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Robust online energy efficiency optimization for distributed multi-cell massive MIMO networks

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Abstract This paper studies the energy efficiency (EE) oriented precoding design in multi-cell massive multiple-input multiple-output (MIMO) systems, with only statistical channel state information (CSI) at the transmitter. During the transmission, as the channel varies dynamically with time and the previously obtained CSI becomes outdated, the base stations must adjust their transmit policies accordingly. To tackle this issue, we propose an online EE maximization algorithm that can achieve a no-regret transmission; i.e., the performance of this online method gradually approaches that of the fixed offline method which has full knowledge of the future CSI. Specifically, we first construct the online EE optimization problem in a distributed way to reduce the information required to be exchanged between cells. Then, we apply the large-dimensional random matrix theory to lower the calculation complexity, and the Charnes-Cooper transform to address the nonconvexity of the problem, respectively. The online gradient ascent method is utilized to perform this no-regret power allocation strategy based on all past CSI. We also assess the robustness of the algorithm to estimation error of statistical CSI under some mild conditions which can usually be satisfied in practice. Numerical results demonstrate the no-regret property and the robustness of the proposed online algorithm for energy efficient multi-cell massive MIMO transmission.

Keywords energy efficiency, statistical CSI, multi-cell MIMO, massive MIMO, online gradient ascent, distributed processing, robust transmission

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1 Introduction

Recently, the 5G networks have been gradually commercialized, and researches on beyond 5G or even 6G cellular networks are emerging [1, 2]. Massive multiple-input multiple-output (MIMO), as a fundamental technology in 5G, greatly improves the link capacity and transmission reliability due to spatial multiplexing and adaptive beamforming gains [3–5]. Moreover, new versions of massive MIMO technologies continue playing an essential role in future wireless communication networks [6, 7].

The dramatic development of data-hungry applications (e.g., Internet of Things (IoT), intelligent driving, cloud-based services) requires higher transmit data rates. Thus, the wireless network is faced with a challenging task: the data rates must be increased to fulfill the increasing needs, yet the energy budget might be extremely tight. With this in mind, energy efficiency (EE) as a design metric is gaining more and more attention [8,9]. Fractional programming methods to address the EE optimization problems were systematically introduced in [8]. EE-oriented precoding strategy exploiting statistical channel state information (CSI) for downlink massive MIMO was investigated in [9]. There also exist investigations that aim to reach an adaptive balance between EE and spectrum efficiency (SE) [10].

In conventional single-cell-based transmission, the user terminals (UTs) at cell-edge tend to undergo inter-cell interference (ICI) and might have poor transmission performance. To tackle this, multi-cell processing based on base station (BS) cooperation was put forward as a solution, which is also called the coordinated multi-cell processing [11]. The main idea of BS cooperation is that a certain number

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of adjacent cells form a cluster where BSs conduct their precoding together, along with information exchange. In this way, the co-channel interferences can be greatly mitigated and the network can attain a preferable data rate. Thus, it has attracted massive investigations [11–13]. However, if the CSI and transmit signals are all shared among cells, especially in massive MIMO scenarios, this kind of cooperation will cause a huge backhaul burden due to the vast data sharing. Contrary to this centralized multi-cell processing, distributed approaches that require limited information sharing among cells are more favorable. In [14], a distributed beamforming method was proposed by converting the transmit beamforming design into a linear minimum mean square error estimation problem. The work in [15] proposed centralized and distributed ways to address the EE optimization problem in multi-cell massive MIMO systems, respectively.

Owing to user mobility and fluctuations in the wireless media, the CSI varies over time in realistic environments. When CSI alters, the transmit strategies that are based on outdated channel states might have poor performance. Thus, the transmitter must try to adapt to the changing channel and update the precoding strategy accordingly. Online optimization method, which is a sequential decision making procedure, can be utilized for this dynamic transmit design [16]. Specifically, at each stage, the transmitter chooses a precoding matrix based on the CSI of all past states. After the decision has been made, the transmitter receives some problem-specific feedback, and continues to make new decisions. One commonly adopted optimization criterion to evaluate the performance of this online precoding design is called regret. It is a concept in game theory that was first put forward in [17]. When applied to wireless communications, regret is defined by the cumulative performance differences between the two policies [18]. One is the online precoding design, and the other is a fixed offline transmit strategy, where the transmitter is assumed to know the CSI before transmission. It is a theoretical benchmark that aims to maximize the system performance over the whole transmission time. Based on the definition of regret, an online algorithm can achieve no-regret transmission if it provides an asymptotic solution to the ideal offline policy mentioned above [16]. Regret-based investigations have attracted much attention recently. For example, a general introduction of no-regret learning and online optimization algorithms was shown in [16]. The regret-based formulation of the power control scheme was applied to IoT systems in [19].

Note that these existing approaches are based on instantaneous CSI, and the performance is highly dependent on the accuracy of the estimation process. However, it is often challenging to acquire sufficiently accurate instantaneous CSI in realistic environments, especially for massive MIMO systems [20]. On the contrary, the statistical CSI, i.e., the statistics of the channels, varies over a much broader time scale. In addition, the uplink and downlink statistics of the channel are usually reciprocal in time division duplex and frequency division duplex systems [21, 22]. Therefore, when acquiring instantaneous CSI is difficult, the statistical CSI can be utilized for transmission strategy design. Of course, even with the statistical CSI, factors such as pilot contamination can still damage the estimation accuracy, and measurement errors cannot be completely eliminated. Thus, schemes needed to be proposed to improve the robustness to estimation imperfections in statistical CSI. In [23], a robust transmission method for mmWave vehicular communications with imperfectly known statistical CSI was studied. A deep learning-based user scheduling algorithm using statistical CSI was proposed in [24].

Contributions. Based on the discussion above, we study the robust energy-efficient downlink transmit policy in a multi-cell massive MIMO system, with only statistical CSI available. We summarize the major contributions of this paper as follows.

- We first formulate an online EE optimization program in a distributed way, considering the backhaul burden resulting from inter-cell communication. Then, we obtain the optimal transmit directions in closed form so that the original problem is converted into a power allocation one, thus reducing the optimization complexity.
- Next, we reduce the computational complexity by applying the deterministic equivalent (DE) method on the ergodic rate, where the expectation calculation is avoided. The Charnes-Cooper transform is utilized to address the nonconvexity of the problem.
- Then, we resort to the online gradient ascent (OGA) method and propose an online precoding strategy. We establish the no-regret properties of the proposed algorithm, and show the algorithm's robustness under estimation errors. Numerical results show the EE performance of our proposed algorithm and its no-regret property.

Organizations. The rest of this paper is organized as follows. In Section 2, we present the adopted channel model and power consumption model. The online EE optimization model is formulated, along with the concept of regret. In Section 3, the DE method and the Charnes-Cooper transform are applied to address the computational complexity and the nonconvexity of the problem, respectively. Moreover, we figure out the online transmit policy for the problem, and show its robustness to estimation errors in statistical CSI. We provide numerical simulations in Section 4 and conclude this paper in Section 5.

Notations. Column vectors and matrices are typeset in lower and upper case boldface letters, respectively. Let $\mathbb{R}^{M\times N}$ and $\mathbb{C}^{M\times N}$ denote the $M\times N$ real and complex vector spaces, respectively. An identity matrix of size $M\times M$ is marked by $\boldsymbol{I}_M$, and let $\boldsymbol{A}\succeq\boldsymbol{0}$ denote a positive semi-definite matrix. The Hadamard product is represented by $\odot$ and $[x]^+$ denotes $\max\{x,0\}$. The expression $\operatorname{diag}\{\boldsymbol{x}\}$ stands for a diagonal matrix where its diagonal elements correspond to the elements of $\boldsymbol{x}$. The operators $\operatorname{tr}\{\cdot\}$, $\det\{\cdot\}$ and $\mathrm{E}\{\cdot\}$ denote the trace, determinant and expectation operations, respectively. Let $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ denote the transpose, conjugate and conjugate-transpose operations on matrices, respectively.

2 System model and problem formulation

2.1 System model

Consider the downlink transmission in a multi-cell massive MIMO scenario. The system consists of U cells, and each cell-u contains one M-antenna BS and one N-antenna UT. Let BS-u and UT-u denote the BS and the UT in cell-u, respectively. With the signal sent by BS-u expressed as $\boldsymbol{x}_u\in\mathbb{C}^{M\times 1}$, we write the received signal at UT-u under an additive Gaussian noise $\boldsymbol{n}_u$ as

$$\boldsymbol{y}_u=\boldsymbol{H}_{u,u}\boldsymbol{x}_u+\underbrace{\sum\nolimits_{j\neq u}\boldsymbol{H}_{u,j}\boldsymbol{x}_j}_{\text{inter-cell interference}}+\boldsymbol{n}_u\in\mathbb{C}^{N\times 1},\tag{1}$$

where $\boldsymbol{H}_{u,v}\in\mathbb{C}^{N\times M}$ denotes the downlink channel matrix from BS-v to UT-u, and $\boldsymbol{n}_u\in\mathbb{C}^{N\times 1}$ is modeled as a complex circularly symmetric distributed Gaussian noise, which is zero-mean and has a covariance matrix $\sigma^2\boldsymbol{I}_N$. The transmitted signal satisfies $\mathrm{E}\{\boldsymbol{x}_u\}=\boldsymbol{0}$, $\mathrm{E}\{\boldsymbol{x}_u\boldsymbol{x}_j^{\mathrm{H}}\}=\boldsymbol{0}$ $(\forall j\neq u)$, and its covariance matrix is $\boldsymbol{Q}_u=\mathrm{E}\{\boldsymbol{x}_u\boldsymbol{x}_u^{\mathrm{H}}\}\in\mathbb{C}^{M\times M}$.

To describe the downlink channel spatial correlations, we follow the Weichselberger's channel model [25], which is a widely adopted model for transmission design [26, 27]. We characterize the structure of the downlink channel matrix $\boldsymbol{H}_{u,v}$ as [28]

$$\boldsymbol{H}_{u,v}=\boldsymbol{U}_{u,v}\boldsymbol{G}_{u,v}\boldsymbol{V}_{u,v}^{\mathrm{H}},\tag{2}$$

where $\boldsymbol{U}_{u,v}\in\mathbb{C}^{N\times N}$ and $\boldsymbol{V}_{u,v}\in\mathbb{C}^{M\times M}$ are deterministic unitary matrices, and $\boldsymbol{G}_{u,v}\in\mathbb{C}^{N\times M}$ is a random matrix with its entries statistically uncorrelated [28], which is also known as the beam domain channel. It is proved in [20] that $\boldsymbol{V}_{u,v}$ becomes asymptotically identical to $\boldsymbol{V}$ as M approaches infinity, i.e.,

$$\boldsymbol{V}_{u,v}\overset{M\to\infty}{=}\boldsymbol{V},\tag{3}$$

where $\boldsymbol{V}$ is irrelevant to the locations of UTs and only relates to the BS antenna array topologies. For example, if the BS antenna configuration is a uniform linear array and the antenna number is fairly large, i.e., $M=128$, $\boldsymbol{V}$ can be well approximated by the discrete Fourier transform matrix [20]. Note that this approximation has been widely adopted in various studies [9, 10, 29].

In this work, we design the precoding policy based on the statistical CSI, i.e., the statistical characteristics of the instantaneous CSI. Specifically, we adopt the concept of eigenmode channel coupling matrix in [28], which is defined as

$$\boldsymbol{\Omega}_{u,v}=\mathrm{E}\{\boldsymbol{G}_{u,v}\odot\boldsymbol{G}_{u,v}^*\}\in\mathbb{R}^{N\times M}.\tag{4}$$

From (1), we can observe that the aggregate interference-plus-noise at UT-u is $\boldsymbol{n}_u'=\sum_{j\neq u}\boldsymbol{H}_{u,j}\boldsymbol{x}_j+\boldsymbol{n}_u$. Consider the worst-case design, which means that if the noise covariance is fixed, the worst case noise follows a Gaussian distribution in the sense that the corresponding rate is a lower bound of the exact rate [30]. Then, $\boldsymbol{n}_u'$ is assumed to be a Gaussian noise with covariance [30].

$$\boldsymbol{K}_u=\mathrm{E}\{\boldsymbol{n}_u'\boldsymbol{n}_u'^{\mathrm{H}}\}=\sigma^2\boldsymbol{I}_N+\sum_{j\neq u}\mathrm{E}\{\boldsymbol{H}_{u,j}\boldsymbol{Q}_j\boldsymbol{H}_{u,j}^{\mathrm{H}}\},\tag{5}$$

where $\boldsymbol{K}_u$ is accessible at each UT-u. Based on the above assumptions, we present the ergodic data rate of UT-u as in [31, 32], i.e.,

$$R_u(\boldsymbol{Q}) = \mathrm{E}\left\{\log\det\left(\boldsymbol{K}_u + \boldsymbol{H}_{u,u}\boldsymbol{Q}_u\boldsymbol{H}_{u,u}^{\mathrm{H}}\right)\right\} - \log\det\left(\boldsymbol{K}_u\right). \tag{6}$$

By rewriting $\boldsymbol{H}_{u,u}$ as $\boldsymbol{H}_{u,u} = \boldsymbol{U}_{u,u}\boldsymbol{G}_{u,u}\boldsymbol{V}$ based on (2) and (3), and applying the Sylvester's determinant identity, the ergodic rate can be rewritten as

$$R_u(\boldsymbol{Q}) = \mathrm{E}\left\{\log\det\left(\widetilde{\boldsymbol{K}}_u + \boldsymbol{G}_{u,u}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{Q}_u\boldsymbol{V}\boldsymbol{G}_{u,u}^{\mathrm{H}}\right)\right\} - \log\det\left(\widetilde{\boldsymbol{K}}_u\right), \tag{7}$$

where $\widetilde{\boldsymbol{K}}_u$ is defined as

$$\widetilde{\boldsymbol{K}}_u \triangleq \sigma^2\boldsymbol{I}_N + \sum_{j\neq u}\boldsymbol{\Pi}_{u,j}\left(\boldsymbol{V}^{\mathrm{H}}\boldsymbol{Q}_j\boldsymbol{V}\right). \tag{8}$$

Moreover, $\boldsymbol{\Pi}_{u,j}(\boldsymbol{X})$ is a diagonal matrix-valued function with its diagonal elements calculated by [9]

$$\left[\boldsymbol{\Pi}_{u,j}(\boldsymbol{X})\right]_{n,n} = \mathrm{tr}\left\{\mathrm{diag}\left\{\left(\left[\boldsymbol{\Omega}_{u,j}\right]_{n,:}\right)^{\mathrm{T}}\right\}\boldsymbol{X}\right\}, \quad n = 1, 2, \ldots, N. \tag{9}$$

In general multi-cell scenarios, the BSs cooperate with each other to design the precoding matrices in a centralized way, which may cause large backhaul burdens, especially in massive MIMO systems. Therefore, in this paper, we develop algorithms in a distributed way, where less information is exchanged compared to the centralized approaches. Based on the distributed scenario, we assume that the BSs only have access to the statistical CSI of their cells. At the same time, the UTs are available to their own instantaneous CSI, which can be achieved through a proper pilot design [33]. Moreover, as can be seen from (8) and (9), $\widetilde{\boldsymbol{K}}_u$ depends on the transmit covariance matrices and the CSI from other cells. Hence, if we set

$$\widetilde{\boldsymbol{G}}_{u,u} = \widetilde{\boldsymbol{K}}_u^{-1/2}\boldsymbol{G}_{u,u} \tag{10}$$

as the effective beam domain channel matrix from BS-u to UT-u, the data rate in (7) can be written as

$$R_u(\boldsymbol{Q}_u) = \mathrm{E}\left\{\log\det\left(\boldsymbol{I}_N + \widetilde{\boldsymbol{G}}_{u,u}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{Q}_u\boldsymbol{V}\widetilde{\boldsymbol{G}}_{u,u}^{\mathrm{H}}\right)\right\}. \tag{11}$$

Moreover, the effective statistical CSI is expressed as

$$\widetilde{\boldsymbol{\Omega}}_{u,u} = \mathrm{E}\left\{\widetilde{\boldsymbol{G}}_{u,u}\odot\widetilde{\boldsymbol{G}}_{u,u}^{*}\right\} = \widetilde{\boldsymbol{K}}_u\boldsymbol{\Omega}_{u,u}. \tag{12}$$

For the consumed power of the system, we apply the widely adopted model as follows [8, 34]:

$$P_u(\boldsymbol{Q}_u) = \xi_u\mathrm{tr}\left\{\boldsymbol{Q}_u\right\} + MP_{\mathrm{c},u} + P_{\mathrm{s},u}, \tag{13}$$

which is the sum of two dynamic terms and one static term. The coefficient ξ_u (> 1) is the inverse of the power amplifier efficiency, $\mathrm{tr}\left\{\boldsymbol{Q}_u\right\}$ denotes the transmit power of UT-u. $P_{\mathrm{s},u}$ and $P_{\mathrm{c},u}$ are the dynamic power consumed per BS antenna and the basic power consumption of BS-u, respectively.

2.2 Problem formulation

With the ergodic rate and power consumption in hand, we construct the EE maximization problem. Recall that we consider the distributed precoding design, where in each iteration of the algorithm, cell-u regards the interferences from other cells as constant terms and conducts its own precoding. After each iteration, information is exchanged among cells, and the interference terms are updated. Then, a new iteration begins until convergence. Next, we focus on the EE optimization problem of cell-u in one iteration as

$$\mathcal{P}: \quad \max_{\boldsymbol{Q}_u}\frac{R_u(\boldsymbol{Q}_u)}{P_u(\boldsymbol{Q}_u)}$$

$$\text{s.t. } \mathrm{tr}\left\{\boldsymbol{Q}_u\right\} \leqslant P_{\max,u}, \quad \boldsymbol{Q}_u \succeq \boldsymbol{0}, \tag{14}$$

where $R_u\left(\boldsymbol{Q}_u\right)$ is shown in (11). Note that the EE of cell-u depends not only on its own transmit covariance matrix, but also on those of other cells, which is reflected by the effective channel $\widetilde{\boldsymbol{G}}_{u,u}$.

Directly handling problem (14) is computationally challenging, which involves solving an $M \times M$ complex transmit covariance matrix. Applying similar technique as in [9, 35], we show that beam domain transmission is favorable for our statistical CSI aided EE optimization problem. Specifically, we first decompose the transmit covariance matrix as $\boldsymbol{Q}_u = \boldsymbol{D}_u \boldsymbol{\Lambda}_u \boldsymbol{D}_u^{\mathrm{H}}$ via eigenvalue decomposition. During the transmission, $\boldsymbol{D}_u$ represents the subspaces which the transmit signals lie in, while the diagonal elements of $\boldsymbol{\Lambda}_u$ stand for the power assigned to corresponding directions. By doing so, we can obtain that the optimal transmit eigenmatrix $\boldsymbol{D}_u$ for problem $\mathcal{P}$ is given by $\boldsymbol{V}$ in (3), i.e., $\boldsymbol{Q}_u^{\mathrm{opt}} = \boldsymbol{V}\boldsymbol{\Lambda}_u\boldsymbol{V}^{\mathrm{H}}$, which is proved in [9, 35] and therefore is omitted here.

As $\boldsymbol{V}$ is the eigenvector of the BS correlation matrix, which only relates to the BS antenna array topology, problem $\mathcal{P}$ in (14) can be equivalently transformed into a new problem in the beam domain as follows:

$$\begin{aligned} \mathcal{P}_1: \quad & \max_{\boldsymbol{\Lambda}_u} \mathrm{EE}_u\left(\boldsymbol{\Lambda}_u\right) = \frac{R_u\left(\boldsymbol{\Lambda}_u\right)}{\xi_u \mathrm{tr}\left\{\boldsymbol{\Lambda}_u\right\} + M P_{\mathrm{c},u} + P_{\mathrm{s},u}} \\ & \text{s.t. } \mathrm{tr}\left\{\boldsymbol{\Lambda}_u\right\} \leqslant P_{\max,u}, \\ & \quad \boldsymbol{\Lambda}_u \succeq \boldsymbol{0}, \quad \boldsymbol{\Lambda}_u \text{ diagonal}, \end{aligned} \tag{15}$$

where the optimization variable is the power allocation matrix with only M real scalars. Thus, the computational complexity is greatly reduced. Moreover, the expression of data rate becomes

$$R_u\left(\boldsymbol{\Lambda}_u\right) = \mathrm{E}\left\{\log\det\left(\boldsymbol{I}_N + \widetilde{\boldsymbol{G}}_{u,u}\boldsymbol{\Lambda}_u\widetilde{\boldsymbol{G}}_{u,u}^{\mathrm{H}}\right)\right\}, \tag{16}$$

and $\widetilde{\boldsymbol{K}}_u$ in (8) can be rewritten as

$$\widetilde{\boldsymbol{K}}_u = \sigma^2 \boldsymbol{I}_N + \sum_{j \neq u} \boldsymbol{\Pi}_{u,j}\left(\boldsymbol{\Lambda}_j\right). \tag{17}$$

Note that the effective channel $\widetilde{\boldsymbol{G}}_{u,u}$ varies over time, and the main focus of our work is to design the EE-oriented transmission in dynamic situations. Let $\widetilde{\boldsymbol{G}}_{u,u}^s$ denote the effective CSI at stage s, the online EE optimization program can be formulated as

$$\begin{aligned} \mathcal{P}_2: \quad & \boldsymbol{\Lambda}_u^{\mathrm{dyn}} = \arg\max_{\boldsymbol{\Lambda}_u} \mathrm{EE}_u^s\left(\boldsymbol{\Lambda}_u\right) = \frac{R_u^s\left(\boldsymbol{\Lambda}_u\right)}{P_u\left(\boldsymbol{\Lambda}_u\right)} \\ & \text{s.t. } \mathrm{tr}\left\{\boldsymbol{\Lambda}_u\right\} \leqslant P_{\max,u}, \\ & \quad \boldsymbol{\Lambda}_u \succeq \boldsymbol{0}, \quad \boldsymbol{\Lambda}_u \text{ diagonal}, \end{aligned} \tag{18}$$

where

$$R_u^s\left(\boldsymbol{\Lambda}_u\right) = \mathrm{E}\left\{\log\det\left(\boldsymbol{I}_N + \widetilde{\boldsymbol{G}}_{u,u}^s\boldsymbol{\Lambda}_u\left(\widetilde{\boldsymbol{G}}_{u,u}^s\right)^{\mathrm{H}}\right)\right\} \tag{19}$$

denotes the achievable rate at stage s. Recall the assumption that the BS cannot predict the channel state ahead of time, the transmit process is presented as follows. Firstly, at each update stage $s = 1, 2, \ldots,$ the BS derives the power allocation matrix $\boldsymbol{\Lambda}_u^s$. Then, the EE at stage s is calculated by the effective channel matrix $\widetilde{\boldsymbol{G}}_{u,u}^s$ and $\boldsymbol{\Lambda}_u^s$. Finally, at the end of stage s, the BS selects a new power allocation matrix $\boldsymbol{\Lambda}_u^{s+1}$, aiming to maximize the objective function $\mathrm{EE}_u^{s+1}(\boldsymbol{\Lambda}_u)$, and the process continues. To conduct this framework, the EE-optimization strategy must try to adapt to the varying CSI to find the power allocation strategy without knowing the next channel state. To evaluate the performance of this strategy, the concept of regret is raised, which has been widely adopted in previous studies, e.g., [18, 36, 37]. The regret is the difference between the performance achieved by the online transmit policy and the payoff that would have been obtained if the best fixed offline strategy is applied.

To define regret, we first consider the ideal case where the BSs have perfect knowledge of how the channel evolves in advance, and the system designs the transmit policy offline before transmission. Then, the offline power allocation strategy would be the solution to the following problem:

$$\begin{aligned}\mathcal{P}_3: \quad & \mathbf{\Lambda}_u^{\mathrm{off}} = \arg\max_{\mathbf{\Lambda}_u} \frac{1}{S}\sum_{s=1}^{S}\frac{R_u^s\left(\mathbf{\Lambda}_u\right)}{P_u\left(\mathbf{\Lambda}_u\right)} \\ & \text{s.t. } \operatorname{tr}\left\{\mathbf{\Lambda}_u\right\} \leqslant P_{\max,u}, \\ & \qquad \mathbf{\Lambda}_u \succeq \mathbf{0}, \quad \mathbf{\Lambda}_u \text{ diagonal},\end{aligned} \tag{20}$$

which aims to maximize the average EE over a period of S stages. Note that during transmission, the transmit policy of the offline strategy stays fixed. We take the solution of problem $\mathcal{P}_3$ as a performance benchmark, and our online algorithm would find solutions that approach $\mathbf{\Lambda}_u^{\mathrm{off}}$ asymptotically over time.

To this end, we now give the definition of regret of cell-u at stage S as the cumulative difference between the solution of $\mathcal{P}_3$ and the EE achieved by $\mathcal{P}_2$ [37, 38]:

$$\operatorname{Reg}\left(S\right) = \sum_{s=1}^{S}\left[\mathrm{EE}_u^s\left(\mathbf{\Lambda}_u^{\mathrm{off}}\right) - \mathrm{EE}_u^s\left(\mathbf{\Lambda}_u^{\mathrm{dyn}}\right)\right]. \tag{21}$$

We further say an online transmit strategy can achieve no-regret transmission if [16, 18]

$$\operatorname{Reg}\left(S\right) = o(S), \tag{22}$$

where $o(S)$ denotes the infinitesimal of higher order of S, i.e., $\lim_{S\to\infty}\operatorname{Reg}\left(S\right)/S = 0$. Note that the regret is independent of the EE function. In this way, a no-regret strategy provides an asymptotic solution to the average EE maximization offline problem $\mathcal{P}_3$ in (20), without knowing the future channel state $\widetilde{\boldsymbol{G}}_{u,u}(s+1)$ at each stage $s = 1, 2, \ldots, S$.

To state the no-regret strategy more clearly, we make a remark here. If the BSs can predict the future CSI without error, the optimal transmit strategy would be given by $\mathbf{\Lambda}_u^{\mathrm{opt},s} = \arg\max_{\mathbf{\Lambda}_u}\mathrm{EE}_u^s\left(\mathbf{\Lambda}_u\right)$, which means that the BSs select the power allocation matrices at each instance time. Nevertheless, for online transmission, it is not a theoretical target [37, 39]. Hence, when defining the regret, we take the averaged EE maximization offline problem $\mathcal{P}_3$ as a designing benchmark, rather than the online instance problem.

3 Robust energy-efficient power allocation

With the aforementioned discussion, in this section, we first propose an online power allocation policy for problem $\mathcal{P}_2$ in (18). Then, considering the inevitable estimation errors of the CSI, we evaluate the robustness of our proposed algorithm against imperfect statistical CSI in Subsection 3.2.

As can be seen in (16), calculating the ergodic rate brings high computational complexity, for it involves expectation operation on large-dimensional matrices. To find a feasible way to calculate the ergodic rate, we derive its DE expression utilizing the large-dimensional matrix theory [40, 41]. When conducting the DE method, the objective function is calculated iteratively while several auxiliary variables are introduced. Note that as the BS antenna number becomes larger, e.g., in massive MIMO systems, the DE expression is an asymptotic approximation to the objective [26]. We temporarily drop the stage index s for notation convenience, and the DE of $R_u\left(\mathbf{\Lambda}\right)$ is given as

$$\overline{R}_u\left(\mathbf{\Lambda}_u\right) = \log\det\left(\boldsymbol{I}_M + \mathbf{\Xi}_u\mathbf{\Lambda}_u\right) + \log\det\left(\mathbf{\Psi}_u + \boldsymbol{I}_N\right) - \boldsymbol{\gamma}_u^{\mathrm{T}}\widetilde{\mathbf{\Omega}}_{u,u}\boldsymbol{\psi}_u, \tag{23}$$

where $\mathbf{\Xi}_u$ and $\mathbf{\Psi}_u$ are expressed as

$$\mathbf{\Xi}_u = \operatorname{diag}\left\{\widetilde{\mathbf{\Omega}}_{u,u}^{\mathrm{T}}\boldsymbol{\gamma}_u\right\} \in \mathbb{C}^{M\times M}, \tag{24}$$

$$\mathbf{\Psi}_u = \boldsymbol{U}_{u,u}\operatorname{diag}\left\{\widetilde{\mathbf{\Omega}}_{u,u}\boldsymbol{\psi}_u\right\}\boldsymbol{U}_{u,u}^{\mathrm{H}} \in \mathbb{C}^{N\times N}, \tag{25}$$

respectively. Moreover, $\boldsymbol{\gamma}_u \triangleq \left[\gamma_{u,1}, \gamma_{u,2}, \ldots, \gamma_{u,M}\right]^{\mathrm{T}}$, $\boldsymbol{\psi}_u \triangleq \left[\psi_{u,1}, \psi_{u,2}, \ldots, \psi_{u,N}\right]^{\mathrm{T}}$. For the auxiliary variables $\gamma_{u,m}, \forall(u,m)$ and $\psi_{u,n}, \forall(u,n)$, they are iteratively calculated by the following equations until convergence:

$$\gamma_{u,n} = \boldsymbol{u}_{u,n}^{\mathrm{H}}\left(\boldsymbol{I}_N + \mathbf{\Psi}_u\right)^{-1}\boldsymbol{u}_{u,n}, \tag{26}$$

$$\psi_{u,m} = \left[\mathbf{\Lambda}_u \left(\boldsymbol{I}_M + \mathbf{\Xi}_u \mathbf{\Lambda}_u\right)^{-1}\right]_{m,m}, \tag{27}$$

where $\boldsymbol{u}_{u,n}$ is the nth column of $\boldsymbol{U}_{u,u}$, i.e., $\boldsymbol{U}_{u,u} = [\boldsymbol{u}_{u,1}, \boldsymbol{u}_{u,2}, \ldots, \boldsymbol{u}_{u,N}]$. Note that after replacing the rate with its DE, the evolution of the channel is reflected by the effective statistical CSI $\widetilde{\mathbf{\Omega}}_{u,u}$ shown in (12).

After applying the DE method to the ergodic rate, the problem objective becomes $\mathrm{EE}_u(\mathbf{\Lambda}_u) = \overline{R}_u(\Lambda_u)/P_u(\mathbf{\Lambda}_u)$, which exhibits a fractional form, thus the problem is non-convex. To facilitate further algorithm design, we convexify the problem. Charnes-Cooper transform and the Dinkelbach's method are two common techniques to tackle this fractional programming [42]. Yet, the Dinkelbach's method solves the problem in an iterative way, while the Charnes-Cooper's transform converts the problem into a concave one directly. Thus, we resort to the Charnes-Cooper's transform. Specifically, we introduce a new variable

$$\boldsymbol{X}_u = \frac{\xi_u P_{\max,u} + P_{\mathrm{B},u}}{P_{\max,u}} \frac{\mathbf{\Lambda}_u}{\xi_u \mathrm{tr}\left\{\mathbf{\Lambda}_u\right\} + P_{\mathrm{B},u}}, \tag{28}$$

where $P_{\mathrm{B},u} = MP_{\mathrm{c},u} + P_{\mathrm{s},u}$ and the coefficient $(\xi_u P_{\max,u} + P_{\mathrm{B},u})/P_{\max,u}$ is used to normalize the transmit power constraint so that $\mathrm{tr}\left\{\boldsymbol{X}_u\right\} \leqslant 1$ equals $\mathrm{tr}\left\{\mathbf{\Lambda}_u\right\} \leqslant P_{\max,u}$. Consequently, the corresponding feasible region for $\boldsymbol{X}_u$ is

$$\mathcal{X}_u = \left\{\boldsymbol{X}_u \mid \boldsymbol{X}_u \succeq \mathbf{0},\ \boldsymbol{X}_u \text{ diagonal and } \mathrm{tr}\left\{\boldsymbol{X}_u\right\} \leqslant 1\right\}. \tag{29}$$

Solving for $\mathbf{\Lambda}_u$ yields

$$\mathbf{\Lambda}_u = \frac{P_{\mathrm{B},u} P_{\max,u}}{P_{\mathrm{B},u} + \xi_u P_{\max,u}\left(1 - \mathrm{tr}\left\{\boldsymbol{X}_u\right\}\right)} \boldsymbol{X}_u. \tag{30}$$

Then, we substitute (30) into (23) and obtain the objective of the Charnes-Cooper's transformed problem as

$$c(\boldsymbol{X}_u) = \frac{P_{\mathrm{B},u} + \xi_u P_{\max,u}\left(1 - \mathrm{tr}\left\{\boldsymbol{X}_u\right\}\right)}{P_{\mathrm{B},u}\left(P_{\mathrm{B},u} + P_{\max,u}\right)} \log\det\left(\boldsymbol{I}_M + \frac{P_{\mathrm{B},u} P_{\max,u} \mathbf{\Xi}_u \boldsymbol{X}_u}{P_{\mathrm{B},u} + \xi_u P_{\max,u}\left(1 - \mathrm{tr}\left\{\boldsymbol{X}_u\right\}\right)}\right). \tag{31}$$

Given that $\overline{R}_u(\mathbf{\Lambda}_u)$ is concave over $\mathbf{\Lambda}_u$, which can be obtained from [26], the function $c(\boldsymbol{X}_u)$ is also a concave function on $\boldsymbol{X}_u$ [43]. In this way, the fractional EE maximization program is equivalently turned into a convex optimization problem as follows [44]:

$$\begin{aligned} \mathcal{P}_4: \quad & \max_{\boldsymbol{X}_u} c(\boldsymbol{X}_u) \\ & \text{s.t. } \boldsymbol{X}_u \in \mathcal{X}_u. \end{aligned} \tag{32}$$

In view of all this, to tackle the EE maximization problem, we will first figure out a no-regret transmit strategy $\boldsymbol{X}_u$ for problem $\mathcal{P}_4$, then calculate the power allocation matrix $\mathbf{\Lambda}_u$ by the inverse transformation in (30).

3.1 Learning with accurate CSI

We first consider the case where the attained statistical CSI is perfect. We resort to the OGA method, and its key idea is to track the gradient of the problem objective on the variable, then project back to the variable's feasible region [18]. Specifically, we write the derivative of $c(\boldsymbol{X}_u)$ to $\boldsymbol{X}_u$ as

$$\begin{aligned} \mathbf{\Gamma}_u &= \frac{\partial c\left(\boldsymbol{X}_u\right)}{\partial \boldsymbol{X}_u} \\ &= \frac{-\xi_u P_{\max,u} \overline{A}(\mathbf{\Lambda}_u)}{P_{\mathrm{B},u}\left(P_{\mathrm{B},u} + P_{\max,u}\right)} \boldsymbol{I}_M + \frac{P_{\mathrm{B},u} + \xi_u P_{\max,u}\left(1 - \mathrm{tr}\left\{\boldsymbol{X}_u\right\}\right)}{P_{\mathrm{B},u}\left(P_{\mathrm{B},u} + P_{\max,u}\right)} \frac{\partial \overline{A}(\mathbf{\Lambda}_u)}{\partial \boldsymbol{X}_u}, \end{aligned} \tag{33}$$

where $\overline{A}(\mathbf{\Lambda}_u) = \log\det\left(\boldsymbol{I}_M + \mathbf{\Xi}_u \mathbf{\Lambda}_u\right)$ denotes the first term of $\overline{R}_u(\mathbf{\Lambda}_u)$, and its gradient on $\boldsymbol{X}_u$ is

$$\frac{\partial \overline{A}(\mathbf{\Lambda}_u)}{\partial \boldsymbol{X}_u} = \frac{\partial \overline{A}(\mathbf{\Lambda}_u)}{\partial \mathbf{\Lambda}_u} \frac{\partial \mathbf{\Lambda}_u}{\partial \boldsymbol{X}_u}$$

$$= \boldsymbol{B}_u \frac{\xi_u P_{\mathrm{B},u} P_{\max,u}^2 \boldsymbol{X}_u}{\left(P_{\mathrm{B},u} + \xi_u P_{\max,u}\left(1 - \mathrm{tr}\left\{\boldsymbol{X}_u\right\}\right)\right)^2} + \boldsymbol{B}_u \frac{P_{\mathrm{B},u} P_{\max,u} \boldsymbol{I}_M}{P_{\mathrm{B},u} + \xi_u P_{\max,u}\left(1 - \mathrm{tr}\left\{\boldsymbol{X}_u\right\}\right)}, \tag{34}$$

and $\boldsymbol{B}_u = \frac{\partial \overline{A}(\boldsymbol{\Lambda}_u)}{\partial \boldsymbol{\Lambda}_u} = \left(\boldsymbol{I}_M + \boldsymbol{\Xi}_u \boldsymbol{\Lambda}_u\right)^{-1} \boldsymbol{\Xi}_u$. Then, by substituting (34) into (33), $\boldsymbol{\Gamma}_u$ can be rewritten as

$$\boldsymbol{\Gamma}_u = \frac{P_{\max,u}}{P_{\mathrm{B},u} + P_{\max,u}} \left(\boldsymbol{B}_u - \frac{\xi_u \overline{A}(\boldsymbol{\Lambda}_u)}{P_{\mathrm{B},u}} \boldsymbol{I}_M + \frac{\xi_u}{P_{\mathrm{B},u}} \boldsymbol{B}_u \boldsymbol{\Lambda}_u \right). \tag{35}$$

With the expression above, and reconsidering the stage index s, the gradient matrix $\boldsymbol{\Gamma}_u^s$ can be calculated at cell-u, utilizing the power allocation matrix of BS-u and the local effective statistical CSI at stage s. Note that $\boldsymbol{\Gamma}_u^s$ is a bounded function, and we assume that there exists a constant Γ_u that satisfies

$$\|\boldsymbol{\Gamma}_u^s\| \leqslant \Gamma_u, \ \forall s = 1, 2, \ldots, \tag{36}$$

where $\|\boldsymbol{\Gamma}\|$ denotes the Frobenius matrix norm of $\boldsymbol{\Gamma}$.

With the expression of the gradient, the OGA method is conducted as

$$\boldsymbol{X}_u^{s+1} = \boldsymbol{\Pi}_u(\boldsymbol{X}_u^s + \alpha_u^s \boldsymbol{\Gamma}_u^s), \tag{37}$$

which denotes that the problem solution at stage $s+1$ is calculated by $\boldsymbol{X}_u^s$ and the gradient $\boldsymbol{\Gamma}_u^s$ at stage s. The step size α_u^s is a non-increasing sequence of s. Note that $\boldsymbol{X}_u^s$ and $\boldsymbol{\Gamma}_u^s$ are both diagonal matrices; thus, the diagonal matrix-valued projection function $\boldsymbol{\Pi}_u\left(\boldsymbol{Y}\right)$ is presented as [18]

$$\boldsymbol{\Pi}_u\left(\boldsymbol{Y}\right) = \mathrm{diag}\left\{\boldsymbol{\pi}_u(y_1), \boldsymbol{\pi}_u(y_2), \ldots, \boldsymbol{\pi}_u(y_{MK_u})\right\}, \tag{38}$$

where

$$\boldsymbol{\pi}_u(y_m) = \begin{cases} 0, & \text{if } y_m < 0, \\ y_m, & \text{if } y_m \geqslant 0 \text{ and } \sum_{j=1}\left[y_j\right]^+ < 1, \\ \left[y_m - \beta\right]^+, & \text{if } y_m \geqslant 0 \text{ and } \sum_{j=1}\left[y_j\right]^+ \geqslant 1, \end{cases} \tag{39}$$

with $\beta > 0$ chosen to let $\sum_{m: y_m \geqslant 0}\left[y_m - \beta\right]^+ = 1$. Just as in [45], β can be obtained by sorting $\boldsymbol{Y}$ and performing the bisection method on β.

Building on the discussion above, we have obtained an online EE maximization algorithm, which is presented in Algorithm 1. Note that the algorithm is distributed. Specifically, each cell does not need to know the transmit covariance matrices or the CSI of other cells. On the contrary, BS-u only requires its actual local CSI $\boldsymbol{\Omega}_{u,u}$ and the aggregate interference-plus-noise covariance matrix $\widetilde{\boldsymbol{K}}_u$, which is a $N \times N$ diagonal matrix. So the exchanged information between cells is $N \times U$ real-valued scalar variables in one iteration. Moreover, in static cases where the UT's effective channel is constant during the transmission, some convex optimization arguments can be used to show that the solution of our algorithm converges to the static EE maximization problem in (15) [37, 46]. Therefore, the OGA method can be adopted as a low-complexity power allocation algorithm in static cases.

We now discuss the regret property of this method. Applying the same technique in [18], we can derive the following proposition.

Proposition 1. If the step sizes are set as $\alpha_u^s \to 0$ and $s\alpha_u^s \to \infty$, the proposed online transmit policy leads to no-regret for the online EE maximization problem $\mathcal{P}_2$ in (18). Specifically, the cumulative regret satisfies

$$\mathrm{Reg}\left(S\right) \leqslant \frac{1}{\alpha_u^S} + \frac{1}{2}\Gamma_u^2 \sum_{s=1}^{S} \alpha_u^s = o(S), \tag{40}$$

where Γ_u is shown in (36).

Given this proposition, we can say that the proposed method can achieve no-regret transmission. This no-regret property is also revealed by numerical results in Section 4. Note that the step sizes can be tuned to adjust the regret bound in (40). Specifically, if we choose the step sizes as $\alpha_u^s = \alpha s^{-b}$ for some $b \in (0, 1)$, the regret is rewritten as

$$\mathrm{Reg}\left(S\right) \leqslant \frac{S^b}{\alpha} + \frac{1}{2}\alpha\Gamma_u^2 \sum_{s=1}^{S} s^{-b}. \tag{41}$$

Algorithm 1 Robust online EE maximization algorithm

```
1: Initialization: Step size sequence α_u^s, s = 0, X_u^s, u = 1, ..., U;
2: while 1 do
3:     Get the statistical CSI Ω^s;
4:     for u = 1 to U do
5:         Get Λ_u^s by (30);
6:         Calculate the derivative Γ_u^s by (35);
7:         Calculate the projection variable β utilizing the bisection method;
8:         Update X_u^{s+1} with (37);
9:     end for
10:    Let s = s + 1.
11: end while
```

Remind that the term $\sum_{s=1}^{S} s^{-b} = \mathcal{O}(S^{1-b})$ for large S, where the symbol $\mathcal{O}$ stands for "order of", i.e., S^{1-b} is the asymptotic upper bound of $\sum_{s=1}^{S} s^{-b}$ when S tends to infinity. Then, the regret in (41) satisfies $\mathrm{Reg}\,(S) = \mathcal{O}(S^{\max(b,1-b)})$. Therefore, when $b = 1/2$, the regret guarantee reaches its optimal convergence behavior as $\mathrm{Reg}\,(S) \leqslant \sqrt{S}\left(1 + \alpha^2\Gamma_u^2\right)/\alpha = \mathcal{O}(\sqrt{S})$.

As for the computational complexity of Algorithm 1, at each stage s, calculating the power allocation matrices involves no iteration. In fact, the main complexity lies in the calculation of the gradient matrices $\mathbf{\Gamma}_u, \forall u \in \mathcal{U}$ and the projection process. Considering the matrix size, the gradient calculation complexity is $\mathcal{O}(M^{\omega})$, where the complexity exponent ω can be reduced to $\omega < 2.374$ if the improved Coppersmith-Winograd method is applied [47]. As for the projection process, the steps in (39) involve a complexity of $\mathcal{O}(M)$ [45]. So the total complexity of the proposed online algorithm is $\mathcal{O}(M^{\omega})$. For the centralized case, the gradient calculation complexity can reach $\mathcal{O}\left((UM)^{\omega}\right)$, and the projection causes a complexity of $\mathcal{O}\left(UM\right)$, which indicates that our distributed algorithm can greatly reduce the computational complexity.

3.2 Learning with imperfect CSI

So far, we have developed an online transmit policy under the assumption that the BSs can attain perfect effective CSI $\widetilde{\mathbf{\Omega}}_{u,u}$. However, in practical situations, factors such as pilot contamination can result in imperfect channel estimations. Therefore, we will estimate the robustness of our algorithm under imperfect CSI and feedback.

Notice that BS-u requires its local statistical CSI $\mathbf{\Omega}_{u,u}$ and the interference-plus-noise matrix $\widetilde{\boldsymbol{K}}_u$ to conduct the online procedure, which mainly involves calculating the gradient $\mathbf{\Gamma}_u^s$ in (37). To formulate the estimation error, we assume that at each stage s of the transmission, the BS obtains $\hat{\mathbf{\Gamma}}_u^s$, a noisy estimate of $\mathbf{\Gamma}_u^s$, which satisfies the following hypotheses:

$$\mathrm{E}\left\{\hat{\mathbf{\Gamma}}_u^s - \mathbf{\Gamma}_u^s\right\} = 0, \tag{42}$$

$$P\left(\|\hat{\mathbf{\Gamma}}_u^s - \mathbf{\Gamma}_u^s\| \geqslant z\right) \leqslant B/z^c, \quad \text{for some } B > 0 \text{ and for some } c > 2. \tag{43}$$

Both hypotheses are fairly mild and practical. The first hypothesis ensures unbiasedness, which means that the measurement of $\mathbf{\Gamma}_u^s$ contains no systematic error. Secondly, hypothesis (43) amounts to asking that the estimation error is bounded in an uncertain region [48]. Note that hypothesis (43) is satisfied by common error distributions, e.g., uniform, Gaussian/sub-Gaussian, log-normal and exponential distributed errors [49–51].

With the noisy estimation $\hat{\mathbf{\Gamma}}_u^s$ satisfying both hypotheses shown above, and with the step sizes set as $\alpha_u^s = \alpha/s^m$ for some $m \in (2/c, 1)$, we have the regret of the online transmit method under imperfect CSI as

$$\mathrm{E}\left\{\mathrm{Reg}\,(S)\right\} \leqslant \frac{1}{\alpha_u^S} + \frac{1}{2}\hat{\Gamma}_u^2 \sum_{s=1}^{S} \alpha_u^s, \tag{44}$$

where $\hat{\Gamma}_u^2 = \sup_s \mathrm{E}\{\|\hat{\mathbf{\Gamma}}_u^s\|^2\}$, i.e., $\hat{\Gamma}_u^2$ is the supremum of the set $\mathrm{E}\{\|\hat{\mathbf{\Gamma}}_u^s\|^2\}, \forall s$. The above regret property can be proved by the same mathematical process as in [18] and is omitted here.

Note that the constraint for step size parameter $m > 2/c$ suggests the tradeoff between the probability of suffering from high errors and attaining low regret. To be more specific, we first consider the case where the error distribution has heavy tails, which indicates that the estimation has a high probability of experiencing large errors. Then, hypothesis (43) will not be satisfied if $c \gg 2$, and the step size sequence

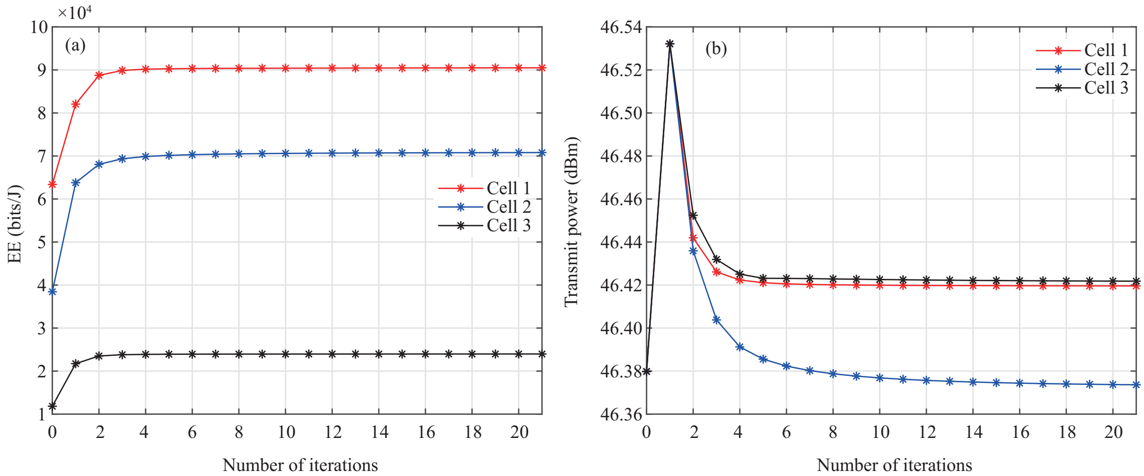

Figure 1 (Color online) EE convergence performance (a) and transmit power convergence performance (b) under $P_{\max,u}$ = 45 dBm.

is $\alpha_u^s = \alpha/s^m$ with the value of $m \in (2/c, 1)$ close to 1. Thus, the first term of (44) is almost linear, so cells will suffer from high regret. On the contrary, if the distribution tails are light, the scope for m is expanded, and the step sizes are more adaptive. Then, the algorithm guarantees a lower regret bound, which can be achieved by a standard Gaussian or exponentially error distribution. Moreover, similar to the accurate CSI case in Subsection 3.1, if the step sizes are set as $\alpha_u^s = \alpha s^{-1/2}$, the optimal regret bound becomes $\mathrm{E}\left\{\mathrm{Reg}\left(S\right)\right\} \leqslant \sqrt{S}(1 + \alpha^2\hat{\Gamma}_u^2)/\alpha$.

4 Numerical results

Numerical analysis is presented in this section to assess the performance of the online transmit algorithm in dynamic situations and its robustness to imperfect CSI. Our simulations consider a multi-cell case with $U = 3$ cells, and each contains one M-antenna BS and a N-antenna UT, with $M = 128$ and $N = 4$. The antenna spacing is half-wavelength. The WINNER II channel model [52] with suburban scenario and non-line-of-sight propagation is employed throughout the simulations. For the power consumption parameters, we set the amplifier inefficiency factor as $\xi_u = 5, \forall u$, $P_{\mathrm{c},u}$ and $P_{\mathrm{s},u}$ as 30 and 40 dBm, respectively. Following [13], the large scale fading factor is given as $10\log_{10}\left(\theta_{u,v}\right) = -38\log_{10}\left(d_{u,v} - 34.5 + c_{u,v}\right)$, where $d_{u,v}$ denotes the distance between BS-v and UT-u and $c_{u,v}$ is the zero-mean log-normal shadow fading.

The first simulation scenario considers the static channel conditions. As is discussed in Subsection 3.1, our online transmit method serves as a convex optimization algorithm in this case. Specifically, in each iteration, after the information exchange among cells, the UTs' effective CSI alters due to the change of the power allocation matrices of other cells. Then, the BSs conduct precoding design based on the new effective CSI until convergence. We choose the step size sequence as $\alpha_u^s \propto 1/\sqrt{s}$. The transmit power constraint is fixed as $P_{\max,u}$ = 45 dBm, $\forall u$. Then, the EE convergence performance is presented by Figure 1(a), and the corresponding transmission power evolution is shown in Figure 1(b). We can see that both the EE performance and the transmission power for all UTs converge within few iterations. The convergence phenomenon indicates that the system reaches a Nash equilibrium; i.e., there is no incentive for the BSs to change their power allocation strategies.

Then, we consider time-varying channels and evaluate the online transmit algorithm performance under mobility. We consider a situation where the UT's statistical CSI converts from $\boldsymbol{\Omega}^a$ to $\boldsymbol{\Omega}^b$, both of which are generated by the WINNER II channel model. Then, following the similar method in [53], a set of statistical CSI is generated as follows:

$$\boldsymbol{\Omega}^s = \rho\boldsymbol{\Omega}^{s-1} + (1-\rho)\boldsymbol{G}^b \odot (\boldsymbol{G}^b)^*,\ s = 1, 2, \ldots, \tag{45}$$

where $\boldsymbol{\Omega}^0 = \boldsymbol{\Omega}^a$, and $\boldsymbol{G}^b$ denotes the corresponding beam domain channel matrix of $\boldsymbol{\Omega}^b$. Moreover, the coefficient ρ is set according to the degree of non-stationarity of the channel. Here, we set it as $\rho = 0.8$. In Figure 2, we plot the average regret of UTs under the evolving channel, which shows that the UTs attain

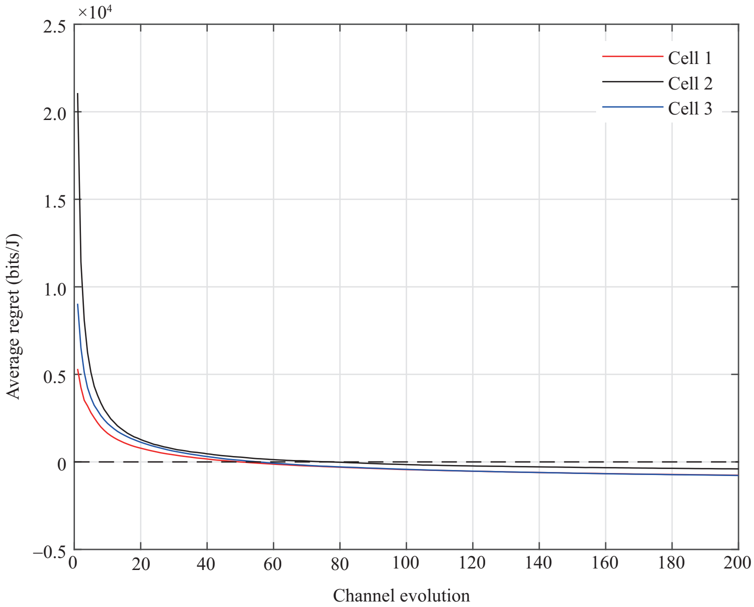

Figure 2 (Color online) User regret under the OGA algorithm.

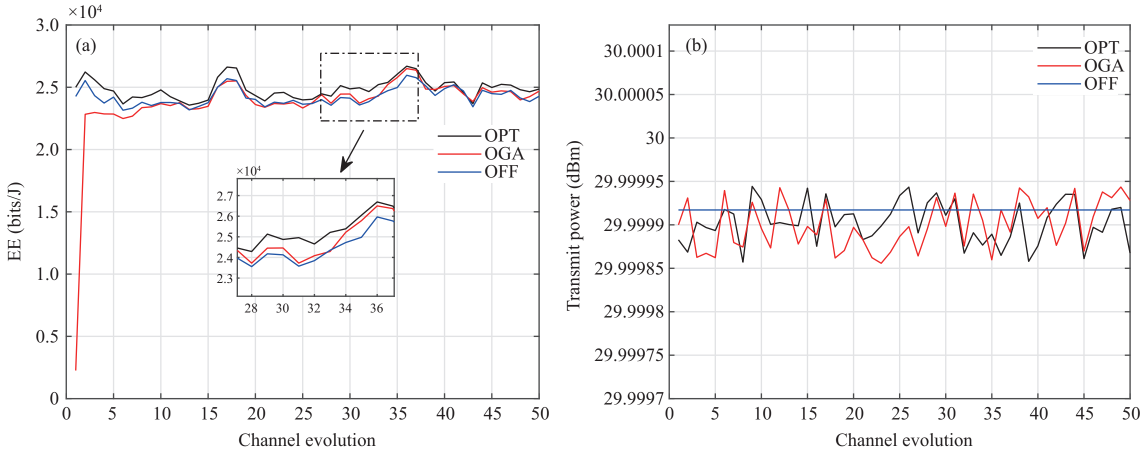

Figure 3 (Color online) Comparison between the OGA-based algorithm and the optimal approach on the EE performance (a) and the transmit power (b).

a no-regret state in a few iterations. After reaching the no-regret state, the regret remains negative, indicating that the proposed algorithm achieves a no-regret transmission.

To assess the EE performance of the proposed online OGA-based method, we compare it to (a) "OFF" approach: the solution to problem $\mathcal{P}_3$ in (20), which aims to maximize the average EE over the transmission time offline, i.e., before transmission; (b) "OPT" approach: the optimal transmit policy, where the BS selects a power allocation matrix at each instance time. Note that both approaches require knowledge of the future CSI, and the OPT method represents an ideal situation where the complicated optimization is done online. In Figures 3(a) and (b), we show the EE and the transmit power of UT-2 under the power constraint $P_{\max,2} = 30$ dBm, respectively. As depicted in Figure 3(a), the OGA method tracks the OPT method well and consistently outperforms the OFF method, which is also revealed by the negative regret shown in Figure 2.

Finally, the algorithm's robustness under estimation imperfection is shown in Figures 4(a) and (b). The simulation considers the static channel case, and the parameter settings align with that in Figure 1(a), except that the BSs only have access to imperfect CSI. To obtain a noisy version of the gradient, we add a Gaussian distributed noise of zero-mean and a standard deviation δ on the gradient $\mathbf{\Gamma}_u^s$. As can be seen in Figure 4(a), when we set δ as 20% of the corresponding gradient deviation, the EE performance is

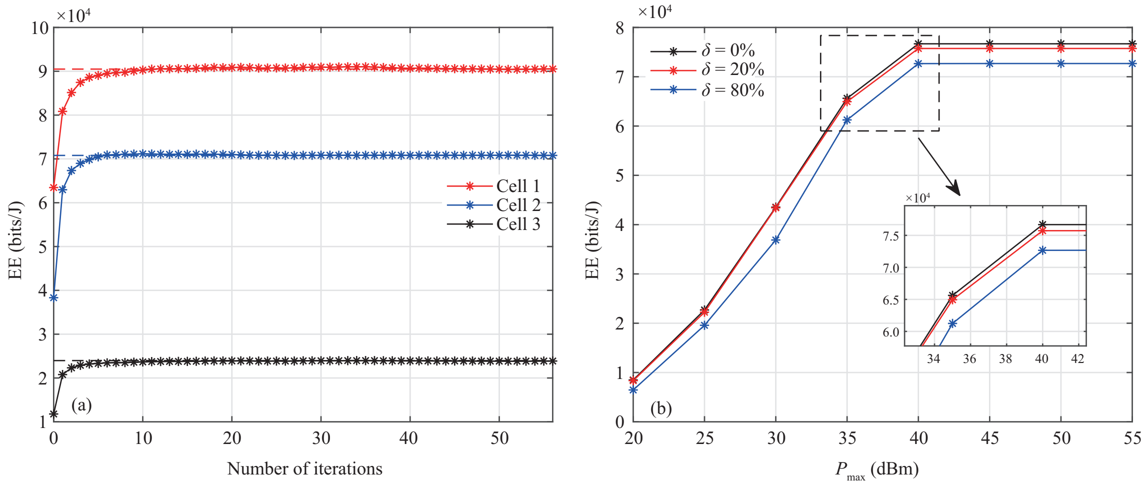

Figure 4 (Color online) (a) Performance of OGA method with Gaussian noise deviation $\delta = 20\%$; (b) the EE performance versus power budget $P_{\max}$ for different noise deviations.

barely affected by the imperfect estimation of the channel. In Figure 4(b), we show the EE performance versus the power budget $P_{\max}$ under different noise deviations, compared with the perfect CSI case. We can see that the proposed algorithm works well under different power budgets. When δ is set as 80% of the corresponding gradient deviation, the EE performance is affected adversely. However, the system still converges within a few iterations, and the EE experiences a drastic increase compared to the initial transmit policy.

5 Conclusion

We have studied the EE maximization precoding design in multi-cell massive MIMO systems with only statistical CSI. To adjust to the varying CSI, we proposed an online EE maximization algorithm and showed its robustness to estimation errors. We first constructed the online EE optimization problem in a distributed way to reduce the backhaul burden. With the Charnes-Cooper transform, we addressed the fractional form of the problem objective, together with the random matrix theory to lower the computational complexity. We further utilized the OGA method to construct this no-regret power allocation strategy based on all past channel information. We showed that the algorithm is robust to estimation error if the error satisfies certain statistical characteristics. Our simulation results showed the no-regret property and the robustness to imperfect CSI of our proposed algorithm.

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