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Adaptive event-trigger-based sampled-data stabilization of complex-valued neural networks: a real and complex LMI approach

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Dear editor,

With the popularization of digital techniques, practical control systems are usually implemented via embedded microprocessors, and sampled-data control is applied. The system state is sampled and received by the controller only at discrete sampling instants [1]. Sampled-data control is fundamentally time-triggered, which may cause unnecessary data transmission if the change of the current sampled data is relatively small compared with the previously released data. As an alternative, an event-trigger (ET) scheme is proposed in which the sampled data is released only when a prescribed triggering condition is satisfied [2–6]. However, the event-trigger threshold in those studies is assumed to be constant. When the system state changes, the constant threshold will lose the ability to respond to the change. To address this limitation, an adaptive event-trigger (AET) scheme whose threshold can be adjusted dynamically according to the system state is proposed [7]. Note that the goal of AET control is to design a sampled-data controller with an AET scheme that can guarantee a longer sampling period or fewer triggering times. The most common way to achieve this goal is to reduce the design conservativeness by using improved Lyapunov functionals. Here, discontinuous Lyapunov functionals that take advantage of the sampling characteristic are proposed [8,9]. However, developing a discontinuous Lyapunov functional to analyze the AET-based sampled-data stabilization of delayed complex-valued neural networks (CVNNs) directly in a complex domain is a challenging problem.

In this study, sampled-data stabilization of delayed CVNNs subject to an AET scheme is addressed. The primary contributions of this study are as follows. (1) To further reduce data transmission, an AET communication scheme is designed to help select the necessary sampled data. (2) A discontinuous Lyapunov functional in a complex domain is developed, and several free matrices are introduced to obtain a relaxed result. (3) A stability analysis of the closed-loop system is carried out directly in the complex domain. A less conservative stabilization criterion expressed into real and complex LMIs is derived. It has been demonstrated that the sampling period can be increased, and the triggering times can be reduced.

Notations. Throughout this study, $\mathbb{R}^n$, $\mathbb{C}^n$, $\mathbb{R}^{n\times n}$, and $\mathbb{C}^{n\times n}$ represent the n-dimensional real and complex Euclidean spaces and the sets of $n\times n$ real and complex matrices, respectively. For $A\in\mathbb{R}^{n\times n}$, A^{-1} denotes the inverse of matrix A and $A>0$ implies that A is a real symmetric positive definite matrix. "T" and "*" denote the transposition and the complex conjugate transposition, respectively. $\mathrm{diag}\{l_1,l_2,\ldots,l_n\}$ represents a diagonal matrix. $\mathrm{Sym}\{X\}$ means $X+X^{\mathrm{T}}$ or $X+X^*$. $\bigstar$ denotes the symmetric block in a real symmetric matrix, and $||z||=\sqrt{z^*z}$ represents the modulus of a vector $z\in\mathbb{C}^n$.

Problem formulation. Consider the delayed CVNN described as follows:

$$\dot{z}(t)=-Dz(t)+Af(z(t))+Bf(z_\tau(t))+u(t),\quad t>0,\quad (1)$$

where $z(t)=[z_1(t),z_2(t),\ldots,z_n(t)]^{\mathrm{T}}\in\mathbb{C}^n$ denotes the system state; diagonal matrix $D>0$ denotes the neuron self-feedback; $z_\tau(t)=z(t-\tau(t))=[z_{1\tau}(t),z_{2\tau}(t),\ldots,z_{n\tau}(t)]^{\mathrm{T}}\in\mathbb{C}^n$ and $0\leqslant\tau(t)\leqslant\overline{\tau}$ is the time delay where $\dot{\tau}(t)\leqslant\mu<1$ is satisfied. $f(z(t))=[f_1(z_1(t)),f_2(z_2(t)),\ldots,f_n(z_n(t))]^{\mathrm{T}}$ and $f(z_\tau(t))=[f_1(z_{1\tau}(t)),f_2(z_{2\tau}(t)),\ldots,f_n(z_{n\tau}(t))]^{\mathrm{T}}$ are the activation functions with $f_i(0)=0$, and $A,B\in\mathbb{C}^{n\times n}$ are the connection weight matrices.

Assumption 1. The component functions $f_i(\cdot)$ are Lipschitz continuous on $\mathbb{C}$. In other words, there exist positive constants F_i $(i=1,2,\ldots,n)$ such that

$$||f_i(u_i)-f_i(v_i)||\leqslant F_i||u_i-v_i||,\quad\forall\, u_i,v_i\in\mathbb{C}.$$

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The stabilization problem of system (1) will be considered under aperiodic sampling. The sampling instants are defined as $0 = l_0 < l_1 < \cdots < l_q < \cdots < \lim_{q\to\infty} l_q = \infty$. The sampling period $\lambda_q = l_q - l_{q-1}$ satisfies $0 < \lambda_m \leqslant \lambda_q \leqslant \lambda_M$, where two constants λ_m and λ_M are the lower and upper bounds of λ_q, respectively. The controller is designed as

$$u(t) = Kz(t_k), \quad t \in [t_k, t_{k+1}), \tag{2}$$

where $K \in \mathbb{R}^{n\times n}$ is the control gain to be designed later.

By taking (2) into (1), system (1) is reduced to

$$\begin{aligned}\dot{z}(t) =& - Dz(t) + Af(z(t)) + Bf(z_\tau(t)) \\ &+ Kz(t_k),\ t \in [t_k, t_{k+1}).\end{aligned} \tag{3}$$

To select the necessary sampled data $z(t_k)$, an AET scheme is introduced. The introduced scheme helps to determine whether the sampled signal $z(l_r)$ should be released. The necessary sampled data $z(t_k)$ is picked out, and the next transmission instant t_{k+1} is determined as follows:

$$\begin{aligned}t_{k+1} = t_k + \sum_{j=1}^{r-1}\lambda_j + \inf_r\{&\lambda_r|e^*(l_r)\Omega e(l_r) \\ &\geqslant \alpha(t)z^*(l_r)\Omega z(l_r)\},\end{aligned} \tag{4}$$

where $e(l_r) = z(l_r) - z(t_k)$, t_k is the latest transmission instant, $l_r = t_k + \sum_{j=1}^{r}\lambda_j$ is the current sampling instant, $r > 0$ is the number of sampling intervals from the former transmission instant t_k to the current sampling instant l_r, λ_j are the variable sampling periods, $\Omega > 0$ is the trigger matrix, and $\alpha(t)$ is the threshold function which is regulated by

$$\dot{\alpha}(t) = \frac{\theta}{\alpha(t)}\left(\frac{1}{\alpha(t)} - \alpha_0\right)e^*(l_r)\Omega e(l_r), \tag{5}$$

where $\alpha(t) \in (0, 1]$ and α_0 and θ are two given positive constants. For simplicity, let $\theta = 1$ in the following Theorem 1 and Corollary 1 in Appendix C.

The signal holding interval $[t_k, t_{k+1}) \triangleq \Psi$ can be divided into subintervals $\Psi_r = [l_r, l_r + \lambda_{r+1})$, i.e., $\Psi = \cup_{r=0}^{p-1}\Psi_r$ with l_r being defined in (4), and $t_{k+1} = l_p$.

Theorem 1. For given scalars γ_1, γ_2, γ_3, and α_0, system (1) with controller (2) and AET scheme (4)-(5) is globally asymptotically stabilized if there exist matrices $P > 0,\ Q > 0,\ R_1 > 0,\ X > 0,\ Z_i > 0\ (i = 1, 2, 3),\ \Omega > 0,\ H_1 > 0,\ H_3 > 0,\ Y_1 > 0,\ Y_3 > 0$, arbitrary matrices $S,\ Y_2,\ H_2,\ N_i,\ M_i\ (i = 1, 2),\ L$, diagonal matrices $\Lambda_1,\ \Lambda_2 > 0$, and invertible matrix G, such that the following LMI conditions are feasible for $\tilde{\lambda} \in \{\lambda_m, \lambda_M\}$:

$$\begin{aligned}&\Xi_1(\tilde{\lambda}) = \Phi_1 + \Phi_2 + \tilde{\lambda}\Phi_3 + \Phi_6 + \Phi_7 + \Phi_8 < 0, \\ &\Xi_2(\tilde{\lambda}) = \Phi_1 + \Phi_2 + \tilde{\lambda}(\Phi_4 + \Phi_5) + \Phi_6 + \Phi_7 + \Phi_8 < 0,\end{aligned} \tag{6}$$

$$U_1 = \begin{pmatrix} Z_1 & S \\ \bigstar & Z_1\end{pmatrix} > 0,\quad U_2 = \begin{pmatrix} Y_1 & Y_2 & N_1 \\ \bigstar & Y_3 & N_2 \\ \bigstar & \bigstar & Z_2\end{pmatrix} > 0,$$

$$U_3 = \begin{pmatrix} H_1 & H_2 & M_1 \\ \bigstar & H_3 & M_2 \\ \bigstar & \bigstar & Z_3\end{pmatrix} > 0,$$

where $\Phi_1 = \mathrm{Sym}\{\epsilon_1^{\mathrm{T}}P\epsilon_6\} + \Pi_1^{\mathrm{T}}Q\Pi_1 - (1-\mu)\Pi_2^{\mathrm{T}}Q\Pi_2 + \epsilon_1^{\mathrm{T}}R_1\epsilon_1 - \epsilon_3^{\mathrm{T}}R_1\epsilon_3 + \overline{\tau}^2\epsilon_6^{\mathrm{T}}Z_1\epsilon_6 - \Pi_3^{\mathrm{T}}U_1\Pi_3, \Phi_2 = -\Pi_4^{\mathrm{T}}X\Pi_4 + \mathrm{Sym}\{\Pi_4^{\mathrm{T}}(N_1\Pi_5 - 2N_2\epsilon_8)\} + \epsilon_1^{\mathrm{T}}F\Lambda_1F\epsilon_1 - \epsilon_4^{\mathrm{T}}\Lambda_1\epsilon_4 + \epsilon_2^{\mathrm{T}}F\Lambda_2F\epsilon_2 - \epsilon_5^{\mathrm{T}}\Lambda_2\epsilon_5, \Phi_3 = \mathrm{Sym}\{\Pi_4^{\mathrm{T}}X\Pi_7\} + \epsilon_6^{\mathrm{T}}Z_2\epsilon_6, \Phi_4 = \Pi_4^{\mathrm{T}}(Y_1 + \frac{\lambda_M^2}{3}Y_3)\Pi_4 + \mathrm{Sym}\{\Pi_4^{\mathrm{T}}N_2\Pi_6\}, \Phi_5 = \mathrm{Sym}\{\Pi_{10}^{\mathrm{T}}(H_1 + \frac{\lambda_M^2}{3}H_3 + \bar{M}_1)\Pi_{11}\}, \Phi_6 = \mathrm{Sym}\{\Pi_8^{\mathrm{T}}\Pi_9\}, \Phi_7 = \Pi_{10}^{\mathrm{T}}(H_1 + \frac{\lambda_M^2}{3}H_3 + \bar{M}_1)\Pi_{10} + \mathrm{Sym}\{\Pi_{10}^{\mathrm{T}}\bar{M}_2\Pi_{11}\} + \epsilon_6^{\mathrm{T}}Z_3\epsilon_6, \Phi_8 = \epsilon_7^{\mathrm{T}}\Omega\epsilon_7 - \alpha_0\epsilon_9^{\mathrm{T}}\Omega\epsilon_9, \Pi_1 = [\epsilon_1^{\mathrm{T}}, \epsilon_4^{\mathrm{T}}]^{\mathrm{T}}, \Pi_2 = [\epsilon_2^{\mathrm{T}}, \epsilon_5^{\mathrm{T}}]^{\mathrm{T}}, \Pi_3 = [\epsilon_1^{\mathrm{T}} - \epsilon_2^{\mathrm{T}}, \epsilon_2^{\mathrm{T}} - \epsilon_3^{\mathrm{T}}]^{\mathrm{T}}, \Pi_4 = [\epsilon_1^{\mathrm{T}} - \epsilon_7^{\mathrm{T}}, \epsilon_8^{\mathrm{T}}]^{\mathrm{T}}, \Pi_5 = [\epsilon_1^{\mathrm{T}} - \epsilon_7^{\mathrm{T}}]^{\mathrm{T}}, \Pi_6 = [\epsilon_1^{\mathrm{T}} + \epsilon_7^{\mathrm{T}}]^{\mathrm{T}}, \Pi_7 = [\epsilon_6^{\mathrm{T}}, \epsilon_1^{\mathrm{T}}]^{\mathrm{T}}, \Pi_8 = [\epsilon_1^{\mathrm{T}} + \gamma_1\epsilon_6^{\mathrm{T}} + \gamma_2\epsilon_7^{\mathrm{T}} + \gamma_3\epsilon_9^{\mathrm{T}}]^{\mathrm{T}}, \Pi_9 = [-\epsilon_6^{\mathrm{T}}G - \epsilon_1^{\mathrm{T}}GD + \epsilon_4^{\mathrm{T}}GA + \epsilon_5^{\mathrm{T}}GB + \epsilon_7^{\mathrm{T}}L - \epsilon_9^{\mathrm{T}}L]^*$, $\Pi_{10} = [\epsilon_1^{\mathrm{T}}, \epsilon_7^{\mathrm{T}}, \epsilon_8^{\mathrm{T}}]^{\mathrm{T}}$, $\Pi_{11} = [\epsilon_6^{\mathrm{T}}, 0, \epsilon_1^{\mathrm{T}}]^{\mathrm{T}}, \bar{M}_1 = [M_2, M_2, 0], \bar{M}_2 = [M_1, -M_1, -2M_2]$, $\epsilon_i = [0_{n\times(i-1)n}, I_n, 0_{n\times(9-i)n}]$, $i = 1, 2, \ldots, 9$. Furthermore, the control gain is solved by $K = G^{-1}L$.

Proof. See Appendix C.

Conclusion. In this study, an AET scheme, covering the traditional ET scheme as a special case, is designed to address the stabilization of delayed CVNNs subject to aperiodic sampling. A discontinuous Lyapunov functional in a complex domain is developed to facilitate the stability analysis, which contains several free matrices and thus may lead to some relaxed stability conditions. A less conservative stabilization criterion is derived by which the control gain and the trigger matrix can be synthesized simultaneously. The simulation comparisons show that the sampling period can be increased and the triggering times can be reduced by using the proposed AET scheme and the constructed discontinuous Lyapunov functional.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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