the unknown parameters. (2) A new truncation-based parameter estimation error reconstruction mechanism is proposed under the sufficient excitation condition rather than the stringent PE condition. It could not only achieve accurate parameter estimation, but also avoid the persistent growth problem encountered while computing the regression matrices. (3) Uncertainties in the drift terms and control coefficients are considered. The singularity problem of the control law is avoided by introducing a projection operator.

The notations are provided in Appendix A.

Consider a class of uncertain nonlinear systems:

$$
\begin{aligned}
x_i^{(m_i)} &= f_{i0}\left(x_j^{(0\sim m_j-1)}|_{j=1\sim i}\right) \\
&\quad + \theta_{fi}^{\mathrm{T}} f_i\left(x_j^{(0\sim m_j-1)}|_{j=1\sim i}\right) \\
&\quad + \left[g_{i0}\left(x_j^{(0\sim m_j-1)}|_{j=1\sim i}\right)\right. \\
&\quad \left. + \theta_{gi}^{\mathrm{T}} g_i\left(x_j^{(0\sim m_j-1)}|_{j=1\sim i}\right)\right] x_{i+1},\ i=1,\dots,n-1, \\
x_n^{(m_n)} &= f_{n0}\left(x_j^{(0\sim m_j-1)}|_{j=1\sim n}\right) \\
&\quad + \theta_{fn}^{\mathrm{T}} f_n\left(x_j^{(0\sim m_j-1)}|_{j=1\sim n}\right) \\
&\quad + \left[g_{n0}\left(x_j^{(0\sim m_j-1)}|_{j=1\sim n}\right)\right. \\
&\quad \left. + \theta_{gn}^{\mathrm{T}} g_n\left(x_j^{(0\sim m_j-1)}|_{j=1\sim n}\right)\right] u, \\
y &= x_1, \qquad (1)
\end{aligned}
$$

where $m_i, i=1,2,\dots,n$ are positive integers, $\theta_{fi}\in\Omega_{fi} = \{\vartheta|\vartheta\in\mathbb{R}^{p_{fi}}, \theta_{fij}^{-}\leqslant\vartheta_j\leqslant\theta_{fij}^{+}, j=1,\dots,p_{fi}\}$ and $\theta_{gi}\in\Omega_{gi}=\{\vartheta|\vartheta\in\mathbb{R}^{p_{gi}},\theta_{gij}^{-}\leqslant\vartheta_j\leqslant\theta_{gij}^{+}, j=1,\dots,p_{gi}\}$, $i=1,2,\dots,n$, are unknown constant parameters, $\theta_{fij}^{-},\theta_{fij}^{+},\theta_{gij}^{-}$ and θ_{gij}^{+} are all known constants. $f_i(x_j^{(0\sim m_j-1)}|_{j=1\sim i})$ and $g_i(x_j^{(0\sim m_j-1)}|_{j=1\sim i})$, $i=1,2,\dots,n$ are continuously differentiable vector functions, where p_{fi} and $p_{gi}, i=1,2,\dots,n$ are positive integers. $f_{i0}(x_j^{(0\sim m_j-1)}|_{j=1\sim i})$ and $g_{i0}(x_j^{(0\sim m_j-1)}|_{j=1\sim i})$, $i=1,2,\dots,n$ are continuously differentiable scalar functions. For convenience, when no ambiguity occurs, the arguments of functions are omitted in the following derivation; e.g., $f_{10}(x_1^{(0\sim m_1-1)})$ will be simply denoted by f_{10}. The control objective of this study is to design a proper control law for system (1) to drive the output of the closed-loop system y to track the given refer-

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ence signal y_d. The system and y_d are assumed to satisfy Assumptions 1 and 2 in Appendix B, respectively.

To facilitate the control design, two preliminary results are given as follows.

Lemma 1 ([1]). For any $\mu_i > 0, i \in \{1, \ldots, n\}$, there exists a vector $a_i^{0\sim m_i-1}$ such that $\lambda_{\min}[A_i(a_i^{0\sim m_i-1})] < -\frac{\mu_i}{2}$, which further implies that there exists a matrix of appropriate dimensions $P_i > 0$ such that $A_i^{\rm T}(a_i^{0\sim m_i-1})P_i + P_iA_i(a_i^{0\sim m_i-1}) < -\mu_i P_i$, where the forms of $a_i^{0\sim m_i-1}$ and $A_i^{\rm T}(a_i^{0\sim m_i-1})$ are given in Appendix A.

Lemma 2. For the ith ($i \in \{1, \ldots, n\}$) subsystem of system (1), construct two low-pass filters: (i) $\dot{\omega}_i = -k_i\omega_i + \varphi_i^{\rm T}, \varphi_i = [f_i, g_ix_{i+1}]^{\rm T}, \omega_i(0) = 0$; (ii) $\dot{\zeta}_i = -k_i\zeta_i + f_{i0} + (\theta_{fi}^0)^{\rm T}f_i + [g_{i0} + (\theta_{gi}^0)^{\rm T}g_i]x_{i+1} + k_ix_i^{(m_i-1)}, \zeta_i(0) = x_i^{(m_i-1)}(0), x_{n+1} = u$, where $k_i > 0$ is a design parameter, θ_{fi}^0 and θ_{gi}^0 are respectively the nominal initial estimations of θ_{fi} and θ_{gi}, and ω_i and ζ_i are respectively the outputs of the two filters. Construct $\dot{C}_i = \omega_i^{\rm T}\omega_i, C_i(0) = 0$, $\dot{D}_i = \omega_i^{\rm T}(\omega_i\theta_i^0 + x_i^{(m_i-1)} - \zeta_i), \theta_i^0 = [(\theta_{fi}^0)^{\rm T}, (\theta_{gi}^0)^{\rm T}]^{\rm T}, D_i(0) = 0$. Let C_i satisfy Assumption 3 in Appendix B. Further, construct the truncations

$$\begin{cases} C_{Ti} = C_i(t_T) \text{ and } D_{Ti} = D_i(t_T), & \text{if Trace}\,[C_i(t_T)] > \pi_i, \\ C_{Ti} = C_i(t) \text{ and } D_{Ti} = D_i(t), & \text{otherwise}, \end{cases}$$

with $\pi_i > 0$ being a design parameter. Let $W_i = C_{Ti}\hat{\theta}_i - D_{Ti}$, where $\hat{\theta}_i^{\rm T} = [\hat{\theta}_{fi}^{\rm T}, \hat{\theta}_{gi}^{\rm T}]$ is the estimation of $\theta_i^{\rm T} = [\theta_{fi}^{\rm T}, \theta_{gi}^{\rm T}]$, and $\hat{\theta}_{fi}$ and $\hat{\theta}_{gi}$ are the estimations of θ_{fi} and θ_{gi}, respectively. Let $\tilde{\theta}_i^{\rm T} = \hat{\theta}_i^{\rm T} - \theta_i^{\rm T} = [\tilde{\theta}_{fi}^{\rm T}, \tilde{\theta}_{gi}^{\rm T}]$ denote the estimation error. Then, one obtains $W_i = C_{Ti}\tilde{\theta}_i$ and $\|C_{Ti}\| \leqslant \pi_i$.

The proof of Lemma 2 and necessary remarks are provided in Appendix C.

Main results. The control algorithm is given as follows.

Step i ($1 \leqslant i \leqslant n-1$).

$$z_i^{(0\sim m_i-1)} = x_i^{(0\sim m_i-1)} - x_{ic}^{(0\sim m_i-1)}, \tag{2}$$

$$\begin{aligned} x_{(i+1)d} = \frac{1}{g_{i0} + \hat{\theta}_{gi}^{\rm T}g_i}\Big[& -f_{i0} - \hat{\theta}_{fi}^{\rm T}f_i \\ & - a_i^{0\sim m_i-1}z_i^{(0\sim m_i-1)} + x_{ic}^{(m_i)}\Big], \end{aligned} \tag{3}$$

$$\dot{\hat{\theta}}_i = \mathrm{Pr}_{\hat{\theta}_i}\left(\left(z_i^{(0\sim m_i-1)}\right)^{\rm T}P_{ilc}\varphi_i - \gamma_iW_i\right), \tag{4}$$

$$\begin{aligned} &\tau_{(i+1)j}\dot{\bar{x}}_{(i+1)j} + \bar{x}_{(i+1)j} = \bar{x}_{(i+1)(j-1)}, \bar{x}_{(i+1)0} = x_{(i+1)d}, \\ &\bar{x}_{(i+1)j}(0) = \bar{x}_{(i+1)(j-1)}(0),\ j = 1, \ldots, (m_{i+1}-1), \\ &\tau_{(i+1)m_{i+1}}\dot{x}_{(i+1)c} + x_{(i+1)c} = \bar{x}_{(i+1)(m_{i+1}-1)}, \\ &x_{(i+1)c}(0) = \bar{x}_{(i+1)(m_{i+1}-1)}(0). \end{aligned} \tag{5}$$

Step n.

$$z_n^{(0\sim m_n-1)} = x_n^{(0\sim m_n-1)} - x_{nc}^{(0\sim m_n-1)}, \tag{6}$$

$$\begin{aligned} u = \frac{1}{g_{n0} + \hat{\theta}_{gn}^{\rm T}g_n}\Big[& -f_{n0} - \hat{\theta}_{fn}^{\rm T}f_n \\ & - a_n^{0\sim m_n-1}z_n^{(0\sim m_n-1)} + x_{nc}^{(m_n)}\Big], \end{aligned} \tag{7}$$

$$\dot{\hat{\theta}}_n = \mathrm{Pr}_{\hat{\theta}_n}\left(\left(z_n^{(0\sim m_n-1)}\right)^{\rm T}P_{nlc}\varphi_n - \gamma_nW_n\right), \tag{8}$$

where $x_{1c} = y_d$, $P_{ilc}, i = 1, \ldots, n$ are respectively the last column of P_i, $\mathrm{Pr}_{\hat{\theta}_i}(\cdot)$ are the projection operators of which the definition and properties are given in Appendix D, $\hat{\theta}_i(0) = \theta_i^0$, γ_i are positive design parameters, and $\tau_{lj} > 0, l = 2, \ldots, n, j = 1, 2, \ldots, m_l$ are filtering constants.

Then, we have the following theorem.

Theorem 1. Consider that the high-order SFNSs (1) satisfying Assumptions 1–3. The adaptive DSC algorithm consisting of the state transformations (2) and (6), virtual control laws (3), low-pass filters (5), adaptive laws (4) and (8), and actual control law (7) can realize that all of the closed-loop states are semi-global uniformly ultimately bounded by selecting proper design parameters. The ultimate bounds of the tracking error z_1, as well as the parameter estimation errors $\tilde{\theta}_i, i = 1, \ldots, n$ can be adjusted by tuning the design parameters.

The proof of Theorem 1 and necessary remarks are given in Appendix E. Furthermore, a numerical example is shown in Appendix F.

Conclusion. In this study, for a class of uncertain high-order SFNSs, an adaptive DSC method is proposed. To avoid the "differential explosion" problem, multiple (if necessary) first-order low-pass filters are introduced at every step of the control design. The parameter estimation errors are reconstructed and integrated into the adaptive laws using a set of low-pass filters. The truncation operation is introduced to avoid the unbounded integral operations when computing the regression matrices. The projection operator is introduced to avoid the singularity problem of the control law. It is proved that the tracking error and the parameter estimation errors ultimately converge to an arbitrarily small neighborhood around zero.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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