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Distributed estimation-based output consensus control of heterogeneous leader-follower systems with antagonistic interactions

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Dear editor,

This study addresses an output bipartite consensus problem for leader-follower heterogeneous multi-agent systems, and thus further generalizes the results in [1, 2]. The main contributions include three aspects. (i) Novel distributed estimation schemes are introduced to estimate the unavailable information of the leader. (ii) Owing to the unknown system matrix of the leader, we also estimate the solution of the regulator equation. (iii) An observer-based controller is proposed to guarantee the output bipartite consensus.

Problem formulation. The agent dynamics is given by

$$\dot{x}_i = A_i x_i + B_i u_i, \quad y_i = C_i x_i, \tag{1}$$

where x_i, u_i, y_i are the state, control input and output, respectively. $A_i = \left(\begin{smallmatrix} 0 & I_{n_i-1} & \\ \bar{a}_{i1} & \cdots & \bar{a}_{in_i} \end{smallmatrix}\right) \in \mathbb{R}^{n_i \times n_i}$, $B_i = \mathrm{col}(0, \dots, 0, 1)$, and $C_i = (1\ 0\ \cdots\ 0)$.

The exosystem (leader) is described by

$$\dot{x}_0 = A_0 x_0, \quad y_0 = C_0 x_0, \tag{2}$$

where x_0 and y_0 are the state and output, respectively. $C_0 = (1\ 0\ \cdots\ 0) \in \mathbb{R}^{1\times n_0}$ and $A_0 = \left(\begin{smallmatrix} 0 & I_{n_0-1} & \\ -a_0 & \cdots & -a_{n_0-1} \end{smallmatrix}\right) \in \mathbb{R}^{n_0 \times n_0}$.

When we put the leader labeled as node 0 into the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ is a set of all the followers, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = [\mathcal{A}_{ij}] \in \mathbb{R}^{n\times n}$ denotes the adjacency matrix, the augmented graph $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$ can be obtained, where $\bar{\mathcal{V}} = \{0, 1, \dots, n\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$.

Assumption 1. All the eigenvalues of A_0 have zero real parts and multiplicity one.

Assumption 2. $\mathrm{rank}\left(\begin{smallmatrix} A_i - \lambda I_{n_i} & B_i \\ C_i & 0 \end{smallmatrix}\right) = n_i + p_i$, $\lambda \in \sigma(A_0)$, where $\sigma(A_0)$ is the spectrum of A_0.

Assumption 3. (C_i, A_i) is detectable.

Assumption 4. The coopetition network $\mathcal{G}$ is structurally balanced and the leader is a root node of the spanning tree in $\bar{\mathcal{G}}$.

Let $e_i = y_i - s_i y_0$ for $i = 1, 2, \dots, n$, where $s_i = 1$ for $i \in \mathcal{V}_1$ and $s_i = -1$ for $i \in \mathcal{V}_2$, $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}, \mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$. The output bipartite consensus problem is solved if $\lim_{t\to\infty} y_i - y_0 = 0$ for $i \in \mathcal{V}_1$, $\lim_{t\to\infty} y_i + y_0 = 0$ for $i \in \mathcal{V}_2$.

Let $\xi_0 = \mathrm{col}(\xi_{01}, \xi_{02}, \dots, \xi_{0n_0}) = T_1 x_0$, where

$$T_1 = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ a_{n_0-1} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ a_1 & a_2 & \cdots & a_{n_0-1} & 1 \end{pmatrix}.$$

A distributed observer is designed to estimate x_0.

$$\dot{\xi}_{i1} = \xi_{i2} + (l_{i2} - s_i \hat{a}_{i,n_0-1})\xi_{i1} - \mu_i(\bar{\xi}_i), \tag{3a}$$

$$\begin{aligned}\dot{\xi}_{ij} = {} & \xi_{i,j+1} + l_{i,j+1}\xi_{i1} - s_i \hat{a}_{i,n_0-j}\xi_{i1} \\ & - l_{ij}(\xi_{i2} + l_{i2}\xi_{i1} - s_i \hat{a}_{i,n_0-1}\xi_{i1}),\ j = 2, \dots, n_0 - 1,\end{aligned} \tag{3b}$$

$$\dot{\xi}_{i,n_0} = -l_{i,n_0}(\xi_{i2} + l_{i2}\xi_{i1} - s_i \hat{a}_{i,n_0-1}\xi_{i1}) - s_i \hat{a}_{i,0}\xi_{i1}, \tag{3c}$$

where $\bar{y}_i = C_0 \xi_i$, $\xi_i = \mathrm{col}(\xi_{i1}, \xi_{i2}, \dots, \xi_{in_0})$, $\mu_i(\bar{\xi}_i) = \rho_i(\bar{\xi}_i)\bar{\xi}_i$, $\bar{\xi}_i = \sum_{j\in N_i} |\mathcal{A}_{ij}|(\xi_{i1} - \mathrm{sign}(\mathcal{A}_{ij})\xi_{j1}) + b_i(\xi_{i1} - s_i \xi_{01})$. $b_i = 1$ if agent i can get information from the leader; otherwise, $b_i = 0$.

Let $\alpha_{il} = \sum_{j\in N_i} |\mathcal{A}_{ij}|(\hat{a}_{il} - \mathrm{sign}(\mathcal{A}_{ij})\hat{a}_{jl}) + b_i(\hat{a}_{il} - s_i a_l)$.

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The adaptive law for a_{ij} is given by

$$\begin{aligned}&\dot{\hat{a}}_{ij}=-c_1\alpha_{ij},\quad j=0,1,\ldots,n_0-2,\\&\dot{\hat{a}}_{i,n_0-1}=s_i\bar{\xi}_i\xi_{i1}-c_1\alpha_{i,n_0-1}.\end{aligned}\tag{4}$$

Theorem 1. Consider (3) and (4). When Assumption 4 holds, one has $\lim_{t\to+\infty}\xi_{i1}-s_i\xi_{01}=0$, $\lim_{t\to+\infty}\xi_{ij}-s_i(\xi_{0j}-l_{ij}\xi_{01})=\xi_{ij}+s_il_{ij}\xi_{01}-s_i\xi_{0j}=0$ and $\lim_{t\to+\infty}\hat{a}_{i,n_0-k}-s_ia_{n_0-k}=0$ exponentially for $j=2,\ldots,n_0$, $k=1,\ldots,n_0$.

The proof of Theorem 1 is given in Appendix A.

There exist matrices Π_i and Γ_i satisfying

$$\Pi_iA_0=A_i\Pi_i+B_i\Gamma_i,\quad 0=C_i\Pi_i-C_0.\tag{5}$$

Let $B_i\hat{\bar{\Gamma}}_i=\hat{\Xi}_i\hat{A}_{i0}-(A_i+B_iF_i)\hat{\Xi}_i$, where $\hat{\Xi}_i=\hat{\Pi}_iT_{1i}^{-1}$, $\hat{\Pi}_i=(\hat{\bar{\Pi}}_{kj}^{[i]})\in\mathbb{R}^{n_i\times n_0}$ satisfies the following. If $n_i<n_0$, $\hat{\Pi}_i=(I_{n_i}\ \ 0_{n_i\times(n_0-n_i)})$; if $n_i=n_0$, $\hat{\Pi}_i=I_{n_i}$; if $n_i>n_0$, then $\hat{\Pi}_i=\binom{I_{n_0}}{\Theta_i}$, where the first column of Θ_i is col$(-s_i\hat{a}_{i0}$, $-s_i\hat{a}_{i0}\hat{\bar{\Pi}}^{[i]}_{(n_0+1),n_0},-s_i\hat{a}_{i0}\hat{\bar{\Pi}}^{[i]}_{(n_0+2),n_0},\ldots,-s_i\hat{a}_{i0}\hat{\bar{\Pi}}^{[i]}_{(n_i-1),n_0})$, the second column of Θ_i is col$(-s_i\hat{a}_{i1},\hat{\bar{\Pi}}^{[i]}_{(n_0+1),1}-s_i\hat{a}_{i1}\hat{\bar{\Pi}}^{[i]}_{(n_0+1),n_0},\hat{\bar{\Pi}}^{[i]}_{(n_0+2),1}-s_i\hat{a}_{i1}\hat{\bar{\Pi}}^{[i]}_{(n_0+2),n_0},\ldots,\hat{\bar{\Pi}}^{[i]}_{(n_i-1),1}-s_i\hat{a}_{i1}\hat{\bar{\Pi}}^{[i]}_{(n_i-1),n_0})$, similarly, the last column of Θ_i is col$(-s_i\hat{a}_{i,n_0-1},\hat{\bar{\Pi}}^{[i]}_{(n_0+1),(n_0-1)}-s_i\hat{a}_{i,n_0-1}\hat{\bar{\Pi}}^{[i]}_{(n_0+1),n_0},\ldots,\hat{\bar{\Pi}}^{[i]}_{(n_i-1),(n_0-1)}-s_i\hat{a}_{i,n_0-1}\hat{\bar{\Pi}}^{[i]}_{(n_i-1),n_0})$,

$$T_{1i}=\begin{pmatrix}1&0&\cdots&0\\ s_i\hat{a}_{i,n_0-1}&1&\cdots&0\\ \vdots&\vdots&\cdots&\vdots\\ s_i\hat{a}_{i1}&s_i\hat{a}_{i2}&\cdots&1\end{pmatrix},$$

$$\hat{A}_{i0}=\begin{pmatrix}\hat{A}_{i00}&I_{n_0-1}\\ -s_i\hat{a}_{i0}&0_{1\times(n_0-1)}\end{pmatrix},$$

and $\hat{A}_{i00}=\mathrm{col}(-s_i\hat{a}_{i,n_0-1},\ldots,-s_i\hat{a}_{i1})$. F_i is selected such that $A_i+B_iF_i$ is a Hurwitz matrix.

Then, the Luenberger type observers are introduced,

$$\begin{aligned}&\dot{\bar{x}}_i=A_i\bar{x}_i+B_iu_i+H_i(\hat{y}_i-y_i),\\&\hat{y}_i=C_i\bar{x}_i,\quad i=1,\ldots,n,\end{aligned}\tag{6}$$

where $\bar{x}_i$ is the estimation of $x_i(t)$.

The controller is designed as follows:

$$u_i=F_i\bar{x}_i+\hat{\bar{\Gamma}}_i\eta_i,\quad i=1,2,\ldots,n,\tag{7}$$

where $\eta_i=T_{2i}\xi_i$, $T_{2i}=\begin{pmatrix}1&0_{1\times(n_0-1)}\\ \bar{T}_{2i}&I_{n_0-1}\end{pmatrix}$, $\bar{T}_{2i}=\mathrm{col}(l_{i2},\ldots,l_{in_0})$.

The proofs of the following theorems are given in Appendixes B and C, respectively.

Theorem 2. Consider (1). Under Assumptions 1–4 and the controller (7), one has $\lim_{t\to+\infty}\bar{x}_i-\Pi_iT_1^{-1}\eta_i=0$ and $\lim_{t\to+\infty}\bar{x}_i-x_i=0$ exponentially.

Theorem 3. Consider (1) and (2). Under Assumptions 1–4, the distributed observer (3), the adaptive law (4), and the protocol (7), the leader-following output bipartite consensus problem is solved, that is, $\lim_{t\to+\infty}e_i(t)=0$.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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