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Stability analysis of a class of systems with periodically varying delay via looped-functional-based Lyapunov functional

Hongbing ZENG[1*], Huichao LIN[1], Yong HE[2] & Wei WANG[1]

[1]*School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou 412007, China;*
[2]*School of Automation, China University of Geosciences, Wuhan 430074, China*

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Dear editor,

Time delay, which typically appears in many industrial processes, is a challenging issue in the control field because delay can cause the system to experience oscillatory responses or even instability. Time-delay systems belong to the wide class of infinite-dimensional systems, which are difficult to handle in theory. Therefore, significant attention has been paid to system stability analysis with time delay [1–4].

In the real-world applications, a class of delay that vary periodically in an interval is usually observed in practical systems. For example, sampled-data systems can be modeled as time-varying delay systems with sawtooth delay. For such systems, a looped functional approach has been proposed in [5], which relaxes the conditions on the Lyapunov functionals commonly employed in systems with time-varying delay. In [6], a two-sided looped functional approach was proposed. In the sampled-data system, the delay function, $d(t)$, is sawtooth and satisfies $\dot{d}(t)=1$ for $\forall t\neq t_k$. However, many functions cannot satisfy this condition. For example, the cutting process in a rotating cutting machine can be modeled as a system with sinusoidal delay [7]. Determining whether the looped function approach can be applied in such a situation is the motivation for the current study.

In this study, we focus on stability analysis of systems with periodically varying delay. Our primary contributions are summarized as follows. (1) By dividing delay into monotonically increasing intervals and monotonically decreasing intervals, separate looped functionals are proposed for these two classes of intervals. (2) A novel looped-functional-based Lyapunov functional is proposed, and the proposed Lyapunov functional yields less conservative conditions than existing ones.

Main results. Consider the following system with time-varying delay:

$$\begin{cases}\dot{x}(t)=A_0x(t)+A_1x(t-d(t)),\\ x(\theta)=\phi(\theta),\ \theta\in[-h_2,0],\end{cases}\tag{1}$$

where $A_0\in\mathbb{R}^{n\times n}$ and $A_1\in\mathbb{R}^{n\times n}$ are system matrices, $x(t)\in\mathbb{R}^n$ is the system state, and the initial condition $\phi(\theta)$ is a continuous function defined on $[-h_2,0]$. Here, $d(t)$ is a continuous bounded function satisfying the following:

$$d(t)\in[h_1,h_2],\ |\dot{d}(t)|\leqslant\mu<1,\tag{2}$$

where h_1 and h_2 are the minimum and maximum of the delay, respectively. It is assumed that the delay varies periodically between h_1 and h_2, and each period comprises one monotonically increasing interval and one monotonically decreasing interval.

To reduce the conservativeness of the derived stability condition, a framework is proposed to construct the Lyapunov functional. Without loss of generality, it is assumed that there exist $t_{2k-1}<t_{2k}, k=\{1,2,3,\ldots\}$, such that $d(t_{2k-1})$ and $d(t_{2k})$ are extreme values of $d(t)$, where $d(t_{2k-1})=h_1$ and $d(t_{2k})=h_2$ are the minimum and the maximum, respectively. Thus, it is concluded that the time-delay function $d(t)$ is monotonically increasing in intervals $[t_{2k-1},t_{2k}]$ and monotonically decreasing in intervals $[t_{2k},t_{2k+1}]$. Then, similar to [6], two looped functionals are constructed separately for each of these intervals.

Case 1. When $t\in[t_{2k-1},t_{2k})$, i.e., $\dot{d}(t)\in[0,\mu]$, a looped functional is defined as follows:

$$\begin{aligned}V_I(t)=&\,2\eta_1^{\mathrm{T}}(t)Q_1\eta_2(t)\\&+(d(t)-h_1)\int_{t-h_2}^{t-d(t)}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)\mathrm{d}s\\&+(d(t)-h_2)\int_{t-d(t)}^{t-h_1}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)\mathrm{d}s,\end{aligned}\tag{3}$$

where $\eta_1(t)=\left[\begin{smallmatrix}(d(t)-h_1)(x(t-h_2)-x(t-d(t)))\\(h_2-d(t))(x(t-d(t))-x(t-h_1))\end{smallmatrix}\right]$ and $\eta_2(t)=[\,x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-h_1)\ \ x^{\mathrm{T}}(t-d(t))\ \ x^{\mathrm{T}}(t-h_2)\,]^{\mathrm{T}}$.

Case 2. When $t\in[t_{2k},t_{2k+1})$, i.e., $\dot{d}(t)\in[-\mu,0]$, a looped functional is defined as follows:

$$V_D(t)=2\eta_1^{\mathrm{T}}(t)P_1\eta_2(t)$$

* Corresponding author (email: 9804zhb@163.com)

$$+ (d(t) - h_1) \int_{t-h_2}^{t-d(t)} \dot{x}^{\mathrm{T}}(s) R_1 \dot{x}(s) \mathrm{d}s$$
$$+ (d(t) - h_2) \int_{t-d(t)}^{t-h_1} \dot{x}^{\mathrm{T}}(s) R_2 \dot{x}(s) \mathrm{d}s. \qquad (4)$$

Remark 1. Considering that $d(t_{2k-1}) = h_1$ and $d(t_{2k}) = h_2$, time $t \in [t_{2k-1}, t_{2k}]$ is mapped to delay $d(t) \in [h_1, h_2]$. Inspired by [6], the two-sided looped functional (3) is constructed for $t \in [t_{2k-1}, t_{2k}]$ by exploiting the relationship between intervals $[h_1, d(t)]$ and $[d(t), h_2]$. Similarly, another functional (4) is constructed for $t \in [t_{2k}, t_{2k+1}]$.

Remark 2. Differing from traditional Lyapunov functionals, $V_I(t)$ and $V_D(t)$ do not need to be positive. This relaxation plays an important role in reducing the conservativeness of the derived stability conditions.

As a result, the following stability criterion is obtained by introducing the proposed looped functionals.

Theorem 1. For given scalars $\mu \in [0, 1)$ and $h_2 > h_1 \geqslant 0$, system (1) is asymptotically stable if there exist $P \in \mathbb{S}_+^{10n}$, $W_1, W_2, W_3 \in \mathbb{S}_+^{2n}$, $U_1, U_2 \in \mathbb{S}_+^{n}$, $P_1, Q_1 \in \mathbb{R}^{2n \times 4n}$, $R_1, R_2, Z_1, Z_2 \in \mathbb{S}^n$, and $E_1, E_2, F_1, F_2 \in \mathbb{R}^{3n \times 13n}$ such that Eqs. (5)–(8) are feasible.

$$\begin{bmatrix} \Phi(h_1, \dot{d}(t)) & \sqrt{h_2 - h_1} E_1^{\mathrm{T}} \\ * & -\hat{U}_{Z1}(\dot{d}(t)) \end{bmatrix}_{\dot{d}(t) \in [0, \mu]} \leqslant 0, \qquad (5)$$

$$\begin{bmatrix} \Phi(h_2, \dot{d}(t)) & \sqrt{h_2 - h_1} E_2^{\mathrm{T}} \\ * & -\hat{U}_{Z2}(\dot{d}(t)) \end{bmatrix}_{\dot{d}(t) \in [0, \mu]} \leqslant 0, \qquad (6)$$

$$\begin{bmatrix} \Psi(h_1, \dot{d}(t)) & \sqrt{h_2 - h_1} F_1^{\mathrm{T}} \\ * & -\hat{U}_{R1}(\dot{d}(t)) \end{bmatrix}_{\dot{d}(t) \in [-\mu, 0]} \leqslant 0, \qquad (7)$$

$$\begin{bmatrix} \Psi(h_2, \dot{d}(t)) & \sqrt{h_2 - h_1} F_2^{\mathrm{T}} \\ * & -\hat{U}_{R2}(\dot{d}(t)) \end{bmatrix}_{\dot{d}(t) \in [-\mu, 0]} \leqslant 0. \qquad (8)$$

Detailed proofs of Theorem 1 and two additional corollaries are given in Appendixes B and C.

Numerical example. Consider system (1) with

$$A_0 = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix}.$$

The delay function is given by $d(t) = h_1 + \frac{h_2 - h_1}{2}(1 + \cos \frac{2\mu}{h_2 - h_1} t)$. For the different h_1 and μ, the maximum allowable upper bounds h_2 computed by Theorem 1 for the example are presented in Table 1. It can be seen that our results are much larger than those presented in [8,9]. Note that additional numerical examples are discussed in Appendix D.

Table 1 Allowable upper bounds of h_2 for various h_1 and μ

	Method	$h_1 = 1$	$h_1 = 2$	$h_1 = 3$	$h_1 = 4$
	[8]	4.193	4.493	4.397	4.197
$\mu = 0.1$	[9]	4.404	4.572	4.540	4.236
	Theorem 1	5.125	5.100	5.178	5.414
	[8]	2.305	2.566	3.340	4.169
$\mu = 0.5$	[9]	2.351	2.698	3.418	4.209
	Theorem 1	3.622	3.703	4.181	4.825

Conclusion. This study has investigated the stability problem for a class of systems with periodic delay. A looped-functional-based Lyapunov functional has been proposed, and the proposed functional takes the periodicity characteristics of the delay into consideration, which yields new stability conditions. A numerical example has demonstrated that the proposed method can reduce the conservativeness of the computed results significantly compared to existing methods.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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