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Robust adaptive time-varying region tracking control of multi-robot systems

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Dear editor,

Multi-robot systems have been employed in more and more areas in recent years due to their high efficiency, low cost and impressive robustness. When the number of agents is large, effective control of the multi-agent systems becomes a challenging problem due to the infeasibility of centralized control. One promising strategy is to control only the shape of the system instead of the precise relative positions of the robots [1]. This is particularly suitable for some navigation tasks where the robots only need to stay close when moving to the destination and do not need to maintain specific formations. Furthermore, one can easily change the desired shape to avoid possible static and dynamic obstacles in the environment.

A region-based formation control problem for a multi-robot system was first formulated in [2], where the robots were controlled to move as a group inside a desired region while maintaining a minimum distance among themselves. Applications on grasping and manipulating micro-particles or cells of the region tracking control method were conducted in [3, 4]. Since in many cases, the desired region may have time-varying and complex shapes, the region-based robot swarm control method was further extended in [5] dealing with the dynamic region shapes and achieving compound region following. Although uncertainties in the agent dynamics have been handled in the previous results, input disturbances which are ubiquitous in practical multi-robot systems [6] have not been properly considered. In [7, 8], the region tracking problem for multi-robot systems in the presence of both model uncertainties and external disturbances was considered. However, only simple time-invariant desired region shapes were studied and the proposed potential function-based tracking controllers may lead to large control input at the initial stage of the region tracking process which exceed the actuator capacities of the real multi-robot systems.

In this study, we aim to study the time-varying region tracking problem for a class of multi-robot system with both unknown parametric uncertainties and external input disturbances. A novel region tracking controller is proposed based on adaptive sliding mode control which has the advantage of producing reduced control input by using a new type of tracking potential function.

Problem formulation. Consider a multi-robot system with the dynamic model [8]:

$$M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + D_i(x_i)\dot{x}_i + g_i(x_i) + \tau_{di} = \tau_i, \quad (1)$$

where $x_i \in \mathbb{R}^n, i = 1, 2, \ldots, N$ is the generalized coordinate of robot i, $M_i(x_i) \in \mathbb{R}^{n\times n}$ is the inertial matrix which is symmetric and positive definite, $C_i(x_i, \dot{x}_i) \in \mathbb{R}^{n\times n}$ is the Coriolis and centripetal term, $D_i(x_i)\dot{x}_i$ represents the damping force where $D_i(x_i) \in \mathbb{R}^{n\times n}$ is positive definite, $g_i(x_i) \in \mathbb{R}^n$ denotes the gravitational force, $\tau_i \in \mathbb{R}^n$ denotes the control input and $\tau_{di} \in \mathbb{R}^n$ is the external input disturbance satisfying $\|\tau_{di}\| \leqslant \beta_i$ with $\|\cdot\|$ being the Euclidean norm and β_i an unknown positive number.

The time-varying region tracking problem for (1) is considered in this study. First we describe the desired time-varying region. Define a set

$$[f_1(x - c_1), f_2(x - c_2), \ldots, f_M(x - c_M)]^{\mathrm{T}} \leqslant 0,$$

which is the union of M subregions $\{x|f_i(x-c_i) \leqslant 0\}$ where $x \in \mathbb{R}^n$. The functions f_i are continuous and twice partially differentiable. Then, the desired time-varying region $\mathcal{S}$ consists of the points $x' = \bar{A}(t)x + x_o(t)$ where $\bar{A}(t)$ is a linear transformation matrix and $x_o(t)$ is a translation. $\bar{A}(t)$ is differentiable and non-singular. Examples of transformation matrices can be found in Appendix A. The new region $\mathcal{S}$ then can be described by

$$\begin{bmatrix} f_1(A(t)(x - x_o(t)) - c_1) \\ f_2(A(t)(x - x_o(t)) - c_2) \\ \cdots \\ f_M(A(t)(x - x_o(t)) - c_M) \end{bmatrix} \leqslant 0, \quad (2)$$

where $A(t) = (\bar{A}(t))^{-1}$. Then, the time-varying region tracking problem for (1) studied in this study is to design

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the control input τ_i for each robot i such that $x_i \to \mathcal{S}$ as $t \to \infty$ while avoiding collision with respect to each other.

Robust adaptive time-varying region tracking. For each robot $i, i = 1, \ldots, N$, let $T(x_i(t)) = A(t)(x_i(t) - x_o(t))$. Then, a potential function with respect to the desired region is defined as $P_G^i = \sum_{l=1}^{M} P_{Gl}^i$ with $P_{Gl}^i = k_g \ln\cosh\{\max[0, f_l(T(x_i) - c_l)]\}$ where $k_g > 0$. Let $\Delta\xi_i$ denote the region tracking force which takes the form of $\Delta\xi_i = (\frac{\partial P_G^i}{\partial T(x_i)})^{\mathrm{T}}$.

Remark 1. Compared with the previous studies [2–5, 7, 8] which designed region tracking potential functions in quadratic or logarithmic forms, a novel potential function P_G^i is presented with hyperbolic cosine function involved. The partial derivative of the potential energy function is globally bounded and thus has the advantage of producing reduced magnitude of control input. Furthermore, it allows Lyapunov analysis of the global stability of the controller.

To achieve collision avoidance between the agents, suppose each robot i can sense the neighbors in the set $\mathcal{N}_i^c = \{j : \|T(x_j) - T(x_i)\| \leqslant R\}$ where $R > 0$ is related to the sensing ability of the robots. The minimum safe distance between each pair of robots requires $\|T(x_j) - T(x_i)\| \geqslant r$ where $0 < r < R$. Then we consider the collision avoidance potential between agents i and j as $U_{Rij} = k_p(\min(0, \frac{\|T(x_i)-T(x_j)\|^2 - R^2}{\|T(x_i)-T(x_j)\|^2 - r^2}))^2$ where $k_p > 0$ and for agent i, the total potential from all the neighbors is $U_R^i = \sum_{j\in\mathcal{N}_i^c} U_{Rij}$. Then let $\Delta\rho_i = (\frac{\partial U_R^i}{\partial(T(x_i)-T(x_j))})^{\mathrm{T}}$ denote the collision avoidance force.

Further let $\Delta\zeta_i = \alpha_i\Delta\xi_i + \Delta\rho_i$ denote the region tracking index with $\alpha_i > 0$ which reflects the two requirements of the region tracking task, i.e., converging to the desired region and avoiding collision. Let $\Delta x_i = x_i - x_o$ and define the reference velocity as $\dot{x}_{ri} = \dot{x}_o - A^{-1}\dot{A}\Delta x_i - A^{-1}\Delta\zeta_i$. Note that we have the following linear dependence property $M_i(x_i)\ddot{x}_{ri} + C_i(x_i, \dot{x}_i)\dot{x}_{ri} + D_i(x_i)\dot{x}_{ri} + g_i(x_i) = Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\theta_i$ where $Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})$ is a known nonlinear function and θ_i is the unknown parameter vector [9]. For each agent, a sliding variable is defined as $s_i = \dot{x}_i - \dot{x}_{ri}$. The time-varying region tracking controller is then designed as follows:

$$\begin{aligned} &\tau_i = -k_{s_i}s_i - A^{\mathrm{T}}\Delta\zeta_i + Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\hat{\theta}_i + v_i, \\ &v_i(t) = -\frac{\hat{\beta}_i^2 s_i}{\hat{\beta}_i\|s_i\| + \delta_i(t)}, \qquad \dot{\hat{\beta}}_i = k_{\beta_i}\|s_i\|, \\ &\dot{\delta}_i = -k_{\delta_i}\delta_i, \qquad \dot{\hat{\theta}}_i = -k_{\theta_i}Y_i^{\mathrm{T}}s_i, \end{aligned} \tag{3}$$

where $k_{s_i}, k_{\beta_i}, k_{\delta_i}, k_{\theta_i} > 0$ are positive controller parameters, $v_i(t)$ is the robust control term, $\hat{\beta}_i$ is an estimation of β_i, and $\hat{\theta}_i$ is the estimation of the unknown parameter.

Theorem 1. For the multi-robot system (1) with the controller (3), the region tracking index $\Delta\zeta_i$ and the sliding mode surface s_i converge to zero and $T(x_i(t))$ converges to some constant vector asymptotically.

Proof. See Appendix B.

In light of the definition of $\Delta\zeta_i$, we see that $\Delta\xi_i = 0$ if the robots converge to a region where the collision avoidance force $\Delta\rho_i = 0$. This is the case when the desired region is large enough to contain all the robots without inducing repelling forces. Then we have that the time-varying region tracking control is achieved. For the case that $\Delta\xi_i \neq 0$, the region tracking forces and the collision avoidance forces need to cancel each other out. This is the case when the desired region is small and the robots will stay close to the desired region while avoiding collision with respect to each other. By noticing that $\sum_{i=1}^{N}\Delta\rho_i = 0$, we further obtain that $\sum_{i=1}^{N}\alpha_i\Delta\xi_i = 0$. It shows that the robots would spread around the desired region in order to have balanced region tracking forces.

From the proof of Theorem 1, it can be seen that larger controller parameters generally lead to faster convergence to the desired region. However, it also leads to larger control efforts which means there is a trade-off in choosing the proper controller parameters.

Simulation. We consider a region tracking problem for a multi-robot system with 49 robots. Each robot is modeled by $M_i\ddot{x}_i + \eta_i\dot{x}_i + \tau_{di} = \tau_i$, where x_i is the position, M_i is the mass, η_i is the damping parameter, τ_i is the control input, and τ_{di} is the input disturbance. The time-varying region tracking simulation results can be found in Appendix C which show that all the robots track the desired time-varying region successfully while having reduced control input comparing with a conventional potential function-based controller. Furthermore, both the cases of $x_i \in \mathbb{R}^2$ and $x_i \in \mathbb{R}^3$ are simulated to show the effectiveness of the proposed controller.

Conclusion. In this study, we have proposed a new adaptive sliding mode control-based time-varying region tracking controller for multi-robot systems with both uncertain dynamics and unknown input disturbances. A new region tracking potential function is introduced to obtain reduced control input magnitude. Future work includes considering collision avoidance with respect to obstacles in the environment and other control methods handling input saturation.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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