

• Supplementary File •

Adaptive event-trigger-based sampled-data stabilization of complex-valued neural networks: a real and complex LMI approach

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Appendix A Lemma 1

Lemma 1. For any real matrix S and real matrix $V > 0$ satisfying $U = \begin{pmatrix} V & S \\ \star & V \end{pmatrix} > 0$, scalar $\bar{\tau} \geq \tau(t) \geq 0$, and function $\dot{z} : [-\bar{\tau}, 0] \rightarrow \mathbb{C}^n$, we have the following inequality:

$$-\bar{\tau} \int_{t-\bar{\tau}}^t \dot{z}^*(s) V \dot{z}(s) ds < -\vartheta^*(t) U \vartheta(t)$$

where $\vartheta(t) = \begin{bmatrix} z(t) - z_\tau(t) \\ z_\tau(t) - z(t - \bar{\tau}) \end{bmatrix}$ and $z_\tau(t) = z(t - \tau(t))$.

Proof: In view of [1], we have

$$\begin{aligned} -\bar{\tau} \int_{t-\bar{\tau}}^t \dot{z}^*(s) V \dot{z}(s) ds &= -\bar{\tau} \int_{t-\bar{\tau}}^t [\dot{z}_R^T(s), \dot{z}_I^T(s)] \begin{pmatrix} V & 0 \\ \star & V \end{pmatrix} \begin{bmatrix} \dot{z}_R(s) \\ \dot{z}_I(s) \end{bmatrix} \\ &< -[\vartheta_R^T(t), \vartheta_I^T(t)] \begin{pmatrix} U & 0 \\ \star & U \end{pmatrix} \begin{bmatrix} \vartheta_R(t) \\ \vartheta_I(t) \end{bmatrix} \\ &= -\vartheta^*(t) U \vartheta(t). \end{aligned} \tag{A1}$$

Appendix B Lemma 2

Lemma 2. Let $z : [c, d] \rightarrow \mathbb{C}^n$ be a differentiable function. For any vector $\xi \in \mathbb{C}^m$, positive definite matrices $Z_1, Z_3 \in \mathbb{R}^{m \times m}$, $V \in \mathbb{R}^{n \times n}$, and any matrices $Z_2 \in \mathbb{R}^{m \times m}$, $N_1, N_2 \in \mathbb{R}^{m \times n}$ satisfying

$$\Pi = \begin{pmatrix} Z_1 & Z_2 & N_1 \\ \star & Z_3 & N_2 \\ \star & \star & V \end{pmatrix} > 0$$

we have the following inequality:

$$\begin{aligned} -\int_c^d \dot{z}^*(s) V \dot{z}(s) ds &< (d-c) \xi^* [Z_1 + \frac{(d-c)^2}{3} Z_3] \xi \\ &+ \text{Sym}\{\xi^* [N_1 [z(d) - z(c)] - 2N_2 \\ &\times \int_c^d z(s) ds] + (d-c) \xi^* N_2 [z(c) + z(d)]\}. \end{aligned}$$

Proof: Let $g(v) : [c, d] \rightarrow \mathbb{R}$, $g(v) = 2v - c - d$, $\xi = [z^*(d), z^*(c), \int_c^d z^*(s) ds]^*$, $\eta(s) = [\xi^*, g(s) \xi^*, \dot{z}^*(s)]^*$,
 $W = \begin{bmatrix} (d-c)[z_R(d) + z_R(c)] - 2 \int_c^d z_R(s) ds \\ (d-c)[z_I(d) + z_I(c)] - 2 \int_c^d z_I(s) ds \end{bmatrix}$.

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It is easy to see that

$$\eta^*(s)\Pi\eta(s) > 0. \tag{B1}$$

By calculation, it yields $\int_c^d g(v)dv = 0$, $\int_c^d g^2(v)dv = \frac{(d-c)^3}{3}$, $\int_c^d g(v)\dot{z}(v)dv = (d-c)[z(d) + z(c)] - 2\int_c^d z(v)dv$. Integrating the left-hand side of (B1) from c to d yields

$$\begin{aligned} \int_c^d \eta^*(s)\Pi\eta(s)ds &= \int_c^d [\eta_R^T(s), \eta_I^T(s)] \begin{pmatrix} \Pi & 0 \\ \star & \Pi \end{pmatrix} \begin{bmatrix} \eta_R(s) \\ \eta_I(s) \end{bmatrix} ds \\ &= (d-c)[\xi_R^T, \xi_I^T] \begin{pmatrix} Z_1 & 0 \\ \star & Z_1 \end{pmatrix} \begin{bmatrix} \xi_R \\ \xi_I \end{bmatrix} + \frac{(d-c)^3}{3}[\xi_R^T, \xi_I^T] \begin{pmatrix} Z_3 & 0 \\ \star & Z_3 \end{pmatrix} \begin{bmatrix} \xi_R \\ \xi_I \end{bmatrix} + 2[\xi_R^T, \xi_I^T] \begin{pmatrix} N_1 & 0 \\ \star & N_1 \end{pmatrix} \\ &\quad \times \begin{bmatrix} z_R(d) - z_R(c) \\ z_I(d) - z_I(c) \end{bmatrix} + 2[\xi_R^T, \xi_I^T] \begin{pmatrix} N_2 & 0 \\ \star & N_2 \end{pmatrix} W + \int_c^d [z_R^T(s), z_I^T(s)] \begin{pmatrix} V & 0 \\ \star & V \end{pmatrix} \begin{bmatrix} z_R(s) \\ z_I(s) \end{bmatrix} ds \\ &= (d-c)\xi^* Z_1 \xi + \frac{(d-c)^3}{3}\xi^* Z_3 \xi + 2\xi^* N_1 [z(d) - z(c)] + 2\xi^* N_2 [(d-c)[z(d) + z(c)] \\ &\quad - 2\int_c^d z(s)ds] + \int_c^d z^*(s)Vz(s)ds. \end{aligned}$$

Hence, we have

$$- \int_c^d z^*(s)Vz(s)ds < (d-c)\xi^* Z_1 \xi + \frac{(d-c)^3}{3}\xi^* Z_3 \xi + \text{Sym}\{\xi^* N_1 [z(d) - z(c)] + \xi^* N_2 [(d-c)[z(d) + z(c)] - 2\int_c^d z(s)ds]\}.$$

Appendix C Proof of Theorem 1

Consider the time-dependent Lyapunov functional

$$V(t) = \sum_{i=1}^5 V_i(t), \quad t \in \Psi_r \tag{C1}$$

with

$$\begin{aligned} V_1(t) &= z^*(t)Pz(t), \\ V_2(t) &= \int_{t-\tau}^t \xi_1^*(s)Q\xi_1(s)ds + \int_{t-\bar{\tau}}^t z^*(s)R_1z(s)ds + \bar{\tau} \int_{-\bar{\tau}}^0 \int_{t+\theta}^t \dot{z}^*(s)Z_1\dot{z}(s)dsd\theta, \\ V_3(t) &= (l_r + \lambda_{r+1} - t)\xi_2^*(t)X\xi_2(t) + (l_r + \lambda_{r+1} - t) \int_{l_r}^t z^*(s)Z_2\dot{z}(s)ds, \\ V_4(t) &= (t - l_r) \left[\xi_3^*(t)(H_1 + \frac{\lambda_M^2}{3}H_3)\xi_3(t) + \text{Sym}\{\xi_3^*(t)M_2[z(t) + z(l_r)]\} \right] + \text{Sym}\{\xi_3^*(t) [M_1[z(t) - z(l_r)] \\ &\quad - 2M_2 \int_{l_r}^t z(s)ds] \} + \int_{l_r}^t \dot{z}^*(s)Z_3\dot{z}(s)ds, \\ V_5(t) &= \frac{1}{2}\alpha^2(t), \end{aligned}$$

where $\xi_1(t) = [z^*(t), f^*(z(t))]^*$, $\xi_2(t) = [z^*(t) - z^*(l_r), \int_{l_r}^t z^*(s)ds]^*$, $\xi_3(t) = [z^*(t), z^*(l_r), \int_{l_r}^t z^*(s)ds]^*$.

Obviously, $V_1(t)$, $V_2(t)$, $V_3(t)$ and $V_5(t)$ are positive definite. Since $U_3 > 0$, in view of Lemma 2, we obtain that

$$\begin{aligned} V_4(t) &> (t - l_r) \left[\xi_3^*(t) \left[H_1 + \frac{(t - l_r)^2}{3}H_3 \right] \xi_3(t) + \text{Sym}\{\xi_3^*(t)M_2[z(t) + z(l_r)]\} \right] + \int_{l_r}^t \dot{z}^*(s)Z_3\dot{z}(s)ds \\ &\quad + \text{Sym}\{\xi_3^*(t) [M_1[z(t) - z(l_r)] - 2M_2 \int_{l_r}^t z(s)ds] \} \\ &= \int_{l_r}^t \xi_4^*(s)U_3\xi_4(s)ds > 0 \end{aligned}$$

where $\xi_4(s) = [\xi_3^*(s), g(s)\xi_3^*(s), \dot{z}^*(s)]^*$ and $g(s) = 2s - l_r - t$. Thus, $V(t)$ is positive definite. Clearly, $V_1(t)$, $V_2(t)$ and $V_5(t)$ are continuous. Besides, we have

$$\lim_{t \rightarrow l_r^+} V_3(t) = \lim_{t \rightarrow l_r^-} V_3(t) = V_3(l_r) = 0.$$

Thus, $V_3(t)$ is continuous. Also, we have

$$\lim_{t \rightarrow l_r^-} V_4(t) > V_4(l_r) = \lim_{t \rightarrow l_r^+} V_4(t) = 0.$$

Hence, $V(t)$ is a discontinuous Lyapunov functional.

Differentiating (C1) along the trajectories of system (3), we have

$$\begin{aligned}
 \dot{V}_1(t) &= \text{Sym}\{z^*(t)P\dot{z}(t)\} = \chi^*(t)\text{Sym}\{\epsilon_1^T P \epsilon_6\}\chi(t), \\
 \dot{V}_2(t) &< \chi^*(t)[\Pi_1^T Q \Pi_1 - (1 - \mu)\Pi_2^T Q \Pi_2 + \epsilon_1^T R_1 \epsilon_1 - \epsilon_3^T R_1 \epsilon_3 + \bar{\tau}^2 \epsilon_6^T Z_1 \epsilon_6]\chi(t) - \bar{\tau} \int_{t-\bar{\tau}}^t \dot{z}^*(s)Z_1 z(s)ds, \\
 \dot{V}_3(t) &< \chi^*(t)[-\Pi_4^T X \Pi_4 + (l_r + \lambda_{r+1} - t)(\text{Sym}\{\Pi_4^T X \Pi_7\} + \epsilon_6^T Z_2 \epsilon_6)]\chi(t) - \int_{l_r}^t \dot{z}^*(s)Z_2 \dot{z}(s)ds \\
 \dot{V}_4(t) &= \chi^*(t)[\Pi_{10}^T (H_1 + \frac{\lambda_M^2}{3} H_3 + \bar{M}_1)\Pi_{10} + \epsilon_6^T Z_3 \epsilon_6 + \text{Sym}\{\Pi_{10}^T \bar{M}_2 \Pi_{11}\} + \text{Sym}\{(t - l_r)\Pi_{10}^T (H_1 + \frac{\lambda_M^2}{3} H_3 \\
 &\quad + \bar{M}_1)\Pi_{11}\}]\chi(t), \\
 \dot{V}_5(t) &\leq \chi^*(t)(\epsilon_7^T \Omega \epsilon_7 - \alpha_0 \epsilon_9^T \Omega \epsilon_9)\chi(t).
 \end{aligned}$$

where $\chi(t) = [\chi_1^*(t), \chi_2^*(t)]^*$, $\chi_1(t) = [z^*(t), z_\tau^*(t), z^*(t - \bar{\tau}), f^*(z(t)), f^*(z_\tau(t))]^*$, $\chi_2(t) = [\dot{z}^*(t), z^*(l_r), \int_{l_r}^t \dot{z}^*(s)ds, e^*(l_r)]^*$.

For the two integral terms in $\dot{V}_2(t)$ and $\dot{V}_3(t)$, according to Lemma 1, Lemma 2 and $U_1, U_2 > 0$, we have

$$\begin{aligned}
 -\bar{\tau} \int_{t-\bar{\tau}}^t \dot{z}^*(s)Z_1 \dot{z}(s)ds &< -\chi^*(t)\Pi_3^T U_1 \Pi_3 \chi(t), \\
 -\int_{l_r}^t \dot{z}^*(s)Z_2 \dot{z}(s)ds &< \chi^*(t)[\text{Sym}\{\Pi_4^T N_1 \Pi_5 - 2\Pi_4^T N_2 \epsilon_8\} + (t - l_r)[\Pi_4^T (Y_1 + \frac{\lambda_M^2}{3} Y_3)\Pi_4 + \text{Sym}\{\Pi_4^T N_2 \Pi_6\}]]\chi(t).
 \end{aligned}$$

Thus, we further obtain

$$\begin{aligned}
 \dot{V}_2(t) &< \chi^*(t)[\Pi_1^T Q \Pi_1 - (1 - \mu)\Pi_2^T Q \Pi_2 + \epsilon_1^T R_1 \epsilon_1 - \epsilon_3^T R_1 \epsilon_3 + \bar{\tau}^2 \epsilon_6^T Z_1 \epsilon_6 - \Pi_3^T U_1 \Pi_3]\chi(t), \\
 \dot{V}_3(t) &< \chi^*(t)[-\Pi_4^T X \Pi_4 + (l_r + \lambda_{r+1} - t)(\text{Sym}\{\Pi_4^T X \Pi_7\} + \epsilon_6^T Z_2 \epsilon_6) + \text{Sym}\{\Pi_4^T N_1 \Pi_5 - 2\Pi_4^T N_2 \epsilon_8\} + (t - l_r) \\
 &\quad \times [\Pi_4^T (Y_1 + \frac{\lambda_M^2}{3} Y_3)\Pi_4 + \text{Sym}\{\Pi_4^T N_2 \Pi_6\}]]\chi(t).
 \end{aligned}$$

In addition, for scalars $\gamma_1, \gamma_2, \gamma_3$ and any matrix G with appropriate dimension, we have

$$\begin{aligned}
 0 &= \text{Sym}\{[z^*(t)G + \gamma_1 \dot{z}^*(t)G + \gamma_2 z^*(l_r)G + \gamma_3 e^*(l_r)G] \times [-\dot{z}(t) - Dz(t) + Af(z(t)) + Bf(z_\tau(t)) + Kz(l_r) - Ke(l_r)]\} \\
 &= \chi^*(t)\text{Sym}\{\Pi_8^T \Pi_9\}\chi(t).
 \end{aligned} \tag{C2}$$

On the other hand, based on Assumption 1, we obtain

$$\begin{aligned}
 0 &\leq z^*(t)F\Lambda_1 Fz(t) - f^*(z(t))\Lambda_1 f(z(t)) + z_\tau^*(t)F\Lambda_2 Fz_\tau(t) - f^*(z_\tau(t))F\Lambda_2 Ff(z_\tau(t)) \\
 &= \chi^*(t)(\epsilon_1^T F\Lambda_1 F \epsilon_1 - \epsilon_4^T \Lambda_1 \epsilon_4 + \epsilon_2^T F\Lambda_2 F \epsilon_2 - \epsilon_5^T \Lambda_2 \epsilon_5)\chi(t),
 \end{aligned} \tag{C3}$$

where $F = \text{diag}\{F_1, F_2, \dots, F_n\}$.

Then, taking (C2)-(C3) into $\dot{V}(t)$, we obtain

$$\dot{V}(t) \leq \chi^*(t) \left[\frac{l_r + \lambda_{r+1} - t}{\lambda_{r+1}} \Xi_1(\lambda_{r+1}) + \frac{t - l_r}{\lambda_{r+1}} \Xi_2(\lambda_{r+1}) \right] \chi(t). \tag{C4}$$

In the light of (6), we can get

$$\Xi_i(\lambda_{r+1}) = \frac{\lambda_{r+1} - \lambda_m}{\lambda_M - \lambda_m} \Xi_i(\lambda_M) + \frac{\lambda_M - \lambda_{r+1}}{\lambda_M - \lambda_m} \Xi_i(\lambda_m) < 0, \quad i = 1, 2. \tag{C5}$$

Combining (C5) with (C4), we have

$$\dot{V}(t) < 0, \quad t \in \Psi_r. \tag{C6}$$

Thus, system (3) is globally asymptotically stable. Namely, system (1) can be globally asymptotically stabilized.

Remark 1. A discontinuous Lyapunov functional $V_4(t)$ in complex domain is developed in this study, which contains several free real matrices $M_1, M_2, H_2, N_1, N_2, Y_2$ and utilizes more information about the sampling characteristic. Therefore, it may result in a less conservativeness stabilization criterion. To ensure the positive definiteness of $V(t)$, Lemma 1 and lemma 2 are introduced. More importantly, the stability analysis can be fulfilled directly in the complex domain. It will be seen in Appendix D, the introduction of these free matrices can effectively reduce the design conservativeness.

Remark 2. Some difficulties are encountered in the study of AET based aperiodic sampled-data stabilization of delayed CVNNs. Note that the studied neural networks are complex-valued. For a n -dimensional CVNN, the traditional method is to convert it into a $2n$ -dimensional real-valued neural network (RVNN). However, it will lead to high computational complexity. For that reason, we deal with the stabilization of CVNNs directly in complex domain, but this work is not a duplicate of RVNNs. There exist many difficulties in the construction of an appropriate Lyapunov functional and the estimate of its derivative. Also, when constructing the Lyapunov functional, it needs to take the AET scheme (4)-(5) into consideration. To this end, a discontinuous Lyapunov functional and some improved integral inequalities are employed. As an result, a less conservative stability criterion consisting of real and complex LMIs are derived.

To make a comparison, another stabilization criterion is given, which is derived based on the traditional continuous Lyapunov functional $\tilde{V}(t) = \sum V_i(t) (i = 1, 2, 3, 5)$. In other words, set $U_3 = 0$ in Theorem 1 and then Corollary 1 can be obtained.

Corollary 1. For given scalars $\gamma_1, \gamma_2, \gamma_3, \alpha_0$, system (1) with controller (2) and AET scheme (4)-(5) is globally asymptotical stabilized, if there exist matrices $P > 0, Q > 0, R_1 > 0, X > 0, Z_i > 0 (i = 1, 2), \Omega > 0, Y_1 > 0, Y_3 > 0$, arbitrary matrices $S, Y_2, N_i (i = 1, 2), L$, diagonal matrix $\Lambda_i > 0 (i = 1, 2)$, and invertible matrix G , such that the following LMI conditions are feasible for $\tilde{\lambda} \in \{\lambda_m, \lambda_M\}$:

$$\begin{aligned}\Xi_1(\tilde{\lambda}) &= \Phi_1 + \Phi_2 + \tilde{\lambda}\Phi_3 + \Phi_6 + \Phi_8 < 0, \\ \Xi_2(\tilde{\lambda}) &= \Phi_1 + \Phi_2 + \tilde{\lambda}\Phi_4 + \Phi_6 + \Phi_8 < 0, \\ U_1 &= \begin{pmatrix} Z_1 & S \\ \star & Z_1 \end{pmatrix} > 0, U_2 = \begin{pmatrix} Y_1 & Y_2 & N_1 \\ \star & Y_3 & N_2 \\ \star & \star & Z_2 \end{pmatrix} > 0,\end{aligned}$$

where $\Phi_i (i = 1, 2, 3, 4, 6, 8)$ are the same as those defined in Theorem 1. Furthermore, the control gain is solved by $K = G^{-1}L$.

Appendix D Simulation

For CVNN (1), consider the following parameters $D = \text{diag}\{0.8, 0.8, 0.8\}, \tau(t) = \frac{e^t}{1+e^t}, f(z(t)) = 0.2\text{tanh}(x(t)) + i0.2\text{tanh}(y(t))$, and $a_{11} = 2.2 - 1.7i, a_{12} = -1 - 0.1i, a_{13} = -0.1 + 0.1i, a_{21} = -1.3 + 1.8i, a_{22} = 0.7 - 0.2i, a_{23} = -0.5 + 0.2i, a_{31} = -1.4 - 1.9i, a_{32} = 0.8 + 0.2i, a_{33} = -0.3 - 0.2i, b_{11} = 1.2 + 1.5i, b_{12} = -0.7 + 0.2i, b_{13} = -0.3 + 0.4i, b_{21} = 1.3 + 1.7i, b_{22} = 0.6 - 0.3i, b_{23} = -0.4 + 0.5i, b_{31} = 1.2 + 1.6i, b_{32} = 0.9 + 0.1i, b_{33} = -0.3 + 0.2i$. Clearly, the activation function satisfies the Assumption 1.

Firstly, a comparison between the AET scheme and the ET scheme is made. To that end, let $\lambda_m = \lambda_M$, and the parameters in Theorem 1 are selected as $\gamma_1 = 1, \gamma_2 = 1, \gamma_3 = 1$ and $\alpha_0 = 5$. With the help of Lemma 2 in [2] and by calculation, as for the AET scheme, the allowable maximum upper bound (AMUB) of λ_M is 0.64. Whereas, as for the ET scheme, it is 0.59. Obviously, the proposed AET scheme can effectively reduce the data transmission frequency.

Secondly, to illustrate the superiority of the discontinuous free-matrix-based Lyapunov functional $V_4(t)$, a comparison between Theorem 1 and Corollary 1 is made. Set the parameters $\gamma_1 = 1, \gamma_2 = 1, \gamma_3 = 1, \alpha_0 = 5, \lambda_m = 0.01$ and $\lambda_M = 0.4$ in Theorem 1. Then, by Theorem 1 the control gain and the trigger matrix are solved as

$$K = \text{diag}\{-0.4490, -0.4630, -0.4695\}, \Omega = \text{diag}\{30.0142, 30.3093, 30.0420\}.$$

The responses of the system state, the threshold function and the controller are depicted in Fig. 1, respectively. As shown in Fig. 1, the CVNN under the sampled-data control with the proposed AET scheme can successfully achieve the global stabilization.

To make a comparison, we take the same parameters as in Theorem 1. Then by Corollary 1, we can get the control gain and the trigger matrix as follows:

$$\bar{K} = \text{diag}\{-0.8773, -0.7356, -0.7675\}, \bar{\Omega} = \text{diag}\{0.0281, 0.0282, 0.0280\}.$$

To show how the sampling period is enlarged and the triggering times is reduced, Fig. 2 and a table are given, in which t_{tr}, t_{av} and m_{tr} represent the triggering times, the average release period and the maximum transmit interval, respectively. From the table, we can see that the triggering times calculated by Theorem 1 is reduced more than 21.8% in contrast with those calculated by Corollary 1. It suggests that the transmitted data can be effectively reduced by the proposed AET scheme.

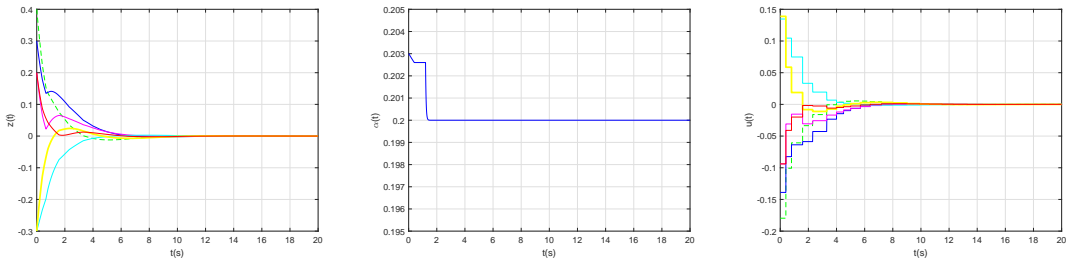


Fig. 1. The responses of the system state $z(t)$, the threshold function $\alpha(t)$ and the controller $u(t)$.

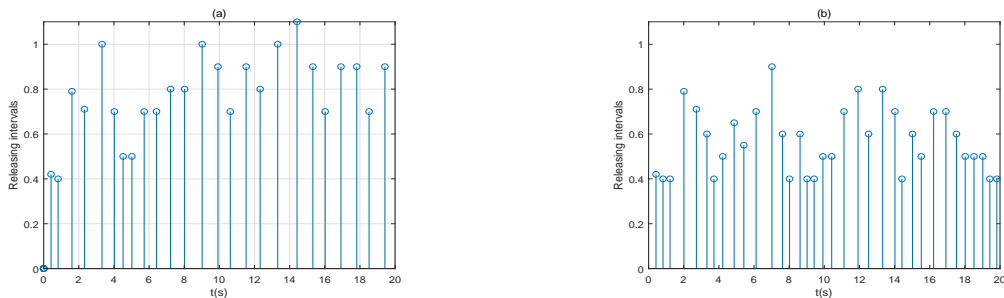


Fig. 2. Release instants and releasing intervals: (a) based on Theorem 1; (b) based on Corollary 1.

Table D1 Comparison on t_{tr} , t_{av} and m_{tr}

	t_{tr}	t_{av}	m_{tr}
Theorem 1	25	0.8	1.1
Corollary 1	32	0.625	0.9

References

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