1 Appendix A

1.1 Stability analysis

Theorem A1. Consider a networked periodic piecewise system with $w(t) = 0$, given scalars $\lambda > 0$, $\rho > 0$, system is α^*-exponentially stable for any delay satifying $0 < d < \min(T_i)$, if there exist matrices $P_{i,1} > 0$, $P_{i,2} > 0$, $P_{s+1,1} = P_{1,1}$, $i \in \mathbf{S}$, and $Q > 0$, $G > 0$, such that

$$\begin{bmatrix} \varepsilon_{i,1} & P_{i,1}B_iK_{i-1} + \frac{e^{-\rho d}}{d}Q & dA_i^TQ \\ * & -e^{-\rho d}G - \frac{e^{-\rho d}}{d}Q & d(B_iK_{i-1})^TQ \\ * & * & -dQ \end{bmatrix} < 0, \tag{1}$$

$$\begin{bmatrix} \varepsilon_{i,2} & P_{i,2}B_iK_{i-1} + \frac{e^{-\rho d}}{d}Q & dA_i^TQ \\ * & -e^{-\rho d}G - \frac{e^{-\rho d}}{d}Q & d(B_iK_{i-1})^TQ \\ * & * & -dQ \end{bmatrix} < 0, \tag{2}$$

$$\begin{bmatrix} \varepsilon_{i,3} & P_{i,2}B_iK_i + \frac{e^{-\lambda d}}{d}Q & dA_i^TQ \\ * & -e^{-\lambda d}G - \frac{e^{-\lambda d}}{d}Q & d(B_iK_i)^TQ \\ * & * & -dQ \end{bmatrix} < 0, \tag{3}$$

$$\begin{bmatrix} \varepsilon_{i,4} & P_{i+1,1}B_iK_i + \frac{e^{-\lambda d}}{d}Q & dA_i^TQ \\ * & -e^{-\lambda d}G - \frac{e^{-\lambda d}}{d}Q & d(B_iK_i)^TQ \\ * & * & -dQ \end{bmatrix} < 0 \tag{4}$$

where

$$\begin{aligned} \varepsilon_{i,1} &= \Theta_{i,1}(P_{i,1}) + \frac{(P_{i,2} - P_{i,1})}{d} + G - \frac{e^{-\rho d}}{d}Q, \\ \varepsilon_{i,2} &= \Theta_{i,1}(P_{i,2}) + \frac{(P_{i,2} - P_{i,1})}{d} + G - \frac{e^{-\rho d}}{d}Q, \\ \varepsilon_{i,3} &= \Theta_{i,2}(P_{i,2}) + \frac{(P_{i+1,1} - P_{i,2})}{T_i - d} + G - \frac{e^{-\lambda d}}{d}Q, \\ \varepsilon_{i,4} &= \Theta_{i,2}(P_{i+1,1}) + \frac{(P_{i+1,1} - P_{i,2})}{T_i - d} + G - \frac{e^{-\lambda d}}{d}Q, \\ \Theta_{i,1}(\Omega) &= A_i^T\Omega + \Omega A_i + \rho\Omega, \\ \Theta_{i.2}(\Omega) &= A_i^T\Omega + \Omega A_i + \lambda\Omega, \end{aligned} \tag{5}$$

and the scalars ρ, λ, satisfying

$$2\alpha^*T_p - (\lambda(T_p - sd) + \rho sd) < 0. \tag{6}$$

Proof. For any $t \in [jT_p + t_{i-1}, jT_p + t_i), i \in \mathbf{S}$, we divide the time interval into two parts, one is the time interval that the controller asynchronously switches with the subsystem switching, and the other is the time interval that the controller synchronously switches with the subsystem switching.
Case1: For $t \in [jT_p + t_{i-1} + d, jT_p + t_i), i \in \mathbf{S}$, the controller synchronously switches with the subsystem switching, that is,

$$\dot{x}(t) = A_ix(t) + B_iK_ix(t-d), \quad t \in [jT_p + t_{i-1} + d, jT_p + t_i).$$

Define a periodic Lyapunov-Krasovskii functional

$$V(x,t) = V_{i,1}(x,t) = V_{1i}(x,t) + V_{2i}(x,t) + V_{3i}(x,t) \tag{7}$$

where

$$V_{1i}(x,t) = x^T(t)P_{i2}(t)x(t),$$

$$\begin{aligned} V_{2i}(x,t) &= \int_{-d}^{0}\int_{t+\theta}^{t} e^{-\lambda(t-\xi)}\dot{x}^T(\xi)Q\dot{x}(\xi)d\xi d\theta, \\ V_{3i}(x,t) &= \int_{t-d}^{t} e^{-\lambda(t-\xi)}x^T(\xi)Gx(\xi)d\xi, \end{aligned}$$

with $P_{i2}(t)$ is periodic time-varying given as

$$P_{i2}(t) = P_{i,2} + \frac{t-jT_p-t_{i-1}-d}{T_i-d}(P_{i+1,1}-P_{i,2}), \quad t\in[jT_p+t_{i-1}+d, jT_p+t_i), \tag{8}$$

and satisfying

$$\min(\min_{i\in\mathbf{S}}\underline{\lambda}(P_{i,2}), \min_{i\in\mathbf{S}}\underline{\lambda}(P_{i+1,1}))I \leqslant P_{i2}(t) \leqslant \max(\max_{i\in\mathbf{S}}\bar{\lambda}(P_{i,2}), \max_{i\in\mathbf{S}}\bar{\lambda}(P_{i+1,1}))I.$$

Since $P_{i+1,1} > 0$, $P_{i,2} > 0$, one has $P_{i2}(t) > 0$. Combining with $Q > 0$, $G > 0$, for any d satisfying $0 < d < T_i$, then one obtains

$$\begin{gathered} V_{1i}(x,t) \geqslant \omega_1\|x(t)\|^2, \\ \omega_1 = \min(\min_{i\in\mathbf{S}}\underline{\lambda}(P_{i,2}), \min_{i\in\mathbf{S}}\underline{\lambda}(P_{i+1,1})) > 0. \end{gathered} \tag{9}$$

On the other hand, one has

$$\begin{aligned} V_{2i}(x,t) &\leqslant \int_{-d}^{0} e^{-\lambda(t-t)}\int_{t+\theta}^{t}\dot{x}^T(\xi)Q\dot{x}^T(\xi)d\xi d\theta \\ &\leqslant d\bar{\lambda}(Q)\int_{t-d}^{t}(\| A_i \|^2\| x(\xi) \|^2 + \| B_iK_i \|^2\| x(\xi-d) \|^2)d\xi \\ &\leqslant 2d^2\bar{\lambda}(Q)(\| A_i \|^2 + \| B_iK_i \|^2)\sup_{t-d\leqslant s\leqslant t}\| x(s) \|^2, \end{aligned} \tag{10}$$

$$\begin{aligned} V_{3i}(x,t) &\leqslant e^{-\lambda(t-t)}\int_{t-d}^{t}x^T(\xi)Gx(\xi)d\xi \\ &\leqslant d\bar{\lambda}(G)\sup_{t-d\leqslant s\leqslant t}\| x(s) \|^2 . \end{aligned} \tag{11}$$

Then, from (9)-(11), the Lyapunov-Krasovskii functional holds that

$$V_{i,1}(x,t) \leqslant \ \omega_2\sup_{t-d\leqslant s\leqslant t}\| x(s) \|^2 \tag{12}$$

where

$$\omega_2 = \max(\bar{\lambda}(P_{i,2})), \bar{\lambda}(P_{i+1,1}) + d\bar{\lambda}(G) + 2d^2\bar{\lambda}(Q)(\| A_i \|^2 + \| B_iK_i \|^2).$$

Then, one has

$$\omega_1\|x(t)\|^2 \leqslant V_{i,1}(x,t) \leqslant \omega_2\sup_{t-d\leqslant s\leqslant t}\| x(s) \|^2 . \tag{13}$$

On the other hand, one has

$$\begin{aligned} \dot{V}_{1i}(x,t)+\dot{V}_{3i}(x,t) &= \dot{x}^T(t)P_{i2}(t)x(t) + x^T(t)P_{i2}(t)\dot{x}(t) + x^T(t)\dot{P}_{i2}(t)x(t) + x^T(t)Gx(t) \\ &\quad -x(t-d)^Te^{-\lambda d}Gx(t-d) - \lambda V_{3i}(x,t), \\ \dot{V}_{2i}(x,t) &= \int_{-d}^{0}[\dot{x}^T(t)Q\dot{x}(t) - e^{-\lambda d}\dot{x}^T(t+\theta)Q\dot{x}(t+\theta) - \lambda\int_{t+\theta}^{t}e^{-\lambda(t-\xi)}\dot{x}^T(\xi)Q\dot{x}(\xi)]d\theta \\ &= -\lambda\int_{-d}^{0}\int_{t+\theta}^{t}e^{-\lambda(t-\xi)}\dot{x}^T(\xi)Q\dot{x}(\xi)d\xi d\theta + d\varsigma^T(t)\begin{bmatrix} A_i^T \\ (B_iK_i)^T \end{bmatrix}Q\begin{bmatrix} A_i & B_iK_i \end{bmatrix}\varsigma(t) \end{aligned}$$

$$-\int_{t-d}^{t}\dot{x}^T(\xi)e^{-\lambda(t-\xi)}Q\dot{x}(\xi)d\xi \tag{14}$$

where $\varsigma(t)=\begin{bmatrix}x^T(t) & x^T(t-d)\end{bmatrix}$, with Newton-Leibniz formula, one has

$$\begin{aligned}\dot{V}_{2i}(x,t) &\leqslant d\dot{x}^T(t)Q\dot{x}(t)-\frac{e^{-\lambda d}}{d}[\int_{t-d}^{t}\dot{x}^T(\upsilon)d\nu]Q[\int_{t-d}^{t}\dot{x}(\upsilon)d\nu]-\lambda V_{2i}(x,t)\\ &\leqslant d[A_ix(t)+B_iK_ix(t-d)]^TQ[A_ix(t)+B_iK_ix(t-d)]\\ &\quad -\frac{e^{-\lambda d}}{d}[x(t)-x(t-d)]^TQ[x(t)-x(t-d)]-\lambda V_{2i}(x,t).\end{aligned} \tag{15}$$

Then, the following inequality holds

$$\begin{aligned}\dot{V}_{i,1}(x,t)+\lambda V_{i,1}(x,t) \leqslant\ & x^T(t)[A_i^TP_{i2}(t)+P_{i2}(t)A_i+G+dA_i^TQA_i\\ &+\dot{P}_{i2}(t)-\frac{e^{-\lambda d}}{d}Q+\lambda P_{i2}(t)]x(t)\\ &+x^T(t)[P_{i2}(t)B_iK_i+dA_i^TQB_iK_i+\frac{e^{-\lambda d}}{d}Q]x(t-d)\\ &+x^T(t-d)[d(B_iK_i)^TQA_i+\frac{e^{-\lambda d}}{d}Q]x(t)\\ &+x^T(t-d)[(B_iK_i)^TP_{i2}(t)+d(B_iK_i)^TQB_iK_i-\frac{e^{-\lambda d}}{d}Q]x(t-d)\\ &-x^T(t-d)e^{-\lambda d}Gx(t-d)\end{aligned} \tag{16}$$

which can be rewritten as

$$\dot{V}_{i,1}(x,t)+\lambda V_{i,1}(x,t)\leqslant\varsigma^T(t)\psi(t,d)\varsigma(t) \tag{17}$$

where

$$\begin{aligned}\psi(t,d) =\ & \begin{bmatrix}A_i^TP_{i2}(t)+P_{i2}(t)A_i+\lambda P_{i2}(t)+\frac{(P_{i+1,1}-P_{i,2})}{T_i-d}+G & P_{i2}(t)B_iK_i\\ (B_iK_i)^TP_{i2}(t) & -e^{-\lambda d}G\end{bmatrix}-\frac{e^{-\lambda d}}{d}\begin{bmatrix}Q & -Q\\ -Q & Q\end{bmatrix}\\ &+d\begin{bmatrix}A_i^T\\ (B_iK_i)^T\end{bmatrix}Q\begin{bmatrix}A_i^T\\ (B_iK_i)^T\end{bmatrix}^T.\end{aligned}$$

With condition (3)-(4) and (17), one knows that $\psi(t,d)<0$, which implies $V_{i,1}(x,t)+\lambda V_{i,1}(x,t)<0$. Then, one has

$$V(x,t_i)<e^{-\lambda(T_i-d)}V(x,t_{i-1+d}). \tag{18}$$

Case 2: For $t\in[jT_p+t_{i-1},jT_p+t_{i-1}+d)$, the controller asynchronously switches with the switching of the subsystem, that is,

$$\dot{x}(t) = A_ix(t)+B_iK_{i-1}x(t-d),\quad t\in[jT_p+t_{i-1},jT_p+t_{i-1}+d).$$

Define another periodic piecewise Lyapunov-Krasovskii functional

$$V(x,t)=V_{i,2}(x,t)=V_{4i}(x,t)+V_{5i}(x,t)+V_{6i}(x,t) \tag{19}$$

where

$$V_{4i}(x,t) = x^T(t)P_{i1}(t)x(t),$$

$$V_{5i}(x,t) = \int_{-d}^{0}\int_{t+\theta}^{t} e^{-\rho(t-\xi)}\dot{x}^T(\xi)Q\dot{x}(\xi)d\xi d\theta,$$
$$V_{6i}(x,t) = \int_{t-d}^{t} e^{-\rho(t-\xi)}x^T(\xi)Gx(\xi)d\xi,$$

with $P_{i1}(t)$ is periodic time-varying given as

$$P_{i1}(t) = P_{i,1} + \frac{t-jT_p-t_{i-1}}{d}(P_{i,2}-P_{i,1}), \quad t \in [jT_p+t_{i-1}, jT_p+t_{i-1}+d). \tag{20}$$

Since $P_{i,1} > 0$, $P_{i,2} > 0$, one has $P_{i1}(t) > 0$. Combining with $Q > 0$, $G > 0$, for any d satisfying $0 < d < T_i$, following the similar arguments in Case 1, then, one has

$$\omega_3\|x(t)\|^2 \leqslant V_{i,2}(x,t) \leqslant \omega_4 \sup_{t-d\leqslant s\leqslant t} \| x(s) \|^2 \tag{21}$$

where

$$\omega_3 = \min(\min_{i\in\mathbf{S}} \underline{\lambda}(P_{i,1}), \min_{i\in\mathbf{S}} \underline{\lambda}(P_{i,2})) > 0,$$
$$\omega_4 = \max(\bar{\lambda}(P_{i,1})), \bar{\lambda}(P_{i,2}) + d\bar{\lambda}(G) + 2d^2\bar{\lambda}(Q)(\| A_i \|^2 + \| B_iK_{i-1} \|^2) > 0.$$

With the condition in Theorem 1, there holds $V(x,t_{i-1+d}) < e^{-\rho d}V(x,t_{i-1})$. Then for any non-negative number j, one has

$$\begin{aligned} V(x_{jT_p^-}) &\leqslant e^{-\lambda(T_i-d)}V(x_{(j-1)T_p+t_{s-1}+d}) \\ &\leqslant e^{-\lambda(T_i-d)}e^{-\rho d}V(x_{(j-1)T_p+t_{s-1}-}) \\ &\leqslant e^{\sum_{i=1}^{s}-\lambda(T_i-d)}e^{-\rho s d}V(x_{(j-1)T_p^-}) \\ &\leqslant e^{-\lambda j(T_p-sd)-\rho j s d}V(x_0) \\ &\leqslant e^{-2j\alpha^* T_p}V(x_0). \end{aligned} \tag{22}$$

From (13) and (21), one obtains that $\| x(t) \|\leqslant \kappa e^{-\alpha^* t} \| x_0 \|_{sup}$, where $\kappa = \min(\sqrt{\frac{\omega_2}{\omega_1}}, \sqrt{\frac{\omega_4}{\omega_3}})$. Therefore, the networked periodic piecewise system is exponentially stable with the decay rate α^*, the proof is completed.

2 Appendix B

Proof. For $t \in [jT_p+t_{i-1}+d, jT_p+t_i)$, construct the Lyapunov function $V(x,t)$ as in Appendix A, and following the similar arguments of the proof of Theorem 1, one has

$$\dot{V}_{i,1}(x,t) + \lambda V_{i,1}(x,t) + \Gamma(t) \leqslant \Phi^T(t)\Upsilon(t,d)\Phi(t) \tag{23}$$

where $\Phi(t) = \begin{bmatrix} x^T(t) & x^T(t-d) & w^T(t)\end{bmatrix}^T$, $\Gamma(t) = z^T(t)z(t) - \gamma^2 w^T(t)w(t)$,

$$\Upsilon(t,d) = \begin{bmatrix} A_i^T P_{i2}(t) + P_{i2}(t)A_i + \lambda P_{i2}(t) + \frac{(P_{i+1,1}-P_{i,2})}{T_i-d} + G & P_{i2}(t)B_iK_i & P_{i2}(t)B_{wi} \\ (B_iK_i)^T P_{i2}(t) & -e^{-\lambda d}G & 0 \\ B_{wi}^T P_{i2}(t) & 0 & -\gamma^2 I \end{bmatrix} - \frac{e^{-\lambda d}}{d}\begin{bmatrix} Q & -Q & 0 \\ -Q & Q & 0 \\ 0 & 0 & 0\end{bmatrix}$$
$$+ d\begin{bmatrix} A_i^T \\ (B_iK_i)^T \\ B_{wi}^T \end{bmatrix} Q \begin{bmatrix} A_i^T \\ (B_iK_i)^T \\ B_{wi}^T \end{bmatrix}^T + \begin{bmatrix} C_i^T \\ 0 \\ 0 \end{bmatrix} I \begin{bmatrix} C_i^T \\ 0 \\ 0 \end{bmatrix}^T.$$

With the convex property, one has $\Upsilon < 0$, then, one obtains

$$\dot{V}_{i,1}(x,t) \leqslant -\lambda V_{i,1}(x,t) - \Gamma(t). \tag{24}$$

For $t \in [jT_p + t_{i-1}, jT_p + t_{i-1} + d)$, similarly, one obtains that

$$\dot{V}_{i,2}(x,t) \leqslant -\rho V_{i,2}(x,t) - \Gamma(t). \tag{25}$$

Assume that $t \in [jT_p + t_{n-1} + d, jT_p + t_n)$, j=1, 2.., n=1, 2..s, then one holds that

$$\begin{aligned}
V(x,t) &\leqslant e^{-\lambda(t-(jT_p+t_{n-1}+d))}V(x, jT_p + t_{n-1} + d) - \int_{jT_p+t_{n-1}+d}^{t} e^{-\lambda(t-\tau)}\Gamma(\tau)d\tau \\
&\leqslant e^{-\lambda(t-(jT_p+t_{n-1}+d))}[e^{-\rho d}V(x, jT_p + t_{n-1}) - \int_{jT_p+t_{n-1}}^{jT_p+t_{n-1}+d} e^{-\rho(jT_p+t_{n-1}+d-\tau)}\Gamma(\tau)d\tau] \\
&\quad - \int_{jT_p+t_{n-1}+d}^{t} e^{-\lambda(t-\tau)}\Gamma(\tau)d\tau \\
&\leqslant e^{-\lambda(t-(jT_p+t_{n-1}+d))}[e^{-\rho d}[e^{-\lambda(jT_p+t_{n-1}-jT_p-t_{n-2}-d)}V(x, jT_p + t_{n-2} + d) \\
&\quad - \int_{jT_p+t_{n-2}+d}^{jT_p+t_{n-1}} e^{-\lambda(jT_p+t_{n-1}-\tau)}\Gamma(\tau)d\tau] - \int_{jT_p+t_{n-1}}^{jT_p+t_{n-1}+d} e^{-\rho(jT_p+t_{n-1}+d-\tau)}\Gamma(\tau)d\tau] \\
&\quad - \int_{jT_p+t_{n-1}+d}^{t} e^{-\lambda(t-\tau)}\Gamma(\tau)d\tau \\
&\leqslant e^{-\lambda(t-(jT_p+nd)-\rho nd)}V(x, jT_p^-) \\
&\quad -\Sigma_{i=1}^{n-1}\int_{jT_p+t_{i-1}+d}^{jT_p+t_i} e^{-\lambda(t-(jT_p+t_i+(n-i)d)-\rho(n-i)d)-\lambda(jT_p+t_i-\tau)}\Gamma(\tau)d\tau \\
&\quad -\Sigma_{i=0}^{n-1}\int_{jT_p+t_i}^{jT_p+t_i+d} e^{-\lambda(t-(jT_p+t_i+(n-i)d)-\rho(n-1-i)d-\rho(jT_p+t_i+d-\tau))}\Gamma(\tau)d\tau \\
&\quad - \int_{jT_p+t_{n-1}+d}^{t} e^{-\lambda(t-\tau)}\Gamma(\tau)d\tau.
\end{aligned}$$

Then, under the zero initial condition, one obtains

$$\begin{aligned}
V(x,t) \leqslant &- \sum_{h=1}^{j}\sum_{l=1}^{s}\int_{(h-1)T_p+t_{l-1}}^{(h-1)T_p+t_{l-1}+d} exp(\Xi_1(h,l))\Gamma(\tau)d\tau \\
&- \sum_{h=1}^{j}\sum_{l=1}^{s}\int_{(h-1)T_p+t_{l-1}+d}^{(h-1)T_p+t_l} exp(\Xi_2(h,l))\Gamma(\tau)d\tau \\
&- \sum_{l=0}^{n-1}\int_{jT_p+t_l}^{jT_p+t_l+d} exp(\Xi_3(l))\Gamma(\tau)d\tau \\
&- \sum_{l=1}^{n-1}\int_{jT_p+t_{l-1}+d}^{jT_p+t_l} exp(\Xi_4(l))\Gamma(\tau)d\tau \\
&- \int_{lT_p+t_{n-1}+d}^{t} e^{-\lambda(t-\tau)}\Gamma(\tau)d\tau
\end{aligned} \tag{26}$$

where
$\Xi_1(h,l) = -\rho((h-1)T_p + t_{l-1} + d - \tau) - (j-h)[\lambda(T_p - sd) + \rho sd] - \Sigma_{i=l}^{s}\lambda(T_i - d) - \rho(s-l)d - \sum_{i=1}^{n-1}\lambda(T_i - d) - \rho nd - \lambda(t - (jT_p + t_{n-1} + d))$,
$\Xi_2(h,l) = -\lambda((h-1)T_p + t_l - \tau) - (j-h)[\lambda(T_p - sd) + \rho sd] - \Sigma_{i=l+1}^{s}\lambda(T_i - d) - \rho(s-l)d - \sum_{i=1}^{n-1}\lambda(T_i - d) - \rho nd - \lambda(t - (jT_p + t_{n-1} + d))$,
$\Xi_3(k) = -\rho(jT_p + t_l + d - \tau) - \Sigma_{i=l+1}^{n-1}\lambda(T_i - d) - \rho(n-l-1)d - \lambda(t - (jT_p + t_{n-1} + d))$,
$\Xi_4(k) = -\lambda(jT_p + t_l - \tau) - \Sigma_{i=l+1}^{n-1}\lambda(T_i - d) - \rho(n-l)d - \lambda(t - (jT_p + t_{n-1} + d))$

which can be rewritten as

$$
\begin{aligned}
&\sum_{h=1}^{j}\sum_{l=1}^{s}\int_{(h-1)T_p+t_{l-1}}^{(h-1)T_p+t_{l-1}+d} exp(\Xi_1(h,l))z^T(\tau)z(\tau)d\tau \\
&+\sum_{h=1}^{j}\sum_{l=1}^{s}\int_{(h-1)T_p+t_{l-1}+d}^{(h-1)T_p+t_l} exp(\Xi_2(h,l))z^T(\tau)z(\tau)d\tau \\
&+\sum_{l=0}^{n-1}\int_{jT_p+t_{l-1}}^{jT_p+t_{l-1}+d} exp(\Xi_3(l))z^T(\tau)z(\tau)d\tau \\
&+\sum_{l=1}^{n-1}\int_{jT_p+t_{l-1}+d}^{jT_p+t_l} exp(\Xi_4(l))z^T(\tau)z(\tau)d\tau \\
&+\int_{jT_p+t_{n-1}+d}^{t} e^{-\lambda(t-\tau)}z^T(\tau)z(\tau)d\tau+V(x,t) \\
\leqslant\ &\gamma^2[\sum_{h=1}^{j}\sum_{l=1}^{s}\int_{(h-1)T_p+t_{l-1}}^{(h-1)T_p+t_{l-1}+d} exp(\Xi_1(h,l))w^T(\tau)w(\tau)d\tau \\
&+\sum_{h=1}^{j}\sum_{l=1}^{s}\int_{(h-1)T_p+t_{l-1}+d}^{(h-1)T_p+t_l} exp(\Xi_2(h,l))w^T(\tau)w(\tau)d\tau \\
&+\sum_{l=0}^{n-1}\int_{jT_p+t_{l-1}}^{jT_p+t_{l-1}+d} exp(\Xi_3(l))w^T(\tau)w(\tau)d\tau \\
&+\sum_{l=1}^{n-1}\int_{jT_p+t_{l-1}+d}^{jT_p+t_l} exp(\Xi_4(l))w^T(\tau)w(\tau)d\tau \\
&+\int_{jT_p+t_{n-1}+d}^{t} e^{-\lambda(t-\tau)}w^T(\tau)w(\tau)d\tau].
\end{aligned} \tag{27}
$$

Moreover, one has

$$
\begin{aligned}
\Xi_1(h,l) &\leqslant -\min(\lambda,\rho)(hT_p-\tau)-(j-h)2\alpha^*T_p-\min(\lambda,\rho)(t-jT_p) \\
&= (-\min(\lambda,\rho)+2\alpha^*)(t-jT_p)-2\alpha^*(t-\tau)+(-\min(\lambda,\rho)+2\alpha^*)(hT_p-\tau) \\
&\leqslant -2\alpha^*(t-\tau)+\max(2\alpha^*-(\min(\lambda,\rho),0))T_p, \\
\Xi_2(h,l) &\leqslant -\min(\lambda,\rho)(hT_p-\tau)-(j-h)2\alpha^*T_p-\min(\lambda,\rho)(t-jT_p) \\
&= (-\min(\lambda,\rho)+2\alpha^*)(t-jT_p)-2\alpha^*(t-\tau)+(-\min(\lambda,\rho)+2\alpha^*)(hT_p-\tau) \\
&\leqslant -2\alpha^*(t-\tau)+\max(2\alpha^*-\min((\lambda,\rho),0))2T_p, \\
\Xi_3(l) &\leqslant -2\alpha^*(t-\tau)+2\alpha^*(t-\tau)-(\min(\lambda,\rho)(t-\tau) \\
&\leqslant -2\alpha^*(t-\tau)+(2\alpha^*-\min((\lambda,\rho),0))2T_p, \\
\Xi_4(l) &\leqslant -2\alpha^*(t-\tau)+2\alpha^*(t-\tau)-(\min(\lambda,\rho)(t-\tau) \\
&\leqslant -2\alpha^*(t-\tau)+\max(2\alpha^*-\min(\lambda,\rho),0)2T_p.
\end{aligned} \tag{28}
$$

On the other hand, one also obtains

$$
\begin{aligned}
\Xi_1(h,l) \geqslant &- \max(\rho,\lambda)((h-1)T_p+t_l-\tau)-\max(\rho,\lambda)(j-h)T_p-\max(\rho,\lambda)(T_p-t_l) \\
&- \lambda(t_{n-1}+d)-\lambda(t-(jT_p+t_{n-1}+d)) \\
= &- \max(\rho,\lambda)(t-\tau), \\
\Xi_2(h,l) \geqslant &- \max(\rho,\lambda)((h-1)T_p+t_l-\tau)-\max(\rho,\lambda)(j-h)T_p-\max(\rho,\lambda)(T_p-t_l) \\
&- \lambda(t_{n-1}+d)-\lambda(t-(jT_p+t_{n-1}+d))
\end{aligned}
$$

$$
\begin{aligned}
&= -\max(\rho,\lambda)(t-\tau),\\
\Xi_3(l) &\geqslant -\max(\rho,\lambda)(t-\tau),\\
\Xi_4(l) &\geqslant -\max(\rho,\lambda)(t-\tau).
\end{aligned}\tag{29}
$$

With $V(x,t)\geqslant 0$, combining (28)-(29), then, one has

$$\int_0^t e^{-\max(\rho,\lambda)(t-\tau)} z^T(\tau)z(\tau)d\tau \leqslant \gamma^2\int_0^t e^{-2\alpha^* t+\max(2\alpha^*-\min(\lambda,\rho),0)2T_p} w^T(\tau)w(\tau)d\tau.$$

Integrating t from $0\to\infty$, for any $0<d<T_i$ and all nonzero disturbance $w(t)\in L_2[0,\infty)$, one obtains

$$\int_0^\infty z^T(\tau)z(\tau)d\tau \leqslant \frac{\max(\lambda,\rho)}{2\alpha^*}e^{\max(2\alpha^*-\min((\lambda,\rho),0))2T_p}\gamma^2\int_0^\infty w^T(\tau)w(\tau)d\tau.$$

Then, the proof is completed.

3 Appendix C

H_∞ controller design algorithm

Step 1. Given a small tolerance $\varrho>0$ and the values of λ, ρ. Set r=1, $\gamma_2^{(0)}=0$ for Hurwitze-stable subsystems, solving $F_i>0$ and Y_i from

$$A_iF_i+B_iY_i+(A_iF_i+B_iY_i)^T<0,\tag{30}$$

then, one can know the initial controller gains according to $K_i=Y_iF_i^{-1}$, i=1,2,...s.

Step 2. For the r-th iteration, fix the $K_i^{(r)}$, $K_0^{(r)}=K_s^{(r)}$ and solve the OPTIMIZE1 for $P_{i,1}^{(r)}>0$, $P_{i,2}^{(r)}>0$, $Q^{(r)}>0$, $G^{(r)}>0$.
OPTIMIZE1: Minimise γ subject to the following conditions

$$\begin{bmatrix}\varpi_{i,1} & P_{i,1}^{(r)}B_iK_{i-1}^{(r)}+\frac{e^{-\rho d}}{d}Q^{(r)} & P_{i,1}^{(r)}B_{wi} & dA_i^TQ^{(r)} & C_i^T\\ * & -e^{-\rho d}G^{(r)}-\frac{e^{-\rho d}}{d}Q^{(r)} & 0 & d(B_iK_{i-1}^{(r)})^TQ^{(r)} & 0\\ * & * & -\gamma^2I & dB_{wi}^TQ^{(r)} & 0\\ * & * & * & -dQ^{(r)} & 0\\ * & * & * & * & -I\end{bmatrix}<0,\tag{31}$$

$$\begin{bmatrix}\varpi_{i,2} & P_{i,2}^{(r)}B_iK_{i-1}^{(r)}+\frac{e^{-\rho d}}{d}Q^{(r)} & P_{i,2}^{(r)}B_{wi} & dA_i^TQ^{(r)} & C_i^T\\ * & -e^{-\rho d}G^{(r)}-\frac{e^{-\rho d}}{d}Q^{(r)} & 0 & d(B_iK_{i-1}^{(r)})^TQ^{(r)} & 0\\ * & * & -\gamma^2I & dB_{wi}^TQ^{(r)} & 0\\ * & * & * & -dQ^{(r)} & 0\\ * & * & * & * & -I\end{bmatrix}<0,\tag{32}$$

$$\begin{bmatrix}\varpi_{i,3} & P_{i,2}^{(r)}B_iK_{i}^{(r)}+\frac{e^{-\lambda d}}{d}Q^{(r)} & P_{i,2}^{(r)}B_{wi} & dA_i^TQ^{(r)} & C_i^T\\ * & -e^{-\lambda d}G^{(r)}-\frac{e^{-\lambda d}}{d}Q^{(r)} & 0 & d(B_iK_{i}^{(r)})^TQ^{(r)} & 0\\ * & * & -\gamma^2I & dB_{wi}^TQ^{(r)} & 0\\ * & * & * & -dQ^{(r)} & 0\\ * & * & * & * & -I\end{bmatrix}<0,\tag{33}$$

$$\begin{bmatrix}\varpi_{i,4} & P_{i+1,1}^{(r)}B_iK_{i}^{(r)}+\frac{e^{-\lambda d}}{d}Q^{(r)} & P_{i+1,1}^{(r)}B_{wi} & dA_i^TQ^{(r)} & C_i^T\\ * & -e^{-\lambda d}G^{(r)}-\frac{e^{-\lambda d}}{d}Q^{(r)} & 0 & d(B_iK_{i}^{(r)})^TQ^{(r)} & 0\\ * & * & -\gamma^2I & dB_{wi}^TQ^{(r)} & 0\\ * & * & * & -dQ^{(r)} & 0\\ * & * & * & * & -I\end{bmatrix}<0\tag{34}$$

where

$$\begin{aligned}
\varpi_{i,1} &= \Theta_{i,1}(P_{i,1})^{(r)} + \frac{(P_{i+1,1}^{(r)} - P_{i,2}^{(r)})}{d} + G^{(r)} - \frac{e^{-\rho d}}{d} Q^{(r)}, \\
\varpi_{i,2} &= \Theta_{i,2}(P_{i,2})^{(r)} + \frac{(P_{i,2}^{(r)} - P_{i,1}^{(r)})}{d} + G^{(r)} - \frac{e^{-\rho d}}{d} Q^{(r)}, \\
\Theta_{i,1}^{r}(\Omega) &= A_i^T \Omega + \Omega A_i + \rho\Omega, \\
\Theta_{i,2}^{r}(\Omega) &= A_i^T \Omega + \Omega A_i + \rho\Omega, \\
\varpi_{i,3} &= \Theta_{i,3}(P_{i,2})^{(r)} + \frac{(P_{i+1,1}^{(r)} - P_{i,2}^{(r)})}{d} + G^{(r)} - \frac{e^{-\lambda d}}{d} Q^{(r)}, \\
\varpi_{i,4} &= \Theta_{i,4}(P_{i+1,1})^{(r)} + \frac{(P_{i+1,1}^{(r)} - P_{i,2}^{(r)})}{d} + G^{(r)} - \frac{e^{-\lambda d}}{d} Q^{(r)}, \\
\Theta_{i,3}^{r}(\Omega) &= A_i^T \Omega + \Omega A_i + \lambda\Omega, \\
\Theta_{i,4}^{r}(\Omega) &= A_i^T \Omega + \Omega A_i + \lambda\Omega.
\end{aligned}$$

And denote $\gamma_1^{(r)} = \gamma$ in OP1, and $P_{i,1}^{(r)} > 0$, $P_{i,2}^{(r)} > 0$, $Q^{(r)} > 0$, $G^{(r)} > 0$ is the solution to OP1.
Step 3. If $\mid \gamma_1^r - \gamma_2^{r-1} \mid < \varrho$, stop, go to Step 6, otherwise, go to Step 4.
Step 4. For fixed $P_{i,1}^{(r)} > 0$, $P_{i,2}^{(r)} > 0$, $Q^{(r)} > 0$, $G^{(r)} > 0$, solve the OPTIMIZE2 for $K_i^{(r)}$ and γ.
OPTIMIZE2: Minimise γ subject to conditions (31)-(34). Denote $\gamma_2^{(r)} = \gamma$, and $K_i^{(r)}$ is the solution to OPTIMIZE2.
Step 5. If $\mid \gamma_2^r - \gamma_1^r \mid < \varrho$, stop, go to Step 6, Otherwise, set $r = r + 1$ and $K_i^{(r)} = K_i^{(r-1)}$, go to Step 2.
Step 6. Output $P_{i,1}^{(r)}$, $P_{i,2}^{(r)}$, $Q^{(r)}$, $G^{(r)}$, γ and $K_i^{(r)}$, i=1,2,...s.

---

4 Appendix D

Numerical examples are used to verify the effectiveness of the proposed method in this section. In Example 1, an open-loop unstable periodic piecewise system is considered, the Hcontrollers gain under different initial condition are obtained. In Example 2, a system with non-stabilizable subsystem is considered

4.1 Example 1

Consider a networked periodic piecewise system with three subsystems given as
$A_1 = \begin{bmatrix} -0.93 & 0 \\ 0 & -5 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B_{w1} = \begin{bmatrix} 0.12 \\ 0.085 \end{bmatrix}$, $C_1 = \begin{bmatrix} 1 & 0.9 \end{bmatrix}$,
$A_2 = \begin{bmatrix} -3.18 & -0.05 \\ 0 & 1.9 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $B_{w2} = \begin{bmatrix} 0.15 \\ 0.1 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 & 0.86 \end{bmatrix}$,
$A_3 = \begin{bmatrix} 3 & 0 \\ 0 & -2.5 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$, $B_{w3} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$, $C_3 = \begin{bmatrix} 1 & 0.87 \end{bmatrix}$,
and period $T_p = 2s$, $T_1 = 0.4s$, $T_2 = 0.6s$, $T_3 = 1s$, $d = 0.1s$. Let λ=1, ρ = 1, and choose the disturbance $w(t)$ as

$$w(t) = \begin{cases} 0.5 + 2cos(t) & \text{t} < 1.5, \\ 0.2 & 1.5 \leqslant \text{t} < 3.5, \\ 0.002 & 3.5 \leqslant \text{t} < 4.5, \\ 0 & \text{t} \geqslant 4.5. \end{cases}$$

The state trajectory of the open-loop system under initial condition $x_0 = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$ is shown in Figure 1, it can be seen that the periodic piecewise system is unstable.

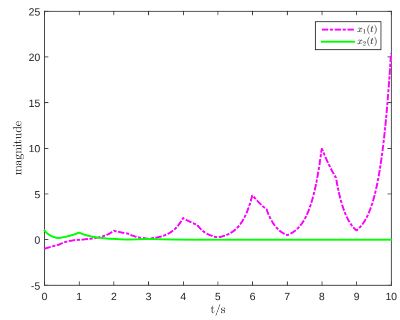

Figure 1 State trajectory of the open-loop system

In the following, we design the initial controller gains according to (30), then given $K_1^{(1)} = \begin{bmatrix} -10.2 & -5 \end{bmatrix}$, $K_2^{(1)} = \begin{bmatrix} -14 & -2 \end{bmatrix}$, $K_3^{(1)} = \begin{bmatrix} -11 & -3 \end{bmatrix}$, after 8 iterations, one obtains the following solution
$P_{1,1} = \begin{bmatrix} 13.7396 & 1.2235 \\ 1.2235 & 3.7791 \end{bmatrix}$, $P_{1,2} = \begin{bmatrix} 12.0190 & 1.7343 \\ 1.7343 & 5.9097 \end{bmatrix}$, $P_{2,1} = \begin{bmatrix} 7.0408 & -0.0282 \\ -0.0282 & 0.7123 \end{bmatrix}$, $P_{2,2} = \begin{bmatrix} 6.7345 & 0.0274 \\ 0.0274 & 0.7184 \end{bmatrix}$
$P_{3,1} = \begin{bmatrix} 11.1404 & 0.4962 \\ 0.4962 & 0.9059 \end{bmatrix}$, $P_{3,2} = \begin{bmatrix} 11.8399 & 0.4574 \\ 0.4574 & 0.9071 \end{bmatrix}$, $Q = \begin{bmatrix} 7.9848 & 0.6913 \\ 0.6913 & 1.0756 \end{bmatrix}$ $G = \begin{bmatrix} 18.0692 & 4.4150 \\ 4.4150 & 1.3339 \end{bmatrix}$,
and the optimal γ=0.3494, and the controller gains are given by
$K_1 = \begin{bmatrix} -4.7793 & -3.1680 \end{bmatrix}$, $K_2 = \begin{bmatrix} -3.2083 & 2.6686 \end{bmatrix}$, $K_3 = \begin{bmatrix} -2.4383 & -1.2935 \end{bmatrix}$.
The control input is shown in Figure 2, and the values of γ trajectory is shown in Figure 3, it can be seen that the γ gradually decreases.

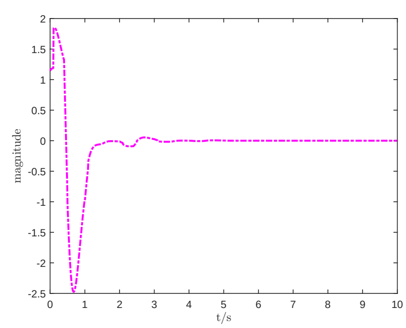

Figure 2 Control input

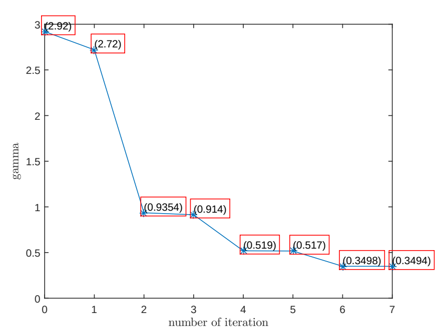

Figure 3 γ trajectory

The state of trajectories under the proposed controller is shown in figure 4, it can be seen that system is stabilized, and the system output with proposed controller is shown in Figure 5.

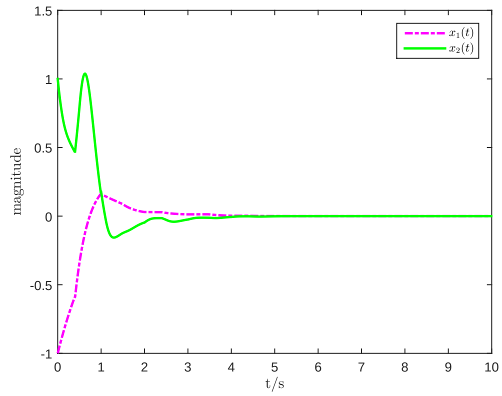

Figure 4 State trajectory of the close-loop system with H_∞ controller

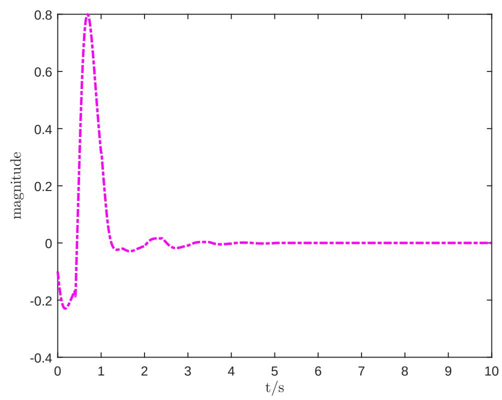

Figure 5 Output of the closed-loop system with H_∞ controller

In addition, controller based on time-invariant Lyapunov functional is used to compare with the controller based on continuous time-varying Lyapunov functional, one can find that the controller under time-invariant Lyapunov functional has no feasible solution, which shows the controller under the proposed method is less conservative.

Moreover, choosing another set of initial gain according to (30), given as $K_1^{(1)} = \begin{bmatrix} -10.2 & -5 \end{bmatrix}$, $K_2^{(1)} = \begin{bmatrix} -14 & -2 \end{bmatrix}$, $K_3^{(1)} = \begin{bmatrix} -10 & -3 \end{bmatrix}$, after 6 iterations, one obtains the following solution

$P_{1,1} = \begin{bmatrix} 35.7776 & 2.9277 \\ 2.9277 & 3.1614 \end{bmatrix}$, $P_{1,2} = \begin{bmatrix} 25.5136 & 0.9379 \\ 0.9379 & 2.9501 \end{bmatrix}$, $P_{2,1} = \begin{bmatrix} 18.1770 & 0.1510 \\ 0.1510 & 1.0565 \end{bmatrix}$, $P_{2,2} = \begin{bmatrix} 16.9984 & -0.2150 \\ -0.2150 & 0.7538 \end{bmatrix}$

$P_{3,1} = \begin{bmatrix} 28.7109 & 0.9889 \\ 0.9889 & 0.9095 \end{bmatrix}$, $P_{3,2} = \begin{bmatrix} 30.4536 & 0.9454 \\ 0.9454 & 0.9096 \end{bmatrix}$, $Q = \begin{bmatrix} 25.1678 & 0.7766 \\ 0.7766 & 1.2621 \end{bmatrix}$, $G = \begin{bmatrix} 38.4538 & 6.1318 \\ 6.1318 & 0.9780 \end{bmatrix}$,

and the optimal γ=0.6334, and the $\hat{\gamma} = 0.7081$. The controller gains are given by

$K_1 = \begin{bmatrix} 5.1268 & -2.6131 \end{bmatrix}$, $K_2 = \begin{bmatrix} -4.5571 & -2.3827 \end{bmatrix}$, $K_3 = \begin{bmatrix} -4.1190 & -1.2529 \end{bmatrix}$.

The values of γ trajectory is shown in Figure 6, and the system output of the closed-loop system with H_∞ controller is shown in Figure 7.

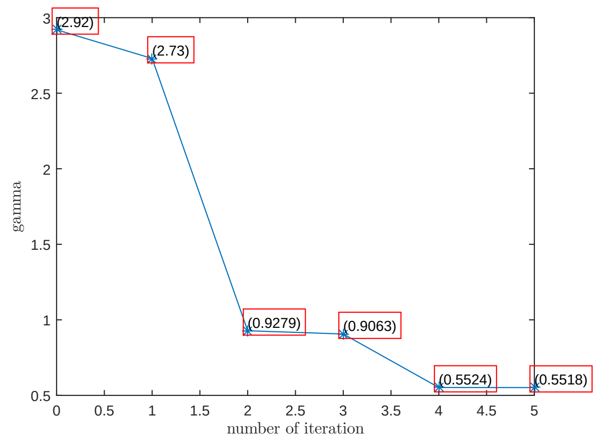

Figure 6 γ trajectory

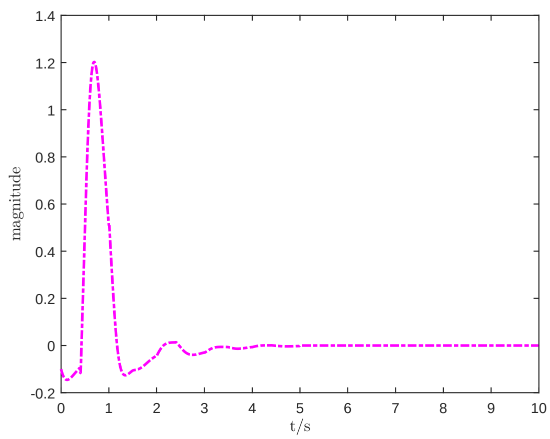

Figure 7 Output of the closed-loop system with H_∞ controller

Notice that choosing different initial gains, the resulting control input and system output would be different. Table 1 lists some controllers under different initial gains.

Table 1 γ optimization

	Intial Gain			*Final Gain*			
	K_1	K_2	K_3	K_1^j	K_2^j	K_3^j	γ
1	$\begin{bmatrix}-10.2 & -5\end{bmatrix}$	$\begin{bmatrix}-14 & -2\end{bmatrix}$	$\begin{bmatrix}-10 & -3\end{bmatrix}$	$\begin{bmatrix}-5.1238 & -2.6131\end{bmatrix}$	$\begin{bmatrix}-4.5571 & -2.3827\end{bmatrix}$	$\begin{bmatrix}-4.1190 & -1.2529\end{bmatrix}$	0.5518
2	$\begin{bmatrix}-10.2 & -5\end{bmatrix}$	$\begin{bmatrix}-13 & -2\end{bmatrix}$	$\begin{bmatrix}-11 & -3\end{bmatrix}$	$\begin{bmatrix}-4.5981 & -3.1390\end{bmatrix}$	$\begin{bmatrix}-3.1854 & -2.6831\end{bmatrix}$	$\begin{bmatrix}-2.4677 & -1.1996\end{bmatrix}$	0.3429
3	$\begin{bmatrix}-10.2 & -5\end{bmatrix}$	$\begin{bmatrix}-12 & -2\end{bmatrix}$	$\begin{bmatrix}-11 & -3\end{bmatrix}$	$\begin{bmatrix}-4.0918 & -2.5668\end{bmatrix}$	$\begin{bmatrix}-3.3271 & -2.7182\end{bmatrix}$	$\begin{bmatrix}-3.0144 & -1.1375\end{bmatrix}$	0.3515
4	$\begin{bmatrix}-10.2 & -5.05\end{bmatrix}$	$\begin{bmatrix}-12 & -2\end{bmatrix}$	$\begin{bmatrix}-11 & -3\end{bmatrix}$	$\begin{bmatrix}-4.0339 & -2.4963\end{bmatrix}$	$\begin{bmatrix}-3.3298 & -2.7256\end{bmatrix}$	$\begin{bmatrix}-3.0142 & -1.1395\end{bmatrix}$	0.3468
5	$\begin{bmatrix}-10.2 & -5.05\end{bmatrix}$	$\begin{bmatrix}-15 & -2\end{bmatrix}$	$\begin{bmatrix}-11 & -3\end{bmatrix}$	$\begin{bmatrix}-4.9262 & -2.5491\end{bmatrix}$	$\begin{bmatrix}-4.6770 & -2.3662\end{bmatrix}$	$\begin{bmatrix}-4.3018 & -1.2626\end{bmatrix}$	0.5774
6	$\begin{bmatrix}-10.2 & -5.05\end{bmatrix}$	$\begin{bmatrix}-13 & -2\end{bmatrix}$	$\begin{bmatrix}-11 & -3\end{bmatrix}$	$\begin{bmatrix}-4.6484 & -3.1596\end{bmatrix}$	$\begin{bmatrix}-3.0546 & -2.6878\end{bmatrix}$	$\begin{bmatrix}-2.4126 & -1.3074\end{bmatrix}$	0.3351

4.2 Example 2

Consider a networked periodic piecewise system with three subsystems given as

$$A_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.85 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{w1} = \begin{bmatrix} 0.15 \\ -0.085 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.15 & 0 \\ 0 & -1.8 \end{bmatrix}, B_2 = \begin{bmatrix} 0.05 \\ 2 \end{bmatrix}, B_{w2} = \begin{bmatrix} 0.15 \\ -0.1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -2.9 & 0 \\ 0 & 1.6 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, B_{w3} = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}, C_3 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$

and period $T_p = 2s$, and $T_1 = 0.4s$, $T_2 = 0.6s$, $T_3 = 1s$, $d = 0.1s$, let λ=0.1, $\rho = 0.1$, it can be observed that all the subsystems are unstable and the first subsystem is non-stabilizable. Introduce the disturbance $w(t)$ as

$$w(t) = \begin{cases} e^{-\lambda t} sint & \text{t} < 1.5, \\ 0.2 & 1.5 \leqslant \text{t} < 3.5. \\ 0.002 & 3.5 \leqslant \text{t} < 4.5, \\ 0 & \text{t} \geqslant 4.5, \end{cases}$$

The state trajectory of the open-loop system is shown in Figure 8, under initial condition $x_0 = \begin{bmatrix} -2 & 1 \end{bmatrix}^T$, it can be seen that the periodic piecewise system is unstable.

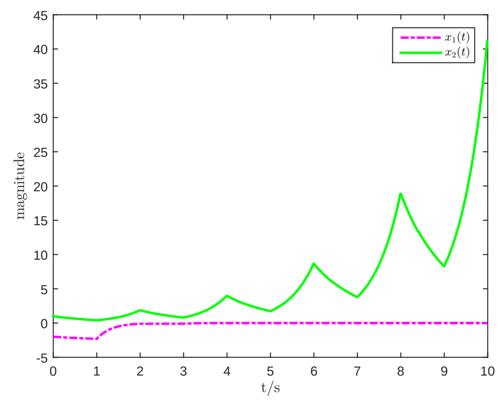

Figure 8 State trajectory of the open-loop system

In the following, according to the H_∞ controller design algorithm, no controller is designed for the first subsystem, with the initial controller gains $K_1^{(1)} = \begin{bmatrix} 0 & 0 \end{bmatrix}$, $K_2^{(1)} = \begin{bmatrix} -12 & -2.5 \end{bmatrix}$, $K_3^{(1)} = \begin{bmatrix} -11 & -2.9 \end{bmatrix}$, and one obtains the following solution after 8 iterations

$$P_{1,1} = \begin{bmatrix} 0.1158 & 0.0023 \\ 0.0023 & 0.0043 \end{bmatrix}, P_{1,2} = \begin{bmatrix} 0.1002 & 0.0023 \\ 0.0023 & 0.0050 \end{bmatrix}, P_{2,1} = \begin{bmatrix} 0.0531 & -0.0022 \\ -0.0022 & 0.0029 \end{bmatrix},$$
$$P_{2,2} = \begin{bmatrix} 0.0476 & -0.0015 \\ -0.0015 & 0.0025 \end{bmatrix}, P_{3,1} = \begin{bmatrix} 0.0162 & 0.0017 \\ 0.0017 & 0.0029 \end{bmatrix}, P_{3,2} = \begin{bmatrix} 0.0212 & 0.0015 \\ 0.0015 & 0.0027 \end{bmatrix},$$
$$Q = \begin{bmatrix} 0.0070 & 0.0035 \\ 0.0035 & 0.0056 \end{bmatrix}, G = \begin{bmatrix} 0.0114 & 0.0036 \\ 0.0036 & 0.0012 \end{bmatrix}$$

and the optimal γ=0.0565, and the controller gains are given by
$K_2 = \begin{bmatrix} -0.8565 & -1.8715 \end{bmatrix}$, $K_3 = \begin{bmatrix} -1.1659 & -2.4896 \end{bmatrix}$.
One may notice that there is no controller designed for the first subsystem because it is non-stabilizable.

The control input is shown in Figure 9, and Figure 10 shows the γ trajectory, which decreases to the final value.

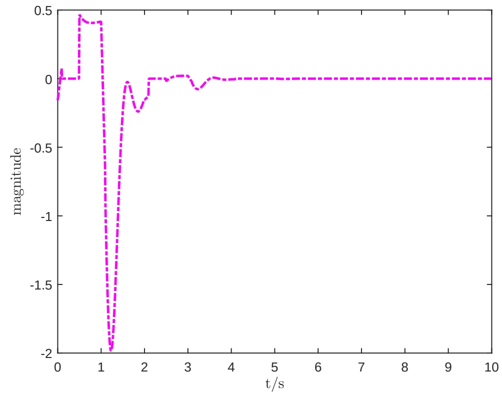

Figure 9 Control input

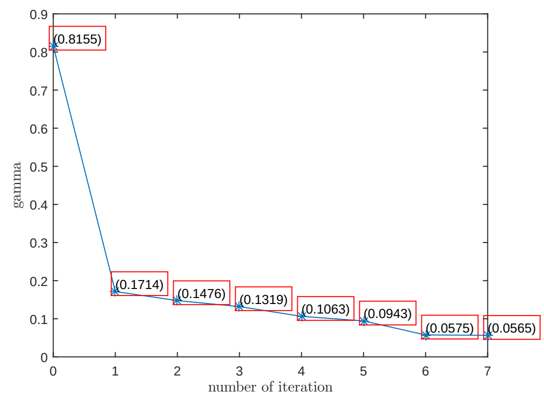

Figure 10 γ trajectory

The state of trajectories of the close-loop system is shown in Figure 11, it can be seen that system is stabilized, and the system output under the proposed controller is shown in Figure 12.

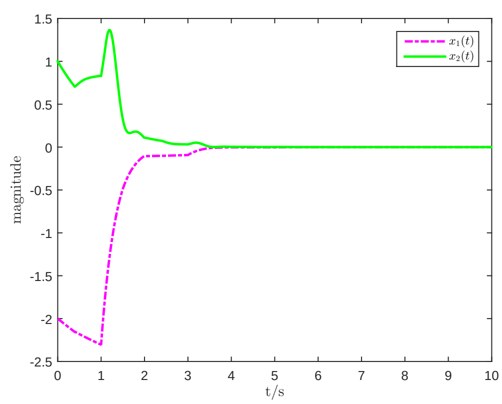

Figure 11 State trajectory of the close-loop system with H_∞ controller

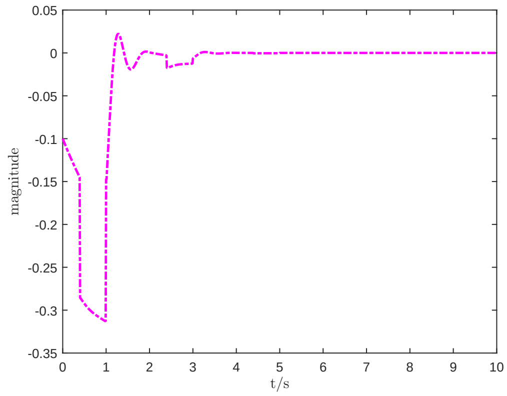

Figure 12 Output of the closed-loop system with H_∞ controller