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H_∞ control of networked periodic piecewise systems under asynchronous switching with input delay

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Dear editor,

Periodic piecewise systems broadly exist in engineering applications, and different control problems of various periodic piecewise systems have been studied [1–3]. With the development of the network technology, the network-induced delay gives rise to instability and performance degradation of systems, and many effective solutions have been proposed to solve it [4]. However, for networked systems with the switched property, the network-induced delay would simultaneously result in the state transmission delay and the controller switching signal transmission delay, which brings the problem of asynchronous control. The weighted L_2-gain index is developed for the asynchronous switched system with network-induced delay in [5]. An asynchronous switching protocol and joint-design method of the switching signal and controller were proposed in [6,7], respectively. However, few studies have focused on networked control of periodic piecewise systems under asynchronous switching, which are encountered in power system applications. Because of the fixed switching law, the joint design method of the switching law and controller adopted in an arbitrary switched system is not applicable to the networked periodic piecewise systems. Motivated by the above observations, this article studies the control problem of networked periodic piecewise systems under asynchronous switching with input delay, which can be combined with distributed systems [8].

Problem formulation. Consider the following continuous-time periodic piecewise linear system, for $t \in [jT_p + t_{i-1}, jT_p + t_i)$, $j = 0, 1, 2, \ldots$, $i \in S \triangleq \{1, 2, \ldots, s\}$:

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) + B_{wi} w(t), \\ z(t) &= C_i x(t). \end{aligned} \tag{1}$$

T_p is the fundamental period of the system, t_i is the switch instant from the ith subsystem to $(i+1)$th subsystem in the first period, and $t_0 = 0$. s is the number of subsystems in one period. T_i is the dwell time of the ith subsystem, and then one has $T_i = t_i - t_{i-1}$. Assume that the network induced total time delay is defined as d and has $0 < d < \min(T_i)$. One could find that the system switching and the controller switching would be asynchronous because of the transmission delay. Without loss of generality, one can use two classes of time-delay systems to describe the networked periodic piecewise systems:

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i K_{i-1} x(t-d) + B_{wi} w(t), \\ &\quad t \in [jT_p + t_{i-1}, jT_p + t_{i-1} + d), \\ \dot{x}(t) &= A_i x(t) + B_i K_i x(t-d) + B_{wi} w(t), \\ &\quad t \in [jT_p + t_{i-1} + d, jT_p + t_i), \\ x(t) &= \varphi(t),\ t \in [-d, 0), \end{aligned} \tag{2}$$

where $\varphi(t)$ is the system initial condition with a delay d. It can be seen that the networked periodic piecewise system (2) is controlled by two classes of controllers for any subsystems $t \in [jT_p + t_{i-1}, jT_p + t_i)$. One class is switching asynchronously with the networked periodic piecewise linear system, and the other class is switching synchronously with the networked periodic piecewise system.

The stability analysis of the networked periodic piecewise system is given in Appendix A. Based on it, the performance analysis of the networked periodic piecewise system is given in the following theorem.

Theorem 1. Consider a networked periodic piecewise system (2), given scalars $\lambda > 0$, $\rho > 0$. If there exist the scalar $\gamma > 0$, and the matrices $P_{i,1} > 0$, $P_{i,2} > 0$, $P_{s+1,1} = P_{1,1}$, $i \in S$, $Q > 0$, $G > 0$, such that

$$\begin{bmatrix} \varepsilon_{i,1} & \chi_{i,1} & P_{i,1}B_{wi} & dA_i^{\mathrm{T}}Q & C_i^{\mathrm{T}} \\ * & \beta_{i1} & 0 & d(B_iK_{i-1})^{\mathrm{T}}Q & 0 \\ * & * & -\gamma^2 I & dB_{wi}^{\mathrm{T}}Q & 0 \\ * & * & * & -dQ & 0, \\ * & * & * & * & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} \varepsilon_{i,2} & \chi_{i,2} & P_{i,2}B_{wi} & dA_i^{\mathrm{T}}Q & C_i^{\mathrm{T}} \\ * & \beta_{i1} & 0 & d(B_iK_{i-1})^{\mathrm{T}}Q & 0 \\ * & * & -\gamma^2 I & dB_{wi}^{\mathrm{T}}Q & 0 \\ * & * & * & -dQ & 0 \\ * & * & * & * & -I \end{bmatrix} < 0,$$

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$$\begin{bmatrix} \varepsilon_{i,3} & \chi_{i,3} & P_{i,2}B_{wi} & dA_i^{\mathrm{T}}Q & C_i^{\mathrm{T}} \\ * & \beta_{i2} & 0 & d(B_iK_i)^{\mathrm{T}}Q & 0 \\ * & * & -\gamma^2 I & dB_{wi}^{\mathrm{T}}Q & 0 \\ * & * & * & -dQ & 0 \\ * & * & * & * & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} \varepsilon_{i,4} & \chi_{i,4} & P_{i+1,1}B_{wi} & dA_i^{\mathrm{T}}Q & C_i^{\mathrm{T}} \\ * & \beta_{i2} & 0 & d(B_iK_i)^{\mathrm{T}}Q & 0 \\ * & * & -\gamma^2 I & dB_{wi}^{\mathrm{T}}Q & 0 \\ * & * & * & -dQ & 0 \\ * & * & * & * & -I \end{bmatrix} < 0,$$

where $\varepsilon_{i,1} = \Theta_{i,1}(P_{i,1}) + \frac{(P_{i,2}-P_{i,1})}{d} + G - \frac{\mathrm{e}^{-\rho d}}{d}Q$, $\varepsilon_{i,2} = \Theta_{i,1}(P_{i,2}) + \frac{(P_{i,2}-P_{i,1})}{d} + G - \frac{\mathrm{e}^{-\rho d}}{d}Q$, $\varepsilon_{i,3} = \Theta_{i,2}(P_{i,2}) + \frac{(P_{i+1,1}-P_{i,2})}{T_i-d} + G - \frac{\mathrm{e}^{-\lambda d}}{d}Q$, $\varepsilon_{i,4} = \Theta_{i,2}(P_{i+1,1}) + \frac{(P_{i+1,1}-P_{i,2})}{T_i-d} + G - \frac{\mathrm{e}^{-\lambda d}}{d}Q$, $\Theta_{i,1}(\Omega) = A_i^{\mathrm{T}}\Omega + \Omega A_i + \rho\Omega$, $\Theta_{i,2}(\Omega) = A_i^{\mathrm{T}}\Omega + \Omega A_i + \lambda\Omega$, $\chi_{i,1} = P_{i,1}B_iK_{i-1} + \frac{\mathrm{e}^{-\rho d}}{d}Q$, $\chi_{i,2} = P_{i,2}B_iK_{i-1} + \frac{\mathrm{e}^{-\rho d}}{d}Q$, $\chi_{i,3} = P_{i,2}B_iK_i + \frac{\mathrm{e}^{-\lambda d}}{d}Q$, $\chi_{i,4} = P_{i+1,1}B_iK_i + \frac{\mathrm{e}^{-\lambda d}}{d}Q$, $\beta_{i1} = -\mathrm{e}^{-\rho d}G - \frac{\mathrm{e}^{-\rho d}}{d}Q$, $\beta_{i2} = -\mathrm{e}^{-\lambda d}G - \frac{\mathrm{e}^{-\lambda d}}{d}Q$, then, for any delay $0 < d < \min(T_i)$, the system (2) follows that $\int_0^\infty z^{\mathrm{T}}(\tau)z(\tau)\mathrm{d}\tau \leqslant \bar{\gamma}^2\int_0^\infty w^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau$, where $\bar{\gamma} = \sqrt{\frac{\max(\lambda,\rho)}{2\alpha^*}}\gamma\mathrm{e}^{\max(2\alpha^*-\min((\lambda,\rho),0))T_p}$.

Proof. See Appendix B for the proof of Theorem 1.

Remark 1. In this study, $P_{i1}(t)$ and $P_{i2}(t)$ are chosen to be continuous time-varying functions. Comparing with the multiple constant Lyapunov matrices in [5], the continuous time-varying Lyapunov matrix obviously has more plentiful dynamic characteristics.

Remark 2. In this study, with adopting a continuous time-varying Lyapunov functional, an unweighted H_∞ performance criterion is obtained, which would be more desirable in applications.

The condition in Theorem 1 is non-convex, which cannot be solved directly. An algorithm is given in Appendix C to obtain the controller gain. And the simulations are provided in Appendix D to illustrate the efficiency of the obtained results.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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