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Quantum algorithm for kernelized correlation filter

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Dear editor,

Kernelized correlation filter (KCF) is a key component of target tracking that is aimed to distinguish the target from the surrounding environment and is a fundamental problem in computer vision. Motivated by the progress in quantum computing, we present a quantum algorithm for KCF based on the core algorithm proposed by Henriques et al. [1, 2]. This algorithm comprises two stages: training and detection. In the training stage, two quantum sub-algorithms, i.e., an improved quantum Fourier transform in computation basis (QFTC) and a quantum algorithm for kernel correlation are presented. Then, based on these two quantum sub-algorithms, a quantum kernelized correlation filter is trained in the frequency domain. In the detection stage, the filter is then used to generate a quantum state whose amplitudes encode the response score for a testing base sample. By theoretical analysis, our quantum algorithm is shown to have an exponential advantage over the classical counterpart under certain conditions. This study shows the power of quantum computing over classical computing in terms of the execution of algorithms based on circulant matrices.

In the two-dimensional (2D) case, given an $M \times N$ training base sample $\boldsymbol{x}$ and performing dense sampling on it, we can obtain a block circulant matrix with circulant blocks (BCCB, i.e., a matrix that is circulant at the block level, and is composed of blocks each of which is also circulant):

$$X = \mathrm{BCCB}(\boldsymbol{x}) = F2^* \mathrm{diag}\left(\mathrm{vec}\left(\hat{\boldsymbol{x}}\right)\right) F2, \tag{1}$$

where $F2^*$ is the complex-conjugate of $F2$ with $F2 = F_M \otimes F_N$, $F_{M(N)}$ is the discrete Fourier transform (DFT) matrix of the order $M(N)$ and $\otimes$ is the Kronecker product. $\mathrm{vec}\left(\cdot\right)$ and $\mathrm{diag}\left(\cdot\right)$ represent vectorization and diagonalization operations, respectively. $\hat{\boldsymbol{x}}$ denotes the DFT of the sample $\boldsymbol{x}$.

In the case of non-linearity, the objective function of ridge regression and its solution can be written as [3]

$$\min_{\boldsymbol{\alpha}} \|K\boldsymbol{\alpha} - \boldsymbol{y}\|_2^2 + \delta \boldsymbol{\alpha}^{\mathrm{T}} K \boldsymbol{\alpha}, \tag{2}$$

and

$$\boldsymbol{\alpha} = \left(K + \delta I\right)^{-1} \boldsymbol{y}, \tag{3}$$

where K is a kernel matrix, and also a BCCB matrix $K = \mathrm{BCCB}\left(\boldsymbol{k}^{xx}\right)$, where the element of the kernel correlation $\boldsymbol{k}^{xx}$ is $\kappa\left(\boldsymbol{x}, P^i \boldsymbol{x} Q^j\right)$, $\kappa(\,)$ represents the kernel function, P and Q are permutation matrices. Therefore, according to the special properties of the circulant matrix, we can rewrite formula (3) as [1, 2, 4]

$$\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{y}} \ominus \left(\hat{\boldsymbol{k}}^{xx} + \delta\right), \tag{4}$$

where $\ominus$ denotes an element-wise division.

Naturally, given a testing base sample $\boldsymbol{z}$, we can calculate its detection response value as [1, 2]

$$\boldsymbol{y}' = \mathcal{F}^{-1}\left(\hat{\boldsymbol{k}}^{zx} \odot \hat{\boldsymbol{\alpha}}\right), \tag{5}$$

where $\hat{\boldsymbol{k}}^{zx}$ is the kernel correlation of $\boldsymbol{z}$ and $\boldsymbol{x}$.

In Appendix A, we have reviewed and supplemented the original classic KCF algorithm and the algorithm program is briefly shown as Algorithm 1.

Algorithm 1 The classical KCF algorithm

Input:
$\boldsymbol{x}$: training base sample, $M \times N$;
$\boldsymbol{y}$: regression target, $M \times N$;
$\boldsymbol{z}$: testing base sample, $M \times N$.
Output:
$\boldsymbol{y}'$: detection score, $M \times N$.
1: Kernel correlation, $\boldsymbol{k}^{xx} = f\left(\mathcal{F}^{-1}\left(\hat{\boldsymbol{x}}^* \odot \hat{\boldsymbol{x}}\right)\right)$ and $\boldsymbol{k}^{zx} = f\left(\mathcal{F}^{-1}\left(\hat{\boldsymbol{z}}^* \odot \hat{\boldsymbol{x}}\right)\right)$;
2: Filtering parameter, $\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{y}} \ominus (\hat{\boldsymbol{k}}^{xx} + \delta)$;
3: Response detection, $\boldsymbol{y}' = \mathcal{F}^{-1}(\hat{\boldsymbol{k}}^{zx} \odot \hat{\boldsymbol{\alpha}})$.

In this study, we present a quantum version of the KCF algorithm. First of all, we improve the QFTC algorithm, which will exist as a sub-algorithm of the subsequent quantum algorithm. That is, we have the following theorem.

Lemma 1. Given an Oracle $O_p : |00\rangle \to \sum_{ij} p_{ij} |ij\rangle$ that can be executed in time $\mathcal{O}\left(\mathrm{polylog}\left(MN\right)\right)$, the quantum

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QFTC algorithm that enables the digital encoding of Fourier transform coefficients

$$\frac{1}{\sqrt{MN}}\sum_{kl}|kl\rangle|0\rangle \rightarrow \frac{1}{\sqrt{MN}}\sum_{kl}|kl\rangle|q_{kl}\rangle, \tag{6}$$

can be implemented in time $\mathcal{O}(\frac{\text{polylog}(MN)}{\epsilon})$ within error ϵ, where $q_{kl}=\frac{1}{\sqrt{MN}}\sum_{ij}p_{ij}\mathrm{e}^{-2\pi\mathrm{i}\left(\frac{ik}{M}+\frac{jl}{N}\right)}$. Actually, this improved algorithm is more simplified in its execution process than the corresponding quantum algorithm [5]. (Appendix B.1 provides the proof of Lemma 1)

Based on the above lemma, we have designed a quantum algorithm to calculate the kernel correlation. In short, we have the following lemma (the proof can be seen in Appendix B.2):

Lemma 2. Suppose we can access $\frac{\sum_{ij}x_{ij}|ij\rangle}{\|\boldsymbol{x}\|_F}$ in time $\mathcal{O}(\text{polylog}(MN))$, and then the kernel correlation can be obtained in the quantum state form by the quantum algorithm

$$|00\rangle \rightarrow \frac{1}{\sqrt{\sum_{ij}\chi_{ij}^{*}\chi_{ij}}}\sum_{ij}\chi_{ij}|ij\rangle \tag{7}$$

with time complexity $\mathcal{O}(\frac{\varrho_0\varrho_X\text{poly}\log(MN)}{\epsilon^2})$, where $\chi_{ij}=f(\frac{1}{\sqrt{MN\sum_{kl}(\hat{x}_{kl}^{*}\hat{x}_{kl})^2}}\sum_{kl}\hat{x}_{kl}^{*}\hat{x}_{kl}\mathrm{e}^{2\pi\mathrm{i}(\frac{ik}{M}+\frac{jl}{N})})$, $\hat{x}_{kl}=\frac{1}{\|\boldsymbol{x}\|_F\sqrt{MN}}\sum_{ij}x_{ij}\mathrm{e}^{-2\pi\mathrm{i}(\frac{ik}{M}+\frac{jl}{N})}$, ϱ_X is the condition number of the data matrix X and $\varrho_0=\frac{\max_{ij}|\chi_{ij}|}{\min_{ij}|\chi_{ij}|}$.

This quantum algorithm for kernel correlation will participate as a sub-algorithm during the parameter training phase and the detection response phase.

Based on the quantum algorithm for kernel correlation, we can train to obtain the filter parameters in the quantum state form, which means we can achieve (details can be found in Appendix B.3)

$$|\boldsymbol{y}\rangle \rightarrow \sum_{kl}\hat{\alpha}_{kl}|kl\rangle \tag{8}$$

in time $\mathcal{O}(\frac{\varrho_x\varrho_0\varrho_X\text{poly}\log(MN)}{\epsilon_x^3})$ within error ϵ_x, where ϱ_x denotes the condition number of the kernel matrix $\text{BCCB}(\boldsymbol{k}^{xx})$.

In the next stage of response detection, we can realize

$$\sum_{kl}\hat{\alpha}_{kl}|kl\rangle \rightarrow \sum_{kl}\hat{y}_{kl}^{'}|kl\rangle, \tag{9}$$

with time complexity $\mathcal{O}(\varrho_z\text{poly}\log(MN)(\frac{\varrho_x\varrho_0\varrho_X}{\epsilon_x^3}+\frac{\varrho_1\varrho_Z}{\epsilon_z^3}))$, where ϱ_z represents the condition number of the kernel matrix $\text{BCCB}(\boldsymbol{k}^{zx})$, ϱ_Z is the condition number of the data matrix Z, $\frac{\max_{ij}|\varkappa_{ij}|}{\min_{ij}|\varkappa_{ij}|}=\varrho_1$ and $\varkappa_{ij}=f(\frac{1}{\sqrt{MN\sum_{kl}(\hat{z}_{kl}^{*}\hat{x}_{kl})^2}}\sum_{kl}\hat{z}_{kl}^{*}\hat{x}_{kl}\mathrm{e}^{2\pi\mathrm{i}(\frac{ik}{M}+\frac{jl}{N})})$. And then a 2D quantum Fourier transform (QFT) is performed on the quantum state in (9) to obtain the quantum state $|\boldsymbol{y}'\rangle$ that encodes the detection response value (see Appendix B.4 for more details).

The procedure of the quantum KCF algorithm can be summarized as Algorithm 2. The specific implementation details of the entire quantum KCF algorithm are given in Appendix B. And the total time complexity is $\mathcal{O}(\varrho_z\text{poly}\log(MN)(\frac{\varrho_x\varrho_0\varrho_X}{\epsilon_x^3}+\frac{\varrho_1\varrho_Z}{\epsilon_z^3}))$.

Algorithm 2 The quantum KCF algorithm

Input:
The data matrices $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$ are stored in the appropriate data structure, respectively.

Output:
$|\boldsymbol{y}'\rangle$: a quantum state that encodes the detection response value in the amplitude.

1: Prepare a quantum state $|\boldsymbol{y}\rangle$;
2: Train parameters to obtain $\sum_{kl}\hat{\alpha}_{kl}|kl\rangle$;
3: Detect response to get $|\boldsymbol{y}'\rangle$.

In contrast, the complexity of the classic KCF algorithm is $\mathcal{O}(MN\log(MN))$. Without loss of generality, let $\epsilon_x=\epsilon_z=\Theta(\epsilon)$, so the complexity of our quantum algorithm can be rewritten as $\mathcal{O}(\frac{\varrho_z\text{poly}\log(MN)}{\epsilon^3}(\varrho_x\varrho_0\varrho_X+\varrho_1\varrho_Z))$. If we have $\epsilon=\varrho_0=\varrho_1=\varrho_x=\varrho_z=\varrho_X=\varrho_Z=\mathcal{O}(\text{poly}\log(MN))$, then it is obvious that the proposed quantum algorithm can achieve an exponential acceleration relative to the classic KCF algorithm. The comparison between the proposed quantum KCF algorithm in this study and the classical KCF algorithm can be seen in Table 1.

Table 1 Comparisons between the classical KCF algorithm and quantum KCF algorithm

Algorithms	Inputs	Outputs	Time complexity
Classical	$\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\delta$	$\boldsymbol{y}'$	$\mathcal{O}(MN\log(MN))$
Quantum	$O_{x,y,z},\delta$	$\|\boldsymbol{y}'\rangle$	$\mathcal{O}(\frac{\varrho_z\text{poly}\log(MN)}{\epsilon^3}(\varrho_x\varrho_0\varrho_X+\varrho_1\varrho_Z))$

In essence, our algorithm mainly takes advantage of the special properties of circulant matrices. We hope that the proposed quantum algorithm can inspire more algorithms related to circulant matrices, and shed some light on the design of quantum algorithms in the field of computer vision.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Henriques J F, Caseiro R, Martins P, et al. Exploiting the circulant structure of tracking-by-detection with kernels. In: Proceedings of European Conference on Computer Vision. Berlin: Springer, 2012. 702–715
2 Henriques J F, Caseiro R, Martins P, et al. High-speed tracking with kernelized correlation filters. IEEE Trans Pattern Anal Mach Intell, 2015, 37: 583–596
3 Rifkin R, Yeo G, Poggio T. Regularized least-squares classification. Nato Science Series Sub Series III Computer and Systems Sciences, 2003, 190: 131–154
4 Gray R M. Toeplitz and circulant matrices: a review. FNT Commun Inf Theor, 2005, 2: 155–239
5 Zhou S S, Loke T, Izaac J A, et al. Quantum Fourier transform in computational basis. Quantum Inf Process, 2017, 16: 82