**SCIENCE CHINA**
Information Sciences

• **Supplementary File** •

# Attitude control of a novel tilt-wing UAV in hovering flight

Jihong ZHU[1], Yunjie YANG[1*], Xiangyang WANG[2], Xiaming YUAN[1] & Xiao YANG[2]

[1]*Department of Precision Instrument, Tsinghua University, Beijing* 100084*, China;*
[2]*Institute for Aero Engine, Tsinghua University, Beijing* 100084*, China*

## Appendix A Further analysis of the linear/square root control law

An intuitive comparison of the simple linear control law and the proposed combined control law is shown in Figure A1. The blue line shows the controller output with the linear control law only, and the red line shows the controller output with the linear/square root control law. For the red line, it can be seen that the controller output is continuous at the switching points $e_0$ and $-e_0$. The sign function $sgn(\cdot)$ here ensures that the controller output and the pitch error keep the same sign. Compared with the linear control, the controller output is smaller with the square root control in the large error zone. This alleviates the control saturation and prevents system oscillations.

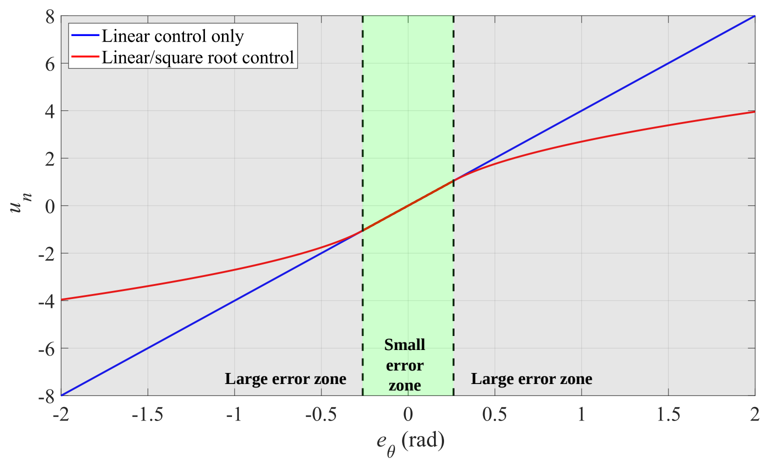


**Figure A1** An illustration of different control laws

Further, by differentiating $u_n$ with respect to $e_\theta$, we have:

$$\frac{du_n}{de_\theta} = \begin{cases} k_\theta\sqrt{\dfrac{-e_0}{2e_\theta + e_0}}, & e_\theta \in (-\infty, -e_0) \\ k_\theta, & e_\theta \in [-e_0, e_0] \\ k_\theta\sqrt{\dfrac{e_0}{2e_\theta - e_0}}, & e_\theta \in (e_0, \infty) \end{cases} \tag{A1}$$

and

$$\begin{aligned} {u'}_{n_+}(e_0) = {u'}_{n_-}(e_0) = k_\theta, \\ {u'}_{n_+}(-e_0) = {u'}_{n_-}(-e_0) = k_\theta \end{aligned} \tag{A2}$$

Similarly, an intuitive illustration of Eqs (A1) and (A2) is shown in Figure A2. It can be seen that the derivative of the controller output with respect to the error $e_\theta$ also changes continuously. And as shown in Eq. (A2)), the left and right derivatives of the linear/square root control law are equal at the switching points $e_0$ and $-e_0$. Therefore, we can conclude that although the different formulations and the sign function $sgn(\cdot)$ are used in the linear/square root control law, the controller output changes smoothly without any interruption. The controller output will not chattering due to the switching of the control law. As mentioned above, the smaller controller output in the large error zone alleviates the control saturation and prevents possible oscillations. Meanwhile, the reserved control margin enables the system to cope with other unknown disturbances. This is more beneficial to the safety of the actual flights.

* Corresponding author (email: yangyj021@tsinghua.edu.cn)

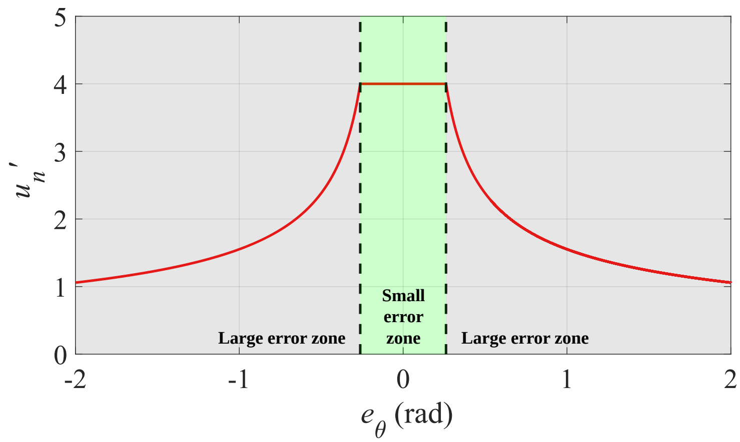


**Figure A2** An illustration of change rate of the controller output

## Appendix B Quantitative analysis of the control performance

To be more intuitive, a quantitative analysis of the control performance is conducted by using the "Mean Squared Error (MSE)" and the "Maximum Absolute Error (MAE)" metrics. The MSE can fairly reflect the tracking error of the different pitching maneuvers, and the MAE can fairly reflect the transient oscillation and overshoot if they happen. The two metrics are defined as:

$$\begin{aligned} \mathrm{MSE} &= \frac{1}{N}\sum_{i=1}^{N}\left(\theta_d\left(i\right)-\theta\left(i\right)\right)^2 \\ \mathrm{MAE} &= \max\left(\left|\theta_d\left(i\right)-\theta\left(i\right)\right|\right) \end{aligned} \quad \text{(B1)}$$

where $N$ denotes the number of sampling points, $\theta_d$ denotes the desired pitch angle command, $\theta_d$ is the actual pitch angle.

A comparison of four experiments is shown in Table B1. For the hanging tethered experiment A, it can be seen that both the MSE and MAE are small without any compensation. This is due to that the UAV is in a low flight velocity, the influence of the main propeller is weak. In Experiment B, namely the free flight experiment without any compensation, both the MSE and MAE are greatly increased. The tracking error is very large. To improve the UAV performance, the integral control is firstly added in Experiment C. As shown in Table B1, the MSE is decreased compared with Experiment B. However, the MAE is also large, which is due to the oscillation after a long-time flight. Finally, with the proposed flight velocity compensation method, both the MSE and MAE are improved. A comparable performance to the hanging tethered experiment A is achieved.

**Table B1** A quantitative analysis of the control performance

| Metrics | Experiment A | Experiment B | Experiment C | Experiment D |
|---|---|---|---|---|
| MSE | 0.63 | 8.79 | 0.90 | 0.65 |
| MAE | 1.67 | 4.45 | 2.68 | 1.63 |

## Appendix C Robustness validation experiments

Robustness to the wind and other external disturbance is also very important to the tilt-wing UAV. Therefore, several other flight experiments are conducted by injecting different types of external disturbances:

**(1) Abrupt disturbances during stationary hovering:** A thin rope was tied at the tail of the THU-TW001 tilt-wing UAV. By manually pulling the rope, an abrupt pitching moment disturbance is injected. The THU-TW001 tilt-wing UAV is hanging tethered for safety (but can move freely). The pitching command remained at zero during the test.

**(2) Abrupt disturbances during pitching maneuvers:** The injection method of disturbances is the same as above, but pitching maneuvers are commanded.

**(3) Wind disturbances during pitching maneuvers:** The flight tests of THU-TW001 tilt-wing UAV is conducted in an outdoor windy weather. According to hand-held anemometer, the wind speed varies between $3 \sim 6m/s$.

Figure C1 shows the flight performance under abrupt disturbances during stationary hovering. The symbol '*' represents the injection of disturbances. It can be seen that even with a large transient pitching moment disturbance, the UAV quickly returns to a stable state in about 2 seconds, and the tracking error tends to be zero after about 4 seconds. Meanwhile, we can find that the UAV can cope with multiple disturbances.

Figure C2 shows the flight performance under abrupt disturbances during pitching maneuvers. The symbol '*' also represents the injection of disturbances. Similarly, the UAV also quickly returns to a stable state even after a larger transient pitching moment disturbance. The pitching command can be well tracked.

Figure C3 shows the flight performance in an outdoor windy weather. Compared with the above manually pulling disturbance, the wind disturbance here is continuous but not that abrupt. It can be seen that the commanded pitch maneuvers are well tracked, although with a small error and slight oscillation. This is acceptable given the complex multidirectional winds of the day.

In summary, we can conclude that the proposed controller is robust to both abrupt shocks and mild persistent disturbances for the THU-TW001 tilt-wing UAV. It has great application potential in actual flights.

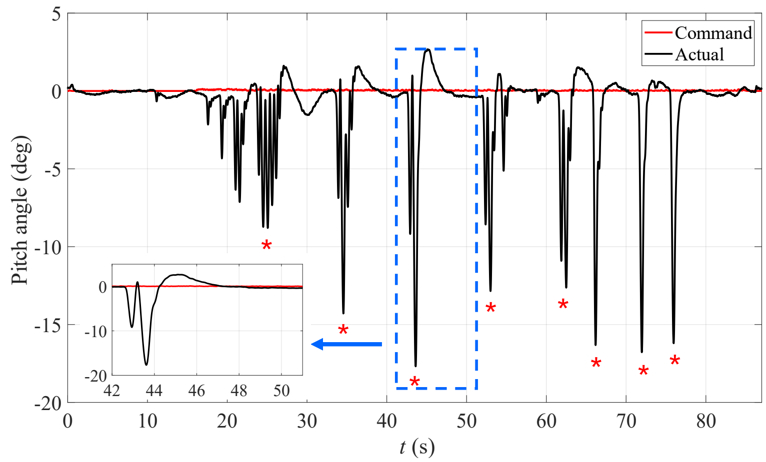


**Figure C1** Abrupt disturbances injection experiment in stationary hovering

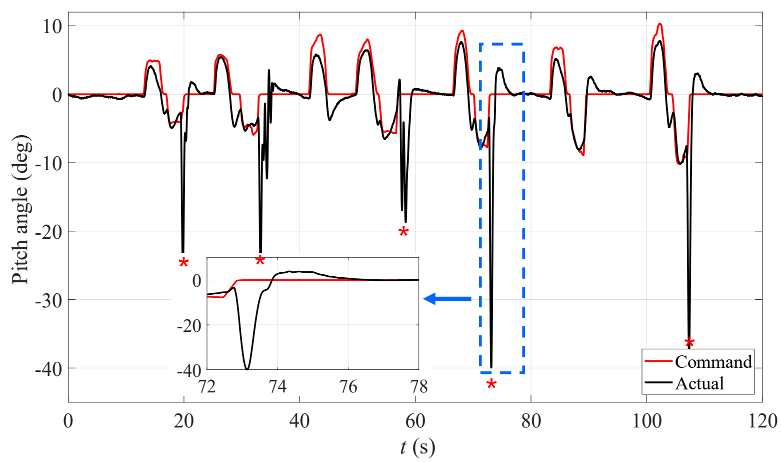


**Figure C2** Abrupt disturbances injection experiment in pitching maneuver

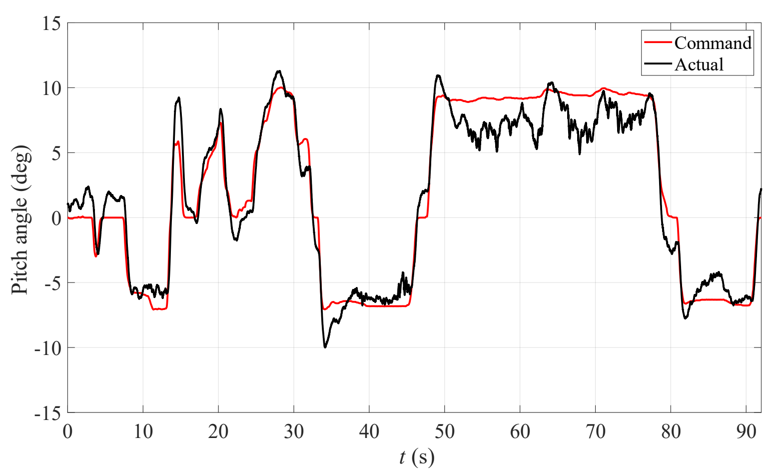


**Figure C3** Outdoor windy weather flight experiment