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Data-driven consensus control of fully distributed event-triggered multi-agent systems

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Abstract This study investigates the consensus control issue in discrete-time linear multi-agent systems (MASs) using data-driven control under undirected communication networks. To alleviate the communication burden, an adaptive event-triggered control strategy involving only local information is proposed and a model-based stability condition is derived that guarantees the asymptotic consensus of MASs. Furthermore, a data-based consensus condition for unknown MASs is established by combining a data-based system representation with the model-based stability condition, using only pre-collected noisy input-state data instead of the accurate system information a priori. Specifically, both model-based and data-driven event-triggered controllers can be utilized without requiring any global information. The validity and correctness of the controllers and associated theoretical results are demonstrated via numerical simulations.

Keywords distributed control, event-triggered control, data-driven control, discrete-time MASs

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1 Introduction

Along with the rapid technological advancement in distributed and networked systems, cooperative control of multi-agent systems (MASs), specifically consensus control, has received considerable attention in the past two decades [1–7]. Moreover, it is well known that communication between agents plays a crucial role in consensus control. Traditional consensus control strategies require communications at all times for continuous-time MASs or all iterations for discrete-time MASs. However, the energy and communication constraints caused by the limited power source and communication bandwidth of a single agent cannot be ignored in cyber-physical systems [8, 9], which consequently shorten the lifespan of the system to a certain degree and cause several issues, including time-varying delays and packet dropouts [10]. These concerns have prompted the study of a viable distributed control strategy for MASs that ensures satisfactory control performance of MASs while conserving limited energy and communication resources. An event-triggering mechanism (ETM) refers to transmitting some information or updating the controller only when a certain event is triggered. On the other hand, event-triggered control (ETC) employs a well-designed ETM to determine whether a data sampling or transmission is due, where the data can be the state, output information, or control signal. Since the seminal contribution [11], ETC has been studied a lot in the context of networked systems and MASs. This is mostly because it saves communication resources and extends the service time of systems in comparison to traditional time-triggered approaches. In the past decade, exciting progress has been made on event-triggered consensus control of both continuous-time [12, 13] and discrete-time [14–16] MASs.

Even though the ETC schemes proposed in the aforementioned results [12–16] rely only on the local information of each agent and its neighbors, they are not fully distributed. This is because there is still

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a need for global information about the communication network, e.g., the eigenvalues of the Laplacian matrix or the total number of agents. This requirement may not be easily accessed, particularly for large-scale MASs, which makes it challenging to apply those ETC schemes. To circumvent the need for global information, many efforts have been devoted to designing adaptive ETMs containing time-varying coupling weights, such as state consensus of homogeneous MASs in [17], cooperative output regulation of heterogeneous MASs in [18], and state estimation of large-scale systems subject to deception attacks in [19]. Note that only ultimate boundedness is achieved in [17] and that Refs. [17–19] addressed the fully distributed event-triggered consensus problem of continuous-time MASs. Such results are still lacking for discrete-time MASs, even though an adaptive control strategy has been constructed in [20] to address the fully distributed consensus control problem of discrete-time MASs. However, communication between agents is needed at each discrete time in [20], resulting in the inevitable waste of communication resources and bandwidth. Therefore, it is important to develop a fully distributed event-triggered protocol for discrete-time linear MASs.

On the other hand, the aforementioned results [12–20] fall within the framework of model-based control, which assumes explicit knowledge of system models. In engineering systems of interest, it is rare to obtain an accurate system model. Conversely, measured trajectories of a system can be acquired more easily. Thus, one may estimate a model via system identification methods, such as [21], and subsequently perform control tasks. Recently, data-driven methods have garnered much attention due to the potential benefit of avoiding the high computing resources required for system identification [22]. The objective of data-driven methods is to design a control law directly from measured data (e.g., input, state, and/or output data) without resorting to an intermediate procedure, such as identifying the system model [23]. One of the basic concerns connected to distributed control of unknown systems is to establish a data-based system representation. Various research results on data-driven control of single systems have been reported to date, such as optimal control [24], aperiodic sampling control [25], robust control [26,27], self-triggered control [28], optimal switching control [29], and robust iterative learning control [30]. In [31], data-driven output synchronization of MASs was addressed; however, Ref. [31] assumed that noise can be directly measured and continuous communications are required at all iterations. Thus, to the best of our knowledge, fully distributed event-triggered consensus control of discrete-time MASs with unknown dynamics has not yet been thoroughly investigated.

This study proposes a model-based and data-driven consensus controller for discrete-time event-triggered MASs under undirected graphs. The difficulty lies in how to construct a data-based stability condition using only data. To achieve this objective, we first study a distributed adaptive ETC strategy and establish a model-based stability condition that ensures the asymptotic consensus of MASs. A sufficient condition is provided to guarantee that events are not triggered during each iteration. Generalizing the data-driven representation in [27] for single systems, a data-based stability condition is reproduced from the model-based condition for MASs. Using this data-based condition, we design the controller and the triggering matrix based on noisy input and state data.

To sum up, the main contributions of this study are epitomized as follows:

(1) An adaptive event-triggered strategy is designed to obtain fully distributed control of discrete-time MASs, which guarantees asymptotic consensus without requiring global information.

(2) Based on a data-based representation for single systems, the model-based controller is realized using data collected offline and locally, along with a data-based asymptotic consensus guarantee.

(3) Leveraging the data-based condition, we offer a method for computing the distributed feedback controller and the triggering matrix from noisy input-state data.

The remainder of this study is outlined in the following. In Section 2, the communication topology is introduced together with the formulation of the consensus problem. A fully distributed ETC strategy is presented in Section 3, along with a model-based stability condition. Section 4 describes a data-driven approach for addressing the consensus problem of unknown event-triggered MASs. Section 5 simulates the proposed control schemes and validates the theoretical results. Finally, Section 6 concludes the study.

Notation. Let $\mathbb{N}$ ($\mathbb{R}$) denote the set of all nonnegative integers (real numbers), and $\mathcal{I}_N = \{1, 2, \ldots, N\}$. Symbols $\mathbf{0}_N$ and $\mathbf{1}_N$ mean the $N \times 1$ column vector of all zeros and ones, respectively. Let $\mathbb{N}_{[a,b]} := \mathbb{N} \cap [a, b]$ for $[a, b] \in \mathbb{N}$. For a symmetric matrix P, $P \succ 0$ shows that P is positive definite and $\lambda_{\max}(P)$ ($\lambda_{\min}(P)$) shows the maximum (minimum) eigenvalues of P. Symbols $(\cdot)^{\mathrm{T}}$ and $\otimes$ represent the transpose for matrices and the Kronecker product, respectively.

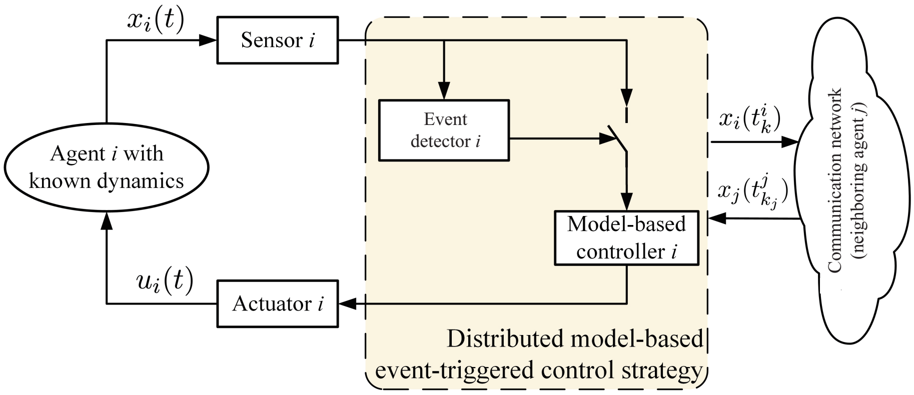

Figure 1 (Color online) Distributed model-based ETC for agent i.

2 Preliminaries and problem formulation

Consider a network with N nodes described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, \dots, v_N\}$ is the nonempty node set, $\mathcal{E} \in (\mathcal{V} \times \mathcal{V})$ is the edge set, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes the weighted adjacency matrix. The edge $(v_i, v_j) \in \mathcal{E}$ indicates that the node v_j can receive information from the node v_i. The adjacency matrix $\mathcal{A}$ is defined such that $a_{ii} = 0$, $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined as $l_{ii} = \sum_{j=1}^{N} a_{ij}$, $l_{ij} = -a_{ij}$ with $i \neq j$. Let $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$ denote the neighbor set of node i.

Consider a discrete-time MAS consisting of N identical agents, indexed by $1, 2, \dots, N$, interacting via a communication network described by the topology $\mathcal{G}$. The dynamics of the ith agent is described by

$$x_i(t+1) = Ax_i(t) + Bu_i(t), \quad t \in \mathbb{N}, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the agent state, $u_i(t) \in \mathbb{R}^p$ is the control input, and $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times p}$ are system matrices.

In this study, the objective is to design fully distributed event-triggered consensus control schemes for discrete-time MASs such that the states of all agents achieve consensus; that is, $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0$, $\forall\, i, j = 1, 2, \dots, N$. For this purpose, we need the following assumptions and lemmas with regard to the MAS (1) and its communication topology.

Assumption 1 (System model)**.** The matrix pair (A, B) in (1) is stabilizable.

Assumption 2 (Communication topology)**.** The graph $\mathcal{G}$ is undirected and connected.

Lemma 1 ([32])**.** For a connected and undirected graph $\mathcal{G}$, zero is a simple eigenvalue of the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ with the corresponding right eigenvector being $\mathbf{1}_N$, i.e., $\mathcal{L}\mathbf{1}_N = 0$. The general algebraic connectivity of $\mathcal{G}$ associated with $\mathcal{L}$ is defined by $\lambda_2(\mathcal{L}) := \min_{\mathbf{1}_N^{\mathrm{T}} x = 0} \frac{x^{\mathrm{T}}\mathcal{L}x}{x^{\mathrm{T}}x}$.

Lemma 2 ([33])**.** Given any $x, y \in \mathbb{R}^N$, Young's inequality states that for any scalar $\phi > 0$, $x^{\mathrm{T}}y \leqslant \frac{x^{\mathrm{T}}x}{2\phi} + \frac{\phi y^{\mathrm{T}}y}{2}$.

3 Distributed event-triggered consensus control: the model-based case

In this section, a distributed adaptive ETC consensus strategy, composed of an ETC protocol and an ETM, is proposed to deal with the consensus problem for discrete-time MAS (1). Under this strategy, a model-based stability condition is established on the premise of an explicit MAS model, which is the essential preparation for deriving our data-driven control scheme as well. Inspired by [10], such a model-based ETC configuration for an MAS is shown in Figure 1.

3.1 Fully distributed ETC strategy

Reflecting the adaptive control law for continuous-time MASs in [17], here we develop an adaptive event-triggered discrete-time controller for each agent in (1) as follows:

$$\begin{aligned} u_i(t) &= K\sum_{j=1}^{N} c_{ij}(t)a_{ij}\left(\bar{x}_i(t)-\bar{x}_j(t)\right), \\ c_{ij}(t+1) &= c_{ij}(t)+\sigma_{ij}a_{ij}\left(\bar{x}_i(t)-\bar{x}_j(t)\right)^{\mathrm{T}}\Phi\left(\bar{x}_i(t)-\bar{x}_j(t)\right), \end{aligned} \tag{2}$$

where a_{ij} is the ijth entry of the adjacency matrix $\mathcal{A}$, $\sigma_{ij}=\sigma_{ji}>0$; $c_{ij}(t)$ denotes the coupling gain for the edge (v_i, v_j), which has the features of adaptively tuning and $c_{ij}(0)=c_{ji}(0)$ are positive constants. $K\in\mathbb{R}^{p\times n}$ and $\Phi\in\mathbb{R}^{n\times n}$ are feedback gain matrices determined later. $\bar{x}_i(t)$ is the last broadcast state of agent i and more specifically it is given by $\bar{x}_i(t)=x_i(t_k^i)$, $\forall t\in\mathbb{N}_{[t_k^i,t_{k+1}^i-1]}$ with t_k^i being the kth ($k\in\mathbb{N}$) triggering time of agent i.

To introduce our ETM, let $e_i(t):=\bar{x}_i(t)-x_i(t)$ for $t\in\mathbb{N}_{[t_k^i,t_{k+1}^i-1]}$ denote the measurement error. We assume without loss of generality that the first triggering time $t_1^i=0$, and subsequent triggering times $\{t_k^i\}_{k=2}^{\infty}$ for agent i are computed as follows:

$$t_{k+1}^i=\inf_{t\geqslant t_k^i}\{t\,|\,f_i(t)\geqslant 0\}, \tag{3}$$

where the triggering function is given by

$$f_i(t)=\sum_{j=1}^{N}\left(1+\varphi c_{ij}(t)\right)a_{ij}e_i^{\mathrm{T}}(t)\Phi e_i(t)-\theta\mathrm{e}^{-\mu t}-\frac{1}{8}\sum_{j=1}^{N}a_{ij}\left(\bar{x}_i(t)-\bar{x}_j(t)\right)^{\mathrm{T}}\Phi\left(\bar{x}_i(t)-\bar{x}_j(t)\right), \tag{4}$$

and φ, θ, and μ are positive constants to be determined. Obviously, the triggering mechanism (3) is checked to determine whether an event is triggered. Once an event is triggered, agent i updates its controller (2) and broadcasts its state to its neighbors. Meanwhile, $e_i(t)$ is reset to zero. It is intuitive that an agent communicates only when $e_i(t)$ is sufficiently large.

Remark 1. Note that the distributed adaptive ETC strategy, relying on local information of each agent and from its neighbors, is designed and implemented in a fully distributed fashion. In other words, the adaptively tuning gain $c_{ij}(t)$ in (2) and (3) based on sampled information circumvents the need for global information $\lambda_2(\mathcal{L})$, which is often required in existing studies, e.g., [12–16].

Remark 2. Note that the term $\mathrm{e}^{-\mu t}$ in (4) is a discrete-time function for $t\in\mathbb{N}_{[t_k^i,t_{k+1}^i-1]}$. Besides, the positivity or negativity of μ affects the convergence of MASs. If $\mu<0$, $\theta\mathrm{e}^{-\mu t}$ is exponentially increasing, which makes it difficult for the triggering error in (4) to exceed the threshold. That is, $f_i(t)\geqslant 0$ may not be satisfied, resulting in no event being triggered. Thus, the consensus of MASs is difficult to be achieved due to the lack of communication between agents. If $\mu=0$, constant θ is included in (4) such that only bounded consensus rather than asymptotic consensus can be reached, which resembles the result in [34]. Only when $\mu>0$, $\theta\mathrm{e}^{-\mu t}$ exponentially decreases to zero. It contributes to limiting the increase of $e_i(t)$ effectively by the threshold, thereby ensuring fast convergence of $e_i(t)$ to zero.

Then, we define $\delta_i(t):=x_i(t)-\sum_{j=1}^{N}x_j(t)/N$ as a disagreement vector for each agent. In light of (1) and (2), the dynamics of $\delta_i(t)$ satisfies

$$\delta_i(t+1)=A\delta_i(t)+BK\sum_{j=1}^{N}c_{ij}(t)a_{ij}(\bar{x}_i(t)-\bar{x}_j(t)). \tag{5}$$

Let $\delta(t)=[\delta_1^{\mathrm{T}}(t),\delta_2^{\mathrm{T}}(t),\ldots,\delta_N^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $x(t)=[x_1^{\mathrm{T}}(t),x_2^{\mathrm{T}}(t),\ldots,x_N^{\mathrm{T}}(t)]^{\mathrm{T}}$. It can be obtained that $\delta(t)=(M\otimes I_n)x(t)$ with $M=I_N-(1/N)\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}$. First, considering that if $x_1(t)=x_2(t)=\cdots=x_N(t)$, one has $x_i(t)=\frac{1}{N}\sum_{j=1}^{N}x_j(t)$ for $i=1,2,\ldots,N$, which implies $\delta_i(t)=0$. Additionally, if $\delta_i(t)=0$ holds for $i=1,2,\ldots,N$, i.e., $\delta(t)=0$, we have that $(\mathcal{L}M\otimes I_n)x(t)=0$ through multiplying $\mathcal{L}$ by both sides of $(M\otimes I_n)x(t)=0$. It follows from Lemma 1 that $M\mathcal{L}=\mathcal{L}M=\mathcal{L}$. Thus, it can be deduced that $(\mathcal{L}\otimes I_n)x(t)=0$, which implies $x_1(t)=x_2(t)=\cdots=x_N(t)$ [35]. Consequently, we conclude that $x_1(t)=x_2(t)=\cdots=x_N(t)$ if and only if $\delta_i(t)=0$ for all $i=1,2,\ldots,N$. In other words, if each agent's disagreement vectors converge to zero asymptotically, the MAS achieves the state asymptotic consensus.

3.2 Model-based consensus analysis

In this part, we develop a model-based stability condition that guarantees state asymptotic consensus of the MAS (1) under the adaptive ETC strategy. In the sequel, the explicit dependence of symbols on t will be omitted if there is no confusion.

Before proceeding, a fact is provided, which will play a vital role in the subsequent consensus analysis.

Lemma 3 ([36])**.** Under Assumption 1, the following discrete-time Riccati-like equation admits a unique positive-definite solution $P \succ 0$:

$$A^{\mathrm{T}}PA - P - A^{\mathrm{T}}PB(B^{\mathrm{T}}PB)^{-1}BPA + Q = 0 \tag{6}$$

for some prescribed matrix $Q \succ 0$.

Theorem 1. Consider the MAS (1) and the ETC protocol (2)–(4) under the graph $\mathcal{G}$. Suppose Assumptions 1 and 2 hold. Let the feedback gain matrices K and Φ be given by $K := -\left(B^{\mathrm{T}}PB\right)^{-1}B^{\mathrm{T}}PA$ and $\Phi := A^{\mathrm{T}}PB\left(B^{\mathrm{T}}PB\right)^{-1}B^{\mathrm{T}}PA$, respectively. For any positive constants σ_{ij}, φ, θ, and μ, the consensus of the MAS has reached asymptotically for any initial states, and all $c_{ij}(t)$ converge to some positive constants.

Proof. See Appendix A.

Proposition 1. For the discrete-time MAS (1) under the ETC protocol (2)–(4), the event is not triggered at each sampling moment if the following function

$$\sum_{s=t_k^i}^{t-1}\|A^{t-s-1}\|\left(\zeta_i + \bar{c}\eta_i\right) = \sqrt{\frac{\theta \mathrm{e}^{-\mu t}}{l_{ii}\|\Phi\|(1+\varphi\bar{c})}} \tag{7}$$

has a solution t^* satisfying $t^* - t_k^i > 1$ for all triggering instants $t_k^i \in \mathbb{N}$, $i = 1, 2, \ldots, N$, where $\bar{c}$, ζ_i, and η_i denote the upper bounds of $c_{ij}(t)$, $\|I_N - A\|\|x_i(t_k^i)\|$, and $\sum_{j=1}^N a_{ij}\|BK\|\|x_i(t_k^i) - x_j(t_{k_j}^j)\|$, respectively.

Proof. See Appendix B.

Remark 3 (Comparison with [17])**.** Difference between our model-based control strategy in (2)–(3) and that of [17] lies in two aspects. First, considering that in digital systems, data can only be transmitted at discrete time instants, our control strategy is designed for discrete-time MASs, which calls for a novel Lyapunov function and stability analysis. Moreover, the asymptotic consensus of MASs, instead of ultimate boundedness in [17], is achieved under the proposed control scheme. Finally, the model-based stability condition established for the proposed control strategy lays a theoretical foundation for our subsequent data-driven stability analysis and controller design in Section 4.

4 Distributed event-triggered consensus control: the data-driven case

This section advocates a data-driven approach to solving the consensus problem of unknown MASs (1) under the fully distributed ETC strategy (2)–(4). To this aim, we begin by reproducing a data-based stability condition from the model-based stability condition in Theorem 1, by developing a data-based representation for the MAS (1). Then, we design feedback gain matrices K and Φ leveraging the data-based stability condition. By this means, the novel data-driven consensus control scheme capitalizes purely on data rather than system matrix pair (A, B). See Figure 2 for a pictorial depiction of the distributed data-driven ETC configuration for an MAS.

4.1 Data-based system representation

In this subsection, let us briefly review the data-based system representation in [27]. Before proceeding, we make the following assumption about the system.

Assumption 3 (Unknown system model)**.** The system matrix pair (A, B) in (1) is unknown. Instead, some input-state data of each agent are locally available.

Suppose that state data $\{x_i(T)\}_{T=0}^{\rho}$ and control input data $\{u_i(T)\}_{T=0}^{\rho-1}$ are measured over the time interval $T \in \{0, 1, \ldots, \rho\}$ from the following disturbed system:

$$x_i(T+1) = Ax_i(T) + Bu_i(T) + Ew_i(T), \tag{8}$$

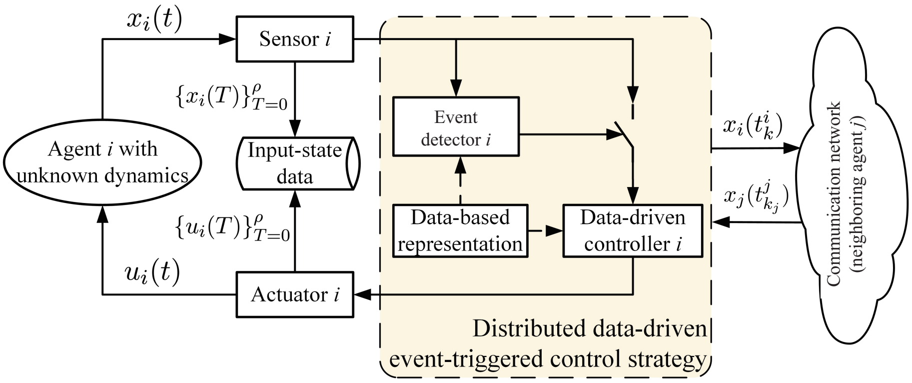

Figure 2 (Color online) Distributed data-driven ETC for agent i.

where $w_i(T) \in \mathbb{R}^n$ stands for the unknown disturbances or unmodeled dynamics and $E \in \mathbb{R}^{n\times n}$ is a known matrix modeling the effect of perturbations on the MAS. It is notable that the collected data are corrupted by the unknown perturbation sequence $\{w_i(T)\}_{T=0}^{\rho-1}$. For subsequent analysis, data collected for agent i are stacked up as follows to form matrices:

$$\begin{aligned} X_{i+} &:= [x(1) \quad x(2) \quad \cdots \quad x(\rho)], \qquad U_i := [u(1) \quad u(2) \quad \cdots \quad u(\rho-1)], \\ X_i &:= [x(1) \quad x(2) \quad \cdots \quad x(\rho-1)], \quad W_i := [w(1) \quad w(2) \quad \cdots \quad w(\rho-1)]. \end{aligned}$$

Further, the collected data and the true system matrix pair (A, B) of the ith agent are related through

$$X_{i+} = AX_i + BU_i + EW_i. \tag{9}$$

It is worth emphasizing that the system matrix pair (A, B) as well as the noise term W_i is unknown, while X_{i+}, X_i, and U_i are measured. To facilitate our analysis, the following assumption is used to model the additive noise, which has also appeared in several existing studies, e.g., [24, 25, 27, 28].

Assumption 4 (Additive noise)**.** The noise matrix W_i satisfies

$$\begin{bmatrix} W_i^{\mathrm{T}} \\ I \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_d & S_d \\ * & R_d \end{bmatrix} \begin{bmatrix} W_i^{\mathrm{T}} \\ I \end{bmatrix} \succeq 0, \tag{10}$$

where $Q_d = Q_d^{\mathrm{T}} \prec 0$, $R_d = R_d^{\mathrm{T}}$, and S_d are of suitable dimensions.

In general, the system with matrices (A, B) in (8) may not be the only system that can interpret the data (X_{i+}, X_i, U_i, W_i) in (9). Put differently, there could be many system matrices (A, B) that can generate the data (X_{i+}, X_i, U_i, W_i). Therefore, we define the set of system matrices (A, B) that can explain the data (X_{i+}, X_i, U_i, W_i) as follows:

$$\Sigma_i := \{[A\ B] \mid X_{i+} = AX_i + BU_i + EW_i\}. \tag{11}$$

Inspired by [27], the set Σ_i of all systems interpreting the data can be equivalently represented in the form of a quadratic matrix inequality by substituting (10) into (11), as asserted by the following lemma.

Lemma 4 (Data-based system representation)**.** The set Σ_i is identical to

$$\Sigma_i = \left\{ [A\ B] \in \mathbb{R}^{n\times(n+p)} \middle| \begin{bmatrix} [A\ B]^{\mathrm{T}} \\ I \end{bmatrix}^{\mathrm{T}} \Theta_i \begin{bmatrix} [A\ B]^{\mathrm{T}} \\ I \end{bmatrix} \succeq 0 \right\}, \tag{12}$$

where

$$\Theta_i := \left[\begin{array}{cc} -X_i & \mathbf{0} \\ -U_i & \mathbf{0} \\ \hline X_{i+} & E \end{array}\right] \begin{bmatrix} Q_d & S_d \\ * & R_d \end{bmatrix} \left[\begin{array}{cc} -X_i & \mathbf{0} \\ -U_i & \mathbf{0} \\ \hline X_{i+} & E \end{array}\right]^{\mathrm{T}}.$$

Indeed, the data-based system representation in Lemma 4 provides a way to design stabilizing controllers using a finite number of measured data X_{i+}, X_i, and U_i without explicit use of parametric models of the system. It is key to tackling the consensus problem for the unknown MAS (1). Different from [25, 27, 28] dealing with a single system, Lemma 4 can be used to characterize all agents in the network based solely on local data of each agent. This further motivates a fully distributed data-driven consensus controller design for MASs along with a data-based ETM, which occupies Subsection 4.2.

4.2 Data-driven consensus analysis

In this subsection, a distributed data-based ETC strategy is designed for unknown MAS (1) using pre-collected noisy data. It can be seen from Section 2 that the distributed controller and ETM are designed under the premise of the explicit system model. However, when the system matrices are unknown, there are two challenges. (c1) How to design a consensus controller as well as ETM using the pre-collected noisy data and the proposed data-based system representation in Lemma 4? (c2) How to develop theoretical consensus guarantees for the resulting data-based event-triggered MAS? To address these challenges, we put forward a data-based stability condition by incorporating the data-based representation with the model-based stability condition. Subsequently, a data-driven method for computing the feedback gain matrices K and Φ is given. The implementation of our ETM is presented in Subsection 3.1. Now, our proposed distributed data-driven ETC is summarized in Algorithm 1 with consensus analysis provided below.

Algorithm 1 Distributed data-driven event-triggered consensus control

Input: initial state $x_i(0) \in \mathbb{R}^n$; adaptive weights $c_{ij}(0) = c_{ji}(0) > 0$, parameters of the triggering function $\sigma_{ij} > 0, \varphi > 0, \theta > 0$, and $\mu > 0$; matrices of the noise model (10) $Q_d = Q_d^{\mathrm{T}} < 0$, $S_d = S_d^{\mathrm{T}}$, and R_d; input-state data $\{x_i(T)\}_{T=0}^{\rho}$ and $\{u_i(T)\}_{T=0}^{\rho-1}$ with $\rho > 1$, for $i, j \in \mathcal{I}_N$;
Construct data-based matrices Θ_i for all $i \in \mathcal{I}_N$ using (12);
Compute feedback gain matrices K and Φ via Theorem 2;
for $i = 1, 2, \ldots, N$ **do**
  if $f_i(t) \geqslant 0$ **then**
    Reset $e_i(t) = 0$, update $\bar{x}_i(t)$ with $x_i(t)$, and broadcast $x_i(t)$ to agent $j \in \mathcal{N}_i$;
  else
    Update $f_i(t)$ in term of (4);
  end if
  if $x_j(t)$ is received from agent j **then**
    Update the control protocol (2);
  end if
  Compute the control protocol (2);
  Update the dynamics of the ith agent (1);
end for

Theorem 2. Consider the MAS (1) and the ETC strategy (2)–(4) under the graph $\mathcal{G}$. Suppose Assumptions 1–4 hold. Choose any positive constants σ_{ij}, φ, θ, and μ. For all $[A\ B] \in \Sigma_i$, if there exist matrices $Y = Y^{\mathrm{T}} \succ 0$, $H = H^{\mathrm{T}} \succ 0$, $Z = Z^{\mathrm{T}}$, and L, and a scalar $\beta > 0$ satisfying (13), the feedback gain matrices are computed as $K := LY^{-1}$ and $\Phi := H^{-1} - Y^{-1}$. Then, the consensus of the MAS (1) is reached asymptotically for any initial states and all $c_{ij}(t)$ converge to some positive constants.

$$M - \beta\Theta_i \succeq 0, \quad N - \beta\Theta_i \succeq 0, \quad \begin{bmatrix} Z & I \\ I & Y \end{bmatrix} \succeq 0, \tag{13}$$

where

$$M := \begin{bmatrix} -Y & -L^{\mathrm{T}} & 0 \\ -L & -LZL^{\mathrm{T}} & 0 \\ 0 & 0 & Y \end{bmatrix}, \quad N := \begin{bmatrix} -H & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y \end{bmatrix}.$$

Proof. According to the Lyapunov function (A2), it can be intuitively deduced that by choosing the state feedback matrices as $K = -(B^{\mathrm{T}}PB)^{-1}BPA$ and $\Phi = A^{\mathrm{T}}PB(B^{\mathrm{T}}PB)^{-1}BPA$ with $P \succ 0$ being the solution of (6), the MAS (1) reaches consensus. Then, by completing the square of (6), one obtains $A^{\mathrm{T}}PA - P + Q - A^{\mathrm{T}}PB(B^{\mathrm{T}}PB)^{-1}BPA = (A + BK)^{\mathrm{T}}P(A + BK) - P + Q = 0$, which implies

$$P - (A + BK)^{\mathrm{T}}P(A + BK) = Q \succ 0. \tag{14}$$

Further, we turn our attention to reconstructing the condition (14) in the form of a quadratic matrix inequality. By a Schur complement argument, the inequality (14) is equivalent to

$$\begin{bmatrix} A^{\mathrm{T}} \\ B^{\mathrm{T}} \\ I \end{bmatrix}^{\mathrm{T}} \left[\begin{array}{cc|c} -P^{-1} & -P^{-1}K^{\mathrm{T}} & 0 \\ -KP^{-1} & -KP^{-1}K^{\mathrm{T}} & 0 \\ \hline 0 & 0 & P^{-1} \end{array}\right] \begin{bmatrix} A^{\mathrm{T}} \\ B^{\mathrm{T}} \\ I \end{bmatrix} \succ 0. \tag{15}$$

Let $Y := P^{-1}$, $L := KY$, and $Z := P$. We rewrite (15) by a standard change of variables as follows:

$$\begin{bmatrix} [A\ B]^{\mathrm{T}} \\ I \end{bmatrix}^{\mathrm{T}} \underbrace{\left[\begin{array}{cc|c} -Y & -L^{\mathrm{T}} & 0 \\ -L & -LZL^{\mathrm{T}} & 0 \\ \hline 0 & 0 & Y \end{array}\right]}_{:=M} \begin{bmatrix} [A\ B]^{\mathrm{T}} \\ I \end{bmatrix} \succ 0. \tag{16}$$

Recall the data-based system representation in (12). Leveraging matrix S-lemma in [27], it can be obtained that there exists a scalar $\beta > 0$ such that

$$M - \beta\Theta_i \succeq 0, \quad \begin{bmatrix} Z & I \\ I & Y \end{bmatrix} \succeq 0. \tag{17}$$

As such, we are in a position to obtain the feedback gain $K = LY^{-1}$ by solving the linear matrix inequality (17). In the same way, we can also calculate the gain matrix Φ in controller (2) and triggering function (4) only from data. In line with Φ satisfying $P + \Phi - A^{\mathrm{T}}PA = Q \succ 0$, we can rewrite it by using a Schur complement argument:

$$\begin{bmatrix} H & A^{\mathrm{T}} \\ A & P^{-1} \end{bmatrix} \succ 0, \tag{18}$$

where $H := (Z + \Phi)^{-1}$. Now, Eq. (18) holds if and only if $H \succ 0$ and $Y - AHA^{\mathrm{T}} \succ 0$ are satisfied. Note that the former inequality is satisfied undoubtedly and independent of matrices (A, B). We represent the latter inequality as

$$\begin{bmatrix} [A\ B]^{\mathrm{T}} \\ I \end{bmatrix}^{\mathrm{T}} \underbrace{\left[\begin{array}{cc|c} -H & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & Y \end{array}\right]}_{:=N} \begin{bmatrix} [A\ B]^{\mathrm{T}} \\ I \end{bmatrix} \succ 0.$$

Considering the data-based representation (12) and matrix S-lemma again, we arrive at $N - \beta\Theta_i \succeq 0$.

To sum up, if there exists a constant $\beta > 0$ such that the above inequality and Eq. (17) hold for any matrix pair $[A\ B] \in \Sigma_i$, then distributed stabilizing controller gain matrices K and Φ can be obtained based on measurement data from the perturbed system (8). Moreover, the forward difference $\triangle V(t)$ in (A8) is guaranteed to be negative. Therefore, reminiscent of the proof of Theorem 1, it can be drawn that the MAS (1) achieves state consensus asymptotically even with unknown A and B, thus completing the proof.

Remark 4. It is intuitive that only input-state data are needed in Theorem 2. Both the adaptive control protocol (2) and the event-triggering law (3) are represented purely using data instead of the system matrices (A, B). Besides, they can be derived and implemented in a fully distributed manner, relying on no global information, e.g., the network topology or scale.

Remark 5. Some comments on Theorem 2 relative to existing results are worth making.

(1) Except for [31], existing studies on data-driven control have been devoted to the stability problem of a single system [23–30]. Theorem 2 addresses the distributed data-driven consensus control problem of MASs under adaptive event-triggering communications. This is made possible by developing a model-based consensus controller and an ETM (refer to Theorem 1), which permits an agent-wise data-based representation in Lemma 4.

(2) As far as consensus control of MASs is concerned, the difference between our data-driven control scheme and that of [31] lies in two aspects. First, an ETM is contained in our data-driven scheme, which

can save communication resources by reducing the frequencies of updates and transmissions. Further, the noise is assumed unknown in this study, while Ref. [31] considered known noise and thus less practical situations.

(3) Theorem 2 features scalability on top of the data-based implementation. Actually, the proposed data-driven approach is applicable to more complex cases of communication time-delays, parameter uncertainties, or arbitrary switching graphs with a positive dwell time. It is straightforward to prove this assertion by integrating a model-based stability condition [37, 38] with the same data-based system representation as (12) in Lemma 4.

Remark 6. The proposed data-driven approach offers a simpler path toward solving the event-triggered consensus control problem of unknown dynamical MASs. Specifically speaking, we design control laws and triggering matrices directly from a finite set of noisy data. Among existing investigations, the only alternative is to first perform system identification followed by model-based control. However, it is generally difficult to provide a stability guarantee for closed-loop systems under such an identification-based control scheme, especially when the system dimension is large (e.g., $n \geqslant 4$) [39], and/or the data are disturbed and limited. Indeed, the proposed approach guarantees the stability and performance of the unknown system despite the inherent uncertainty caused by disturbance. Comparative tests of the proposed data-driven method and the identification-based one can be found in Section 5.

5 Numerical examples

In this section, we demonstrate the model-based and data-driven theoretical results by examples and numerical simulations, respectively.

Consider a two-mass-spring system consisting of two masses sliding freely on a frictionless surface [40] (see Figure 3), where m_1 and m_2 are two masses, and k_1 and k_2 are the stiffness of two springs. The force $u(t) \in \mathbb{R}$ applied to m_1 controls the whole system. Let $x(t) = [y_1(t),\ \dot{y}_1(t),\ y_2(t),\ \dot{y}_2(t)]^{\mathrm{T}}$, where $y_1(t)$ and $\dot{y}_1(t)$ denote the displacement and velocity of mass m_1, respectively; $y_2(t)$ and $\dot{y}_2(t)$ denote the displacement and velocity of mass m_2, respectively. The two-mass-spring system is modeled by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & 0 & -\frac{k_2}{m_2} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u(t), \tag{19}$$

where $m_1 = m_2 = 0.75$ kg and $k_1 = k_2 = 2$ N/m. Consider an MAS consisting of six two-mass-spring systems described above, treating as agent $i, i = 1, 2, \ldots, 6$. Note that the continuous-time linear MAS (19) in a periodic sampled-data setting can be transformed into a discrete-time system equivalently. Setting the sampling interval as $h = 0.1$, Eq. (19) can be represented as a discrete-time MAS (1) with

$$A = \begin{bmatrix} 0.9735 & 0.0991 & 0.0132 & 0.0004 \\ -0.5274 & 0.9735 & 0.2631 & 0.0132 \\ 0.0132 & 0.0004 & 0.9867 & 0.0996 \\ 0.2631 & 0.0132 & -0.2643 & 0.9867 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0066 \\ 0.1321 \\ 0 \\ 0.0006 \end{bmatrix}.$$

The communication graph among these six agents is depicted in Figure 4. It is easy to check that Assumptions 1–2 hold. Consequently, the control objective translates into making the displacements $y_{1i}(t)$ and $y_{2i}(t)$, $i = 1, 2, \ldots, 6$ reach asymptotic consensus, respectively.

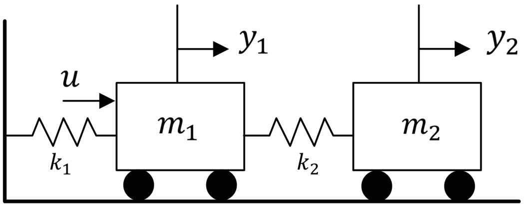

Figure 3 Two-mass-spring system.

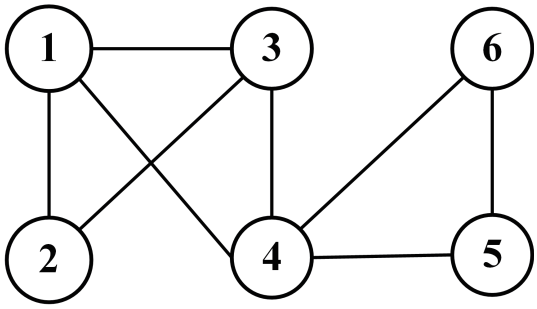

Figure 4 Communication topology $\mathcal{G}$ between agents.

5.1 Model-based setting

In this subsection, we consider the system matrices A and B are known. By Theorem 1, the feedback gain matrices in the model-based ETC protocol (2)–(3) are obtained as

$$K = \begin{bmatrix} -3.1108 & -2.1122 & 1.6866 & -0.6273 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 32.6720 & -10.5955 & -13.4874 & 5.7447 \\ -10.5955 & 3.4361 & 4.3739 & -1.8630 \\ -13.4874 & 4.3739 & 5.5678 & -2.3715 \\ 5.7447 & -1.8630 & -2.3715 & 1.0101 \end{bmatrix}.$$

Then, the parameters are selected as $\varphi = 1$, $\mu = 0.5$, $\theta = 2$, and $\sigma_{ij} = 0.2$, where $\forall\,(i,j) \in \mathcal{E}$. Besides, the initial states for the agents $x_i(0)$ and adaptive gains $c_{ij}(0)$ are selected randomly from the interval $(0,\,1)$ and $(0,\,2)$ for all $i,\,j = 1,2,\ldots,6$, respectively.

The simulation results are depicted in Figure 5. The trajectories of the displacements for two masses, i.e., $y_{1i}(t)$ and $y_{2i}(t)$, $i = 1,2,\ldots,6$, are shown in Figure 5(a). Obviously, the consensus of $y_{1i}(t)$ and $y_{2i}(t)$ is achieved asymptotically. Figure 5(b) depicts the evolution of the adaptive coupling gains $c_{ij}(t)$ in (2). It can be seen that all $c_{ij}(t)$ finally converge to some positive constants.

5.2 Data-driven setting

We evaluate the proposed data-driven consensus controller, where the matrices A and B are assumed unknown now. Setting $\rho = 80$, the measurements $\{x_i(T)\}_{T=0}^{\rho}$ and $\{u_i(T)\}_{T=0}^{\rho-1}$ for each agent can be collected according to (8), where the data-generating inputs are randomly chosen on the interval $u_i(T) \in [-1,\,1]$ with the matrix $E = 0.01I$. Next, the noise samples $w_i(T)$ are bounded in the form of $w_i(T) \in [-\bar{w},\,\bar{w}]^2$ for all $t \in \mathbb{N}$. As interpreted in Assumption 4, we can capture this prior knowledge using the noise model (10) with $Q_d = -I$, $S_d = 0$, and $R_d = \bar{w}^2\rho I$ ($\bar{w} = 0.001$). By solving the data-based linear matrix inequalities in Theorem 2, the feedback gain matrices in (2) and (4) are designed as

$$K = \begin{bmatrix} 0.3580 & -7.1880 & -1.1496 & -1.8863 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 36.1132 & -28.2432 & 3.2236 & -2.6877 \\ -28.2432 & 30.5832 & -0.4806 & 1.1350 \\ 3.2236 & -0.4806 & 18.7429 & -0.0012 \\ -2.6877 & 1.1350 & -0.0012 & 16.9758 \end{bmatrix}.$$

The simulation results are presented in Figure 6 for Algorithm 1. It is obvious that the consensus of the MAS is achieved under the matrices K and Φ found purely from the measured data, validating the correctness of the data-driven design. In addition, the adaptive coupling gains $c_{ij}(t)$ all converge to positive steady-state values. By comparing the plots in Figure 5 with those in Figure 6, it can be seen that when choosing the same parameters and initial values, the trajectories from the model-based and data-driven systems evolve similarly, which demonstrates the effectiveness of our data-driven controller.

5.3 Compared with the system-identification-based method

In this subsection, comparative studies between the proposed data-driven method and the system-identification-based one are performed with the same lack of accurate system models. Leveraging the

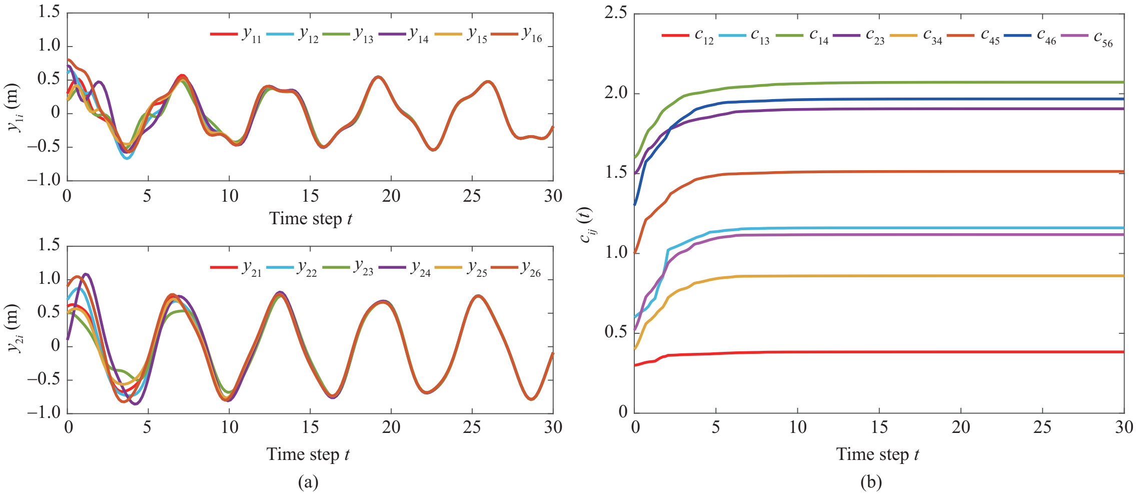

Figure 5 (Color online) Trajectories under the model-based controller. (a) States of the MAS; (b) coupling weights $c_{ij}(t)$.

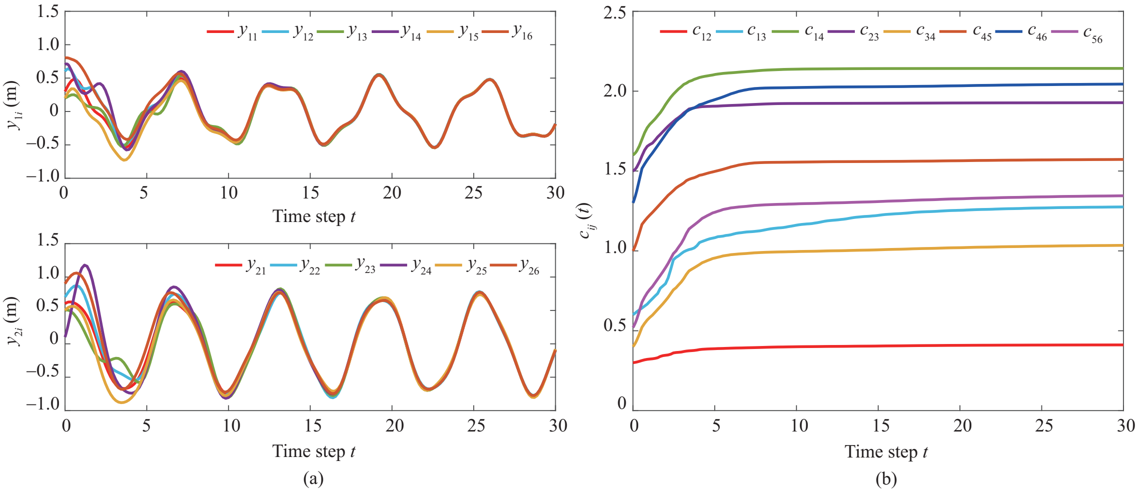

Figure 6 (Color online) Trajectories under the data-driven controller. (a) States of the MAS; (b) coupling weights $c_{ij}(t)$.

pre-collected input data $\{x_i(T)\}_{T=0}^{\rho}$ and state data $\{u_i(T)\}_{T=0}^{\rho-1}$ ($\rho = 80$) in Subsection 5.2, we estimate the discrete-time state-space model through numerical algorithms for subspace state-space system identification (N4SID) [41]. Then, the system matrices with As and Bs are given by

$$\mathrm{As} = \begin{bmatrix} 0.5239 & -1.5698 & -1.3276 & 0.6924 \\ 1.5939 & 0.0182 & 1.0938 & -0.1405 \\ 1.5504 & -0.9113 & 0.5236 & -0.7535 \\ -1.8750 & 0.3793 & 1.8127 & -0.0179 \end{bmatrix}, \quad \mathrm{Bs} = \begin{bmatrix} 0.4439 \\ -0.0254 \\ -0.3287 \\ 0.0471 \end{bmatrix}.$$

Further, the controller gain and triggering matrices are computed by our model-based consensus method (refer to Theorem 1) as follows:

$$K = \begin{bmatrix} -1.4948 & -0.5074 & 0.4491 & 0.0172 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0.5111 & 1.5487 & 1.3576 & -1.0427 \\ 1.5487 & 4.6927 & 4.1137 & -3.1595 \\ 1.3576 & 4.1137 & 3.6061 & -2.7696 \\ -1.0427 & -3.1595 & -2.7696 & 2.1272 \end{bmatrix}.$$

Based on Figures 6 and 7, the comparison between the proposed data-driven approach and the system-identification-based alternative is addressed, focusing on the convergence speed and communication efficiency. First, the steady-state instant determined by our data-driven approach ($t = 15$ s) is approximately

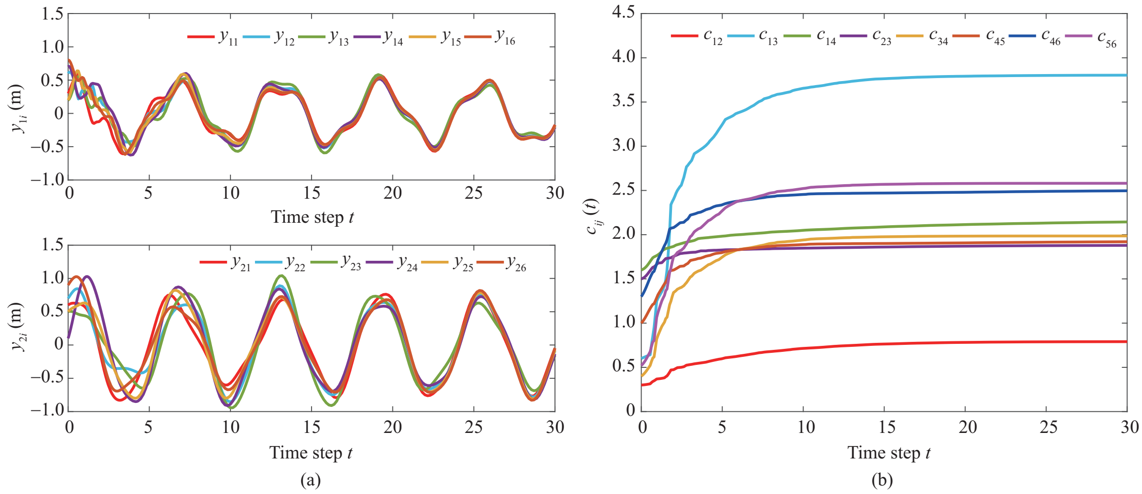

Figure 7 (Color online) Trajectories under the system-identification-based controller. (a) States of the MAS; (b) coupling weights $c_{ij}(t)$.

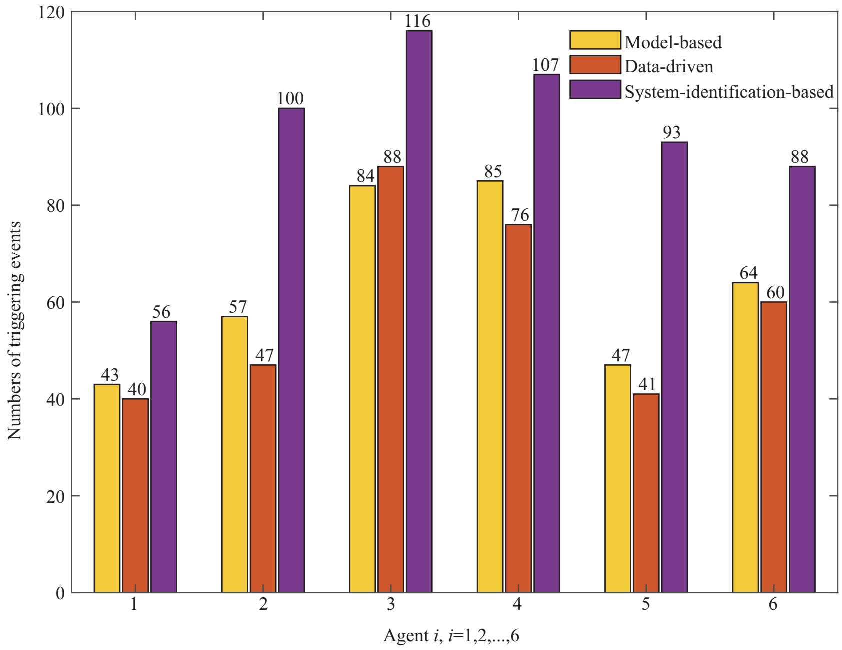

Figure 8 (Color online) Comparison of communication efficiency of different approaches.

half of that determined by the system-identification-based one ($t = 30$ s). It indicates that our data-driven approach exhibits a faster convergence speed. Second, to make a quantitative comparison of communication efficiency, we count the numbers of triggering events over the whole MAS within 30 s for all approaches, and report them in Figure 8. A maximum of 116 (Agent 3 in the system identification case) out of 300 samples are required, which showcases the effectiveness of the proposed ETM. Additionally, compared with the proposed data-driven approach, the system-identification-based approach requires more frequent communication between agents to achieve consensus. This is mainly because as the system dimension grows larger, the system-identification-based approach tends to overfit the noise in the data [39], especially when the amount of available data is limited, and results in poor performance. In words, when facing a large system dimension as well as limited and noisy data, the proposed data-driven method is superior to the system-identification-based one in terms of stability and system performance.

6 Conclusion

In this study, the consensus problem for discrete-time linear MASs under undirected graphs is addressed. Initially, a model-based adaptive control law and an asynchronous ETM were proposed, both of which were fully distributed and independent of global information. To address the consensus of unknown MASs, a data-driven approach for computing the controller gain and triggering matrices directly from data was developed. It was demonstrated that the proposed model-based and data-driven consensus controllers achieve asymptotic stability under standard conditions through intermittent communications. The efficiency and merits of the controllers were corroborated by numerical examples. Generalizing the results to heterogeneous MASs and self-triggered MASs constitutes interesting directions for future study.

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Appendix A Proof of Theorem 1

Proof. Consider the following candidate Lyapunov function:

$$V(t)=\sum_{i=1}^{N}\delta_i^{\mathrm{T}}P\delta_i+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{\alpha c_{ij}}{4\sigma_{ij}}, \tag{A1}$$

where α is a positive constant to be defined later and $P\succ 0$ is the unique solution of (6). Evidently, $V(t)$ is positive definite. The forward difference $\triangle V(t):=V(t+1)-V(t)$ along the trajectory of (5) yields that

$$\begin{aligned}\triangle V(t)=&\sum_{i=1}^{N}\delta_i^{\mathrm{T}}(A^{\mathrm{T}}PA-P)\delta_i+2\sum_{i=1}^{N}\delta_i^{\mathrm{T}}A^{\mathrm{T}}PBK\sum_{j=1}^{N}c_{ij}a_{ij}(\bar{x}_i-\bar{x}_j)+\left(\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(\bar{x}_i-\bar{x}_j)\right)^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\\&\times PBK\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(\bar{x}_i-\bar{x}_j)+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{1}{4}\alpha a_{ij}(\bar{x}_i-\bar{x}_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j).\end{aligned} \tag{A2}$$

In what follows, each term in (17) will be analyzed.

Owing to $c_{ij}(t)=c_{ji}(t)$ and $a_{ij}=a_{ji}$, the second term of (A2) can be handled as

$$2\sum_{i=1}^{N}\delta_i^{\mathrm{T}}A^{\mathrm{T}}PBK\sum_{j=1}^{N}c_{ij}a_{ij}(\bar{x}_i-\bar{x}_j)=-\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(\bar{x}_i-\bar{x}_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j)+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(e_i-e_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j) \tag{A3}$$

by employing the facts $(\bar{x}_i-\bar{x}_j)=(\delta_i-\delta_j)+(e_i-e_j)$ and $\Phi=A^{\mathrm{T}}PB(B^{\mathrm{T}}PB)^{-1}B^{\mathrm{T}}PA$.

By leveraging Lemma 2, one derives

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(e_i-e_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j)&\leqslant\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(e_i-e_j)^{\mathrm{T}}\Phi(e_i-e_j)+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(\bar{x}_i-\bar{x}_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j)\\&\leqslant\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}e_i^{\mathrm{T}}\Phi e_i+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(\bar{x}_i-\bar{x}_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j),\end{aligned} \tag{A4}$$

which follows from $\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(e_i-e_j)^{\mathrm{T}}\Phi(e_i-e_j)\leqslant 4\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}e_i^{\mathrm{T}}\Phi e_i$.

Again using the symmetry of $c_{ij}(t)$ and a_{ij}, it can be deduced that $\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(\bar{x}_i-\bar{x}_j)=0$. Thus, the third term in (A2) is equal to zero.

By substituting (A3) and (A4) into (A2), $\triangle V(t)$ is bounded by

$$\triangle V(t)\leqslant\sum_{i=1}^{N}\delta_i^{\mathrm{T}}(A^{\mathrm{T}}PA-P)\delta_i+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}e_i^{\mathrm{T}}\Phi e_i+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{1}{4}\alpha a_{ij}(\bar{x}_i-\bar{x}_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j). \tag{A5}$$

It can be confirmed that

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(\bar{x}_i-\bar{x}_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j)\\&=\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(\delta_i-\delta_j)^{\mathrm{T}}\Phi(\delta_i-\delta_j)+2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(\delta_i-\delta_j)^{\mathrm{T}}\Phi(e_i-e_j)+\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(e_i-e_j)^{\mathrm{T}}\Phi(e_i-e_j).\end{aligned} \tag{A6}$$

Similarly, it follows from Lemma 2 again that

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\delta_i-\delta_j)^{\mathrm{T}}\Phi(e_i-e_j) \leqslant \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\delta_i-\delta_j)^{\mathrm{T}}\Phi(\delta_i-\delta_j)+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(e_i-e_j)^{\mathrm{T}}\Phi(e_i-e_j). \tag{A7}$$

Then, substituting (A6) and (A7) into (A5) yields $\triangle V(t) \leqslant \delta^{\mathrm{T}}[I_N\otimes(A^{\mathrm{T}}PA-P)-\frac{1}{4}\alpha(\mathcal{L}\otimes\Phi)]\delta+\sum_{i=1}^{N}\sum_{j=1}^{N}(c_{ij}a_{ij}+\alpha)e_i^{\mathrm{T}}\Phi e_i-\frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha a_{ij}(\bar{x}_i-\bar{x}_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j)$. Capitalizing on Lemma 1, it leads to $\delta^{\mathrm{T}}(\mathcal{L}\otimes\Phi)\delta \geqslant \lambda_2(\mathcal{L})\delta^{\mathrm{T}}(I_N\otimes\Phi)\delta$. Then, by recalling the triggering function (4) and selecting a large enough $\alpha \geqslant \max\{1/\varphi, 4/\lambda_2(\mathcal{L})\}$, it follows from Lemma 3 that

$$\begin{aligned}\triangle V(t) \leqslant & -\sum_{i=1}^{N}\delta_i^{\mathrm{T}}Q\delta_i+\alpha\sum_{i=1}^{N}\left[\sum_{j=1}^{N}\left(\frac{1}{\varphi\alpha}\varphi c_{ij}+1\right)a_{ij}e_i^{\mathrm{T}}\Phi e_i-\frac{1}{8}\sum_{j=1}^{N}a_{ij}(\bar{x}_i-\bar{x}_j)^{\mathrm{T}}\Phi(\bar{x}_i-\bar{x}_j)\right]\\ \leqslant & -\lambda_{\min}(Q)\sum_{i=1}^{N}\delta_i^{\mathrm{T}}\delta_i+\alpha N\theta\mathrm{e}^{-\mu t}<0,\end{aligned} \tag{A8}$$

where $A^{\mathrm{T}}PA-P-A^{\mathrm{T}}PB(B^{\mathrm{T}}PB)^{-1}BPA = A^{\mathrm{T}}PA-P-\Phi=-Q\prec 0$.

Hence, it can be verified that $V(t)$ is monotonically decreasing over each time interval $t\in\mathbb{N}_{[t_k^i,t_{k+1}^i-1]}$, $\forall k\in\mathbb{N}, i\in\mathcal{I}_N$. According to (A1), it holds that $V(t)\geqslant 0$ for all $t\geqslant 0$. Thus, $V(t)$ is bounded for all times $t\in\mathbb{N}$. It follows from the definition of $V(t)$ that the boundedness of $V(t)$ implies the boundedness of $c_{ij}(t)$. According to the adaptive law in (2), $c_{ij}(t)$ is monotonically increasing. Thus, every $c_{ij}(t)$ converges to some positive constant. Above all, the consensus error of each agent $\delta_i(t)\to 0$ as $t\to\infty$, which shows that the MAS (1) achieves asymptotic consensus. The proof is completed.

Appendix B Proof of Proposition 1

Proof. Consider the definition of the measurement error $e_i(t)$ of agent i for $t\in\mathbb{N}_{[t_k^i,t_{k+1}^i-1]}$. It follows from (1) and (2) that

$$e_i(t+1)=Ae_i(t)+(I_N-A)\bar{x}_i(t)-BK\sum_{j=1}^{N}c_{ij}(t)a_{ij}\left(x_i(t_k^i)-x_j(t_{k_j}^j)\right). \tag{B1}$$

The solution of (B1) is given by

$$e_i(t)=\sum_{s=t_k^i}^{t-1}A^{t-s-1}\left[(I_N-A)x_i(t_k^i)-BK\sum_{j=1}^{N}c_{ij}(s)a_{ij}\left(x_i(t_k^i)-x_j(t_{k_j}^j)\right)\right],$$

where we have used the fact that $e_i(t_k^i)=0$. As shown in Theorem 1, $c_{ij}(t)$ and $\delta_i(t)$ are bounded. Besides, the boundedness of $\delta_i(t)$ implies $x(t)$ is finite for any finite t. Without loss of generality, assume that $c_{ij}\leqslant\bar{c}$ for some positive constant $\bar{c}$. Then, it can be derived that $\|e_i(t)\|\leqslant\sum_{s=t_k^i}^{t-1}\|A^{t-s-1}\|\big(\zeta_i+\bar{c}\eta_i\big)$, where ζ_i and η_i denote the upper bound of $\|I_N-A\|\|x_i(t_k^i)\|$ and $\sum_{j=1}^{N}a_{ij}\|BK\|\|x_i(t_k^i)-x_j(t_{k_j}^j)\|$, respectively, for $t\in\mathbb{N}_{[t_k^i,t_{k+1}^i-1]}$.

By recalling the ETM (3)–(4), it can be seen that one sufficient condition to guarantee $f_i(t)\leqslant 0$ is $\|e_i(t)\|^2\leqslant\frac{\theta\mathrm{e}^{-\mu t}}{l_{ii}\|\Phi\|(1+\varphi\bar{c})}$. Therefore, the events will not be triggered until

$$\sum_{s=t_k^i}^{t-1}\|A^{t-s-1}\|\big(\zeta_i+\bar{c}\eta_i\big)=\sqrt{\frac{\theta\mathrm{e}^{-\mu t}}{l_{ii}\|\Phi\|(1+\varphi\bar{c})}}. \tag{B2}$$

Let t^* denote the solution of (B2). Thus, a lower bound τ_k^i of $t_{k+1}^i-t_k^i$ can be obtained by $t_{k+1}^i-t_k^i\geqslant\tau_k^i=t^*-t_k^i$. If τ_k^i is greater than 1, the lower bound of the inter-event interval between each agent's two consecutive triggered events is greater than one sampling period. According to [16, Theorem 1], there must exist a sampling interval $h>0$ such that $\tau_k^i>1$. Therefore, the event would not be triggered at each sampling if Eq. (B2) has a solution $t^*-t_k^i>1$ for all triggering instants t_k^i, $i=1,2,\ldots,N$. The proof is completed.