SCIENCE CHINA
Information Sciences

• LETTER •

February 2023, Vol. 66 129301:1–129301:2
https://doi.org/10.1007/s11432-022-3623-y

Reduced-search guessing random additive noise decoding of polar codes

Kefan WANG[1], Yuejun WEI[2], Zhenyuan CHEN[2], Huarui YIN[3], Liping LI[1*] & Wenyi ZHANG[3*]

[1]*Key Laboratory of Intelligent Computing and Signal Processing of the Ministry of Education of China, Anhui University, Hefei* 230601*, China;*
[2]*Huawei Technologies, Shanghai* 201206*, China;*
[3]*Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei* 230022*, China*

Received 3 June 2022/Revised 14 September 2022/Accepted 16 November 2022/Published online 11 January 2023

Citation Wang K F, Wei Y J, Chen Z Y, et al. Reduced-search guessing random additive noise decoding of polar codes. Sci China Inf Sci, 2023, 66(2): 129301, https://doi.org/10.1007/s11432-022-3623-y

Guessing random additive noise decoding (GRAND) has attracted researchers since its introduction in [1]. For an additive noise channel with hard decision output, the algorithm checks all error patterns, and it schedules its search in descending likelihood order. Ordered reliability bit GRAND (ORBGRAND) [2] is a variant of GRAND. When soft channel output is available, it sorts the received symbols, calculates logistic weights, and schedules the error patterns in the order of their reliability values. For high-rate codes, GRAND and its variants can find a valid codeword quickly [1, 2]; but for low-rate codes, they typically require an exceedingly large number of queries even for small block lengths.

In this study, an inherent nature of polar codes identified in a previous work [3] is leveraged to re-design GRAND decoders. We prove that the error patterns do not need to be tested over all the N code symbols. Instead, NR code symbols are sufficient to generate information sequences, where R is the code rate. In this way, the search space of error patterns is reduced from 2^N to 2^{NR}. This reduction of the search space renders GRAND-family decoders of polar codes the ability to decode low-rate codes as well as high-rate codes. We emphasize that there is no performance loss as the reduction of the search space only removes redundant error patterns, without loss of completeness.

Model and background. Let x, $\boldsymbol{x}$, $\boldsymbol{X}$ denote a scalar, a vector, and a matrix, respectively. All vectors are row vectors, and x_i denotes the i-th element of $\boldsymbol{x}$. For a set $\mathcal{A}$, the subvector $\boldsymbol{u}_\mathcal{A}$ of a vector $\boldsymbol{u}$ takes elements of $\boldsymbol{u}$ with indices from $\mathcal{A}$. The information $\boldsymbol{u} \in \boldsymbol{F}_2^K$ is encoded into a codeword $\boldsymbol{x} \in \boldsymbol{F}_2^N$. When a code word $\boldsymbol{x}$ is transmitted over a noisy channel, the channel output is denoted by $\boldsymbol{y}$ and the hard-decision vector is denoted by $\tilde{\boldsymbol{y}} = \mathrm{sign}(\boldsymbol{y})$.

For polar codes, a codeword can be expressed by $\boldsymbol{x} = \boldsymbol{u}_\mathcal{A}\boldsymbol{G}_\mathcal{A} + \boldsymbol{u}_{\bar{\mathcal{A}}}\boldsymbol{G}_{\bar{\mathcal{A}}}$, where $\boldsymbol{G}_\mathcal{A}$ is the submatrix of the generating matrix $\boldsymbol{G}$ with rows selected from the set $\mathcal{A}$. The codeword $\boldsymbol{x}$ can be split into two subvectors $\boldsymbol{x}_\mathcal{A}$ and $\boldsymbol{x}_{\bar{\mathcal{A}}}$: $(\boldsymbol{x}_\mathcal{A}, \boldsymbol{x}_{\bar{\mathcal{A}}}) = (\boldsymbol{u}_\mathcal{A}\boldsymbol{G}_{\mathcal{A}\mathcal{A}} + \boldsymbol{u}_{\bar{\mathcal{A}}}\boldsymbol{G}_{\bar{\mathcal{A}}\mathcal{A}},\ \boldsymbol{u}_\mathcal{A}\boldsymbol{G}_{\mathcal{A}\bar{\mathcal{A}}} + \boldsymbol{u}_{\bar{\mathcal{A}}}\boldsymbol{G}_{\bar{\mathcal{A}}\bar{\mathcal{A}}})$. The matrix $\boldsymbol{G}_{\mathcal{A}\mathcal{A}}$ with elements $\{\boldsymbol{G}_{i,j}\}$ $(i, j \in \mathcal{A})$ is a submatrix selected from $\boldsymbol{G}$. It has been proven in [3] that the inverse matrix of $\boldsymbol{G}_{\mathcal{A}\mathcal{A}}$ is itself and that the submatrix $\boldsymbol{G}_{\bar{\mathcal{A}}\mathcal{A}} = \boldsymbol{0}$, resulting in $\boldsymbol{x}_\mathcal{A} = \boldsymbol{u}_\mathcal{A}\boldsymbol{G}_{\mathcal{A}\mathcal{A}}$. This means that the information bits $\boldsymbol{u}_\mathcal{A}$ can be calculated by

$$\boldsymbol{u}_\mathcal{A} = \boldsymbol{x}_\mathcal{A}\boldsymbol{G}_{\mathcal{A}\mathcal{A}}^{-1} = \boldsymbol{x}_\mathcal{A}\boldsymbol{G}_{\mathcal{A}\mathcal{A}}. \quad (1)$$

Theorem 1 reveals how the relationship (1) can improve the GRAND-family polar decoding.

Theorem 1. Given a guessed error sequence $\boldsymbol{e} = (e_1, e_2, \ldots, e_N)$, $e_i \in \{0, 1\}$, to decode $\hat{\boldsymbol{u}} = (\tilde{\boldsymbol{y}} \oplus \boldsymbol{e})\boldsymbol{G}$, it is sufficient to restrict the errors in $\boldsymbol{e}$ to occur within the subvector $\tilde{\boldsymbol{y}}_\mathcal{A}$.

The proof of Theorem 1 is provided in Appendix A. What Theorem 1 implies is that instead of querying the errors through the N received samples, it is sufficient to query the errors at positions in $\mathcal{A}$, thus reducing the search space from N positions to $K = NR$ positions. With Theorem 1, Corollary 1 follows immediately.

Corollary 1. To decode the information bits $\hat{\boldsymbol{u}}_\mathcal{A}$, the dimension of the search space is 2^{NR}.

Corollary 1 provides a tool for GRAND-family decoders since it enables these decoders the capability of handling low-rate codes. For comparison, the original GRAND-family decoders were declared to be efficient only for high-rate codes [1, 2].

Corollary 2. Define set $\mathcal{E}_I$ containing error patterns with errors only within information bit positions, and set $\mathcal{E}_F$ containing error patterns with errors occurring only within frozen bit positions. Consider three error patterns: $\boldsymbol{e}' \in \mathcal{E}_I$, $\boldsymbol{e}'' \in \mathcal{E}_F$, and $\boldsymbol{e} = \boldsymbol{e}' + \boldsymbol{e}''$. If $\boldsymbol{e}'$ produces a correct

* Corresponding author (email: liping_li@ahu.edu.cn, wenyizha@ustc.edu.cn)

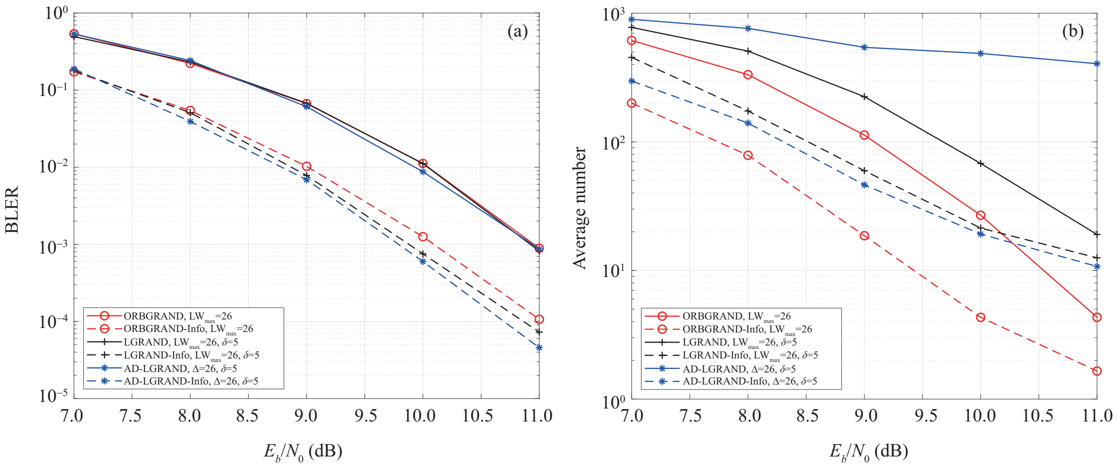

Figure 1 (Color online) Performance of ORBGRAND and its variants for (128, 32) polar codes with 11 CRC bits, where AD-LGRAND is an adaptive searching algorithm in Appendix C, $\mathrm{LW}_{\max}$ and Δ are maximum allowed logistic weights, and δ is the step size to increase the logistic weight (for their specific definitions see Appendix C). (a) BLER; (b) the average number of queries.

information sequence $\hat{\boldsymbol{u}}_{\mathcal{A}}$, then $\boldsymbol{e}$ also produces the same correct information sequence $\hat{\boldsymbol{u}}_{\mathcal{A}}$.

The proof of Corollary 2 is provided in Appendix B. What Corollary 2 shows is that if an error pattern $\boldsymbol{e}' \in \mathcal{E}_I$ can produce a valid information sequence and if the querying process does not stop, then all the error patterns like $\boldsymbol{e} = \boldsymbol{e}' + \boldsymbol{e}''$ still produce the same valid information sequence. Therefore, for list versions of GRAND-family (LGRAND) [4] decoders of polar codes, these redundant queries can be avoided, and it is sufficient to restrict the queries within information bit positions only, as Theorem 1 suggests. The improved list versions of GRAND-family decoders of polar codes are provided in Appendix C.

Simulation. The simulation parameters are listed in Appendix D. Decoders with reduced search space according to Theorem 1 are denoted by the suffix 'Info'. Figure 1(a) shows the block error rate (BLER) performance of ORBGRAND variants. The average numbers of queries are shown in Figure 1(b). Polar code is a (128, 32) code with 11 CRC bits. At a target BLER of 10^{-3}, the proposed ORBGRAND-Info achieves a 1 dB gain compared with the original ORBGRAND. In the mean time, at $E_b/N_0 = 11$ dB (corresponding to BLER of 10^{-3} for ORBGRAND), the average number of queries of ORBGRAND-Info is only half of that of ORBGRAND. By searching over positions within information bits, both LGRAND-Info and AD-LGRAND-Info provide stable improvements compared with corresponding decoders searching over the entire codeword. More simulation results can be found in Appendix D.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant No. 62071002) and National Key Research and Development Program of China (Grant No. 2018YFA0701603).

Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Duffy K R, Li J, Medard M. Capacity-achieving guessing random additive noise decoding. IEEE Trans Inform Theor, 2019, 65: 4023–4040

2 Duffy K R. Ordered reliability bits guessing random additive noise decoding. In: Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing, 2021

3 Li L, Zhang W. On the encoding complexity of systematic polar codes. In: Proceedings of the 28th IEEE International System-on-Chip Conference, 2015. 415–420

4 Abbas S M, Jalaleddine M, Gross W J. List-GRAND: a practical way to achieve maximum likelihood decoding. IEEE Trans Very Large Scale Integr Syst, 2021, doi:10.1109/TVLSI.2022.3223692