SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

February 2023, Vol. 66 122201:1–122201:18
https://doi.org/10.1007/s11432-021-3460-y

Fully actuated system approaches for continuous-time delay systems: part 2. Systems with input delays

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Received 3 January 2022/Revised 23 February 2022/Accepted 14 March 2022/Published online 5 January 2023

Abstract In this paper, the fully actuated system (FAS) approaches for continuous-time systems with time-varying state delays and a constant input delay are presented. Two types of continuous-time high-order FASs are proposed: single-order FASs with both state and input delays and multi-order FASs with both state and input delays. Controllers for both types of time-delay FASs are designed based on the full-actuation features of the systems. Unlike the case of FASs with state delays only, a prediction scheme is required and constructed for both types of FASs with input delays. Similar to the case of FASs with state delays only, constant linear closed-loop systems with arbitrarily assignable eigenstructures are also developed. Illustrative examples are provided to demonstrate the effect of the proposed theories.

Keywords time-delay systems, fully actuated systems, controller designs, input delays, time-varying delays

Citation Duan G R. Fully actuated system approaches for continuous-time delay systems: part 2. Systems with input delays. Sci China Inf Sci, 2023, 66(2): 122201, https://doi.org/10.1007/s11432-021-3460-y

1 Introduction

First, let us present a brief sketch of the development of dynamical control systems with time delays.

1.1 Time-delay systems with input delays

Delays widely exist in many practical systems, such as chemical process systems, hydraulically actuated systems, and combustion systems. In general, delays occur in all systems where instant signal transmission is not realizable.

Time-delay systems are very challenging [1, 2]. To control a system with input delays, there are mainly two ways: to construct a predictor with which the required future states are produced and to compensate for the input delays via certain mechanisms (e.g., the Smith predictor).

For linear systems, when state and/or input delays are relatively small compared with the plant's time scale, the finite dimensional feedback laws designed by standard algorithms (e.g., pole assignment and linear quadratic regulator) are still applicable [3]. If the input delays are relatively large, a Smith predictor can be applied for the compensation. For a single-input single-output system with a pure (long) input delay, a Smith predictor is proposed in [4, 5] to compensate for the delay effect so that the closed-loop system acts like a delay-free system. However, the system is required to be open-loop stable; otherwise, the Smith predictor may fail to work. In [6, 7], Smith's work is extended to the multi-input multi-output and the open-loop unstable systems, and improved predictor feedback approaches have been proposed. Recent advances in this direction can be found in [8].

Stabilization for nonlinear systems has always been a challenging problem, whereas this problem for nonlinear time-delay systems is even more challenging and thus has attracted much attention. Particularly, for nonlinear systems with input delays, the problems of stability analysis and stabilization have been investigated in [9–15]. Moreover, some efforts [16, 17] have considered nonlinear systems with both

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input delays and state delays. To control nonlinear systems with input delays, rather than conducting the prediction of required future states, the idea of designing the control laws that compensate for the input delay has also been widely applied (see [9–11]). Moreover, Refs. [16, 18, 19] use the nonlinear small-gain theorem and Lyapunov-Krasovskii methodology to guarantee input-to-state stability for nonlinear systems with input delays.

Investigations on the analysis and control of time-delay systems have mostly concentrated on the case of constant delays [20], whereas the problems for systems with time-varying delays definitely turn out to be more challenging [21]. Perhaps the earliest results on dynamical systems with time-varying input delays are presented by Nihtila [22, 23]. Quite some attempts have been made to tackle dynamical systems with time-varying delays; among them, the problem of compensation of long time-varying input delays has been treated for both linear systems [24, 25] and nonlinear systems [9, 26, 27].

1.2 Fully actuated system approach

In contrast to the well-known state-space approaches, a new approach was recently proposed for nonlinear system control, called the fully actuated system (FAS) approach and originally initiated by two series of studies (see [28–30] and [31–40]).

Due to various existing physical laws, many dynamical systems are originally modeled as FASs, and those which are not originally modeled as FASs can also be converted into FAS models, provided that they are controllable (in a certain sense). Among state-space systems without time delays, it has been proven that all the following can be converted into FASs:

(1) linear controllable systems, including both the continuous-time systems [29, 41] and discrete-time systems [40];

(2) feedback linearizable systems, including both the continuous-time systems [28] and discrete-time systems [40];

(3) strict-feedback systems, including both the continuous-time systems [28, 32] and discrete-time systems [40], both with time-varying delays; and

(4) some more general types of nonlinear systems [29, 31, 40].

It has been shown that once an FAS model of a dynamical system is derived, a controller can then be easily designed to cancel out the nonlinearities in the system and thus results in a constant linear closed-loop system [28, 31, 37].

Very recently, the FAS approach is generalized to the case of discrete-time systems with delays [42, 43], as well as the case of continuous-time systems with state delays only [44]. Thus far, among time-delay systems in a state-space form, it has been proven that the following can be converted into FASs:

(1) all linear controllable discrete-time systems with both state and input constant delays [42, 43];

(2) all discrete-time strict-feedback systems with time-varying state and input delays [42, 43]; and

(3) all continuous-time strict-feedback systems with time-varying state delays [44].

This study aims to further generalize the FAS approach to continuous-time systems with both state and input delays. It is shown that all linear controllable continuous-time systems with a constant input delay can also be converted into time-delay FASs, and so are all continuous-time strict-feedback systems with time-varying state and input delays. With these motivations and inspired by previous work on the FAS approach, two general continuous-time nonlinear FASs with time delays are proposed: one is the single-order affine and non-affine FASs with time-varying state delays and a constant input delay, the other is the multi-order affine and non-affine FASs with time-varying state delays and a constant input delay.

Different from the work in the first part of this paper series, that is, the work on FASs with state delays only, the feedback controller of an FAS with input delays involves future information of the states, which should be either predicted or compensated. In this paper, a general prediction scheme is presented, which produces the future states required in the controller. Similar to the case of FASs with state delays only, it is shown that a controller for an FAS with time-varying state delays and a constant input delay can also be easily designed with the help of the designed predictor, such that a linear closed-loop system with a completely assignable eigenstructure is obtained.

In the sequential sections, the notations introduced in Section 1 of [44] are also used. Furthermore, some basic facts given in [44] are also used. Therefore, readers are strongly recommended to go to the first part of the paper [44] before reading this second part.

The remainder of this paper is organized as follows. Section 2 provides the motivations for proposing time-delay FASs. Section 3 proposes continuous-time time-delay FASs with single-order and multi-order. Controllers for the proposed single-order and multi-order time-delay FASs are provided in Sections 4 and 5, respectively. An illustrative example is presented in Section 6, followed by a brief concluding remark in Section 7.

2 Motivations

Before presenting the general FAS models with time delays, let us first mention the motivations.

State-space models may not be the best ones to control dynamical systems. It has been demonstrated that FAS models are much more effective and convenient [28–40]. Very recently, the FAS models of discrete-time delay systems [42,43] and those of continuous-time systems with state delays only [44] have been proposed. Before presenting the FAS models for continuous-time systems with input delays, we state here two aspects of our motivations.

2.1 Case of linear systems

In this subsection, it is shown that a continuous-time system with an input delay can be converted into the form of a continuous-time FAS.

Consider the following continuous-time linear delay system in a state-space form:

$$\dot{x}\left(t\right)=A_{0}x\left(t\right)+B_{0}u\left(t-h\right), \tag{1}$$

where $x\in\mathbb{R}^{n}$ and $u\in\mathbb{R}^{r}$ are the state vector and input vector, respectively, h is a nonnegative scalar which represents the time delay in the control vector, and $A_0\in\mathbb{R}^{n\times n}$ and $B_0\in\mathbb{R}^{n\times r}$ are two coefficient matrices such that (A_0,B_0) is a controllable pair.

Note that the controllability of the system (1) with $h\neq 0$ is equivalent to that of the system with $h=0$. Applying the Theorem 3.1 in [40] to the above system (1) immediately gives the following result.

Proposition 1. The linear time-delay system (1) is controllable if and only if it can be converted equivalently into a system in the form of

$$\begin{bmatrix} x_1^{(\mu_1)}\left(t\right) \\ x_2^{(\mu_2)}\left(t\right) \\ \vdots \\ x_\eta^{(\mu_\eta)}\left(t\right) \end{bmatrix} = \begin{bmatrix} L_1\left(x_p^{(0\sim\mu_p-1)}\left(t\right)|_{p=1\sim\eta}\right) \\ L_2\left(x_p^{(0\sim\mu_p-1)}\left(t\right)|_{p=1\sim\eta}\right) \\ \vdots \\ L_\eta\left(x_p^{(0\sim\mu_p-1)}\left(t\right)|_{p=1\sim\eta}\right) \end{bmatrix} + Bu\left(t-h\right), \tag{2}$$

where $\eta\geqslant 1$ is an integer, μ_p, $p=1,2,\ldots,\eta$ are a set of integers, $x_p\in\mathbb{R}^{r_p}$, $p=1,2,\ldots,\eta$ are a set of state vectors with r_p, $p=1,2,\ldots,\eta$ being a set of distinct integers satisfying

$$r_1+r_2+\cdots+r_\eta=r. \tag{3}$$

Further, $L_p\left(\cdot\right)$, $p=1,2,\ldots,\eta$ are a set of linear functions, and $B\in\mathbb{R}^{r\times r}$ is a square upper-triangular matrix with diagonal elements all being 1.

Generally, a system in the form of (2) is called a linear time-delay FAS if $B\in\mathbb{R}^{r\times r}$ is nonsingular, that is, $\det B\neq 0$. Since the system (2) has a set of sub-states with different differential orders, it is often called a time-delay FAS with multi-orders.

With our notations, the above FAS (2) can also be compactly written in the form of

$$x^{(\mu_p)}\left(t\right)|_{p=1\sim\eta}=L\left(x_p^{(0\sim\mu_p-1)}\left(t\right)|_{p=1\sim\eta}\right)+Bu\left(t-h\right), \tag{4}$$

where

$$L\left(\cdot\right)=\begin{bmatrix} L_1\left(\cdot\right) \\ L_2\left(\cdot\right) \\ \vdots \\ L_\eta\left(\cdot\right) \end{bmatrix}.$$

A typical special form of the above linear continuous-time multi-order FAS (2) with delays is the following linear time-delay single-order FAS:

$$x^{(n)}(t) = L\left(x^{(0\sim n-1)}(t)\right) + Bu(t-h), \tag{5}$$

where $L(\cdot)$ is a linear function, and the matrix $B \in \mathbb{R}^{r\times r}$ is nonsingular.

2.2 Case of nonlinear systems

Let us consider the following two-stage strict-feedback system with time delays:

$$\begin{cases} \dot{x}_1(t) = \alpha(x_1(t-\tau_j(t))|_{j=1\sim\zeta}, t) + B_1(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t)x_2(t-\delta(t)), \\ \dot{x}_2(t) = \beta(x_k(t-\tau_j(t))|_{j=1\sim\zeta,k=1,2}, t) + B_2(x_k(t-\sigma_j(t))|_{j=1\sim\gamma,k=1,2}, t)u(t-h), \end{cases} \tag{6}$$

where $\delta(t)$ is a bounded nonnegative function, h is a nonnegative constant, and

$$x_i(t-\tau_j(t))|_{j=1\sim\zeta} = \begin{bmatrix} x_i(t-\tau_1(t)) \\ x_i(t-\tau_2(t)) \\ \vdots \\ x_i(t-\tau_\zeta(t)) \end{bmatrix},\ i=1,2, \tag{7}$$

$$x_i(t-\sigma_j(t))|_{j=1\sim\gamma} = \begin{bmatrix} x_i(t-\sigma_1(t)) \\ x_i(t-\sigma_2(t)) \\ \vdots \\ x_i(t-\sigma_\gamma(t)) \end{bmatrix},\ i=1,2, \tag{8}$$

and

$$\begin{aligned} x_k(t-\tau_j(t))|_{j=1\sim\zeta,k=1,2} &= \begin{bmatrix} x_1(t-\tau_j(t))|_{j=1\sim\zeta} \\ x_2(t-\tau_j(t))|_{j=1\sim\zeta} \end{bmatrix}, \\ x_k(t-\sigma_j(t))|_{j=1\sim\gamma,k=1,2} &= \begin{bmatrix} x_1(t-\sigma_j(t))|_{j=1\sim\gamma} \\ x_2(t-\sigma_j(t))|_{j=1\sim\gamma} \end{bmatrix}. \end{aligned}$$

$B_1(\cdot,t) \in \mathbb{R}^{r\times r}$ and $B_2(\cdot,t) \in \mathbb{R}^{r\times r}$ are two matrix functions and satisfy the following full-actuation conditions.

Condition C1. $\det B_1(X,t) \neq 0$ or ∞, for all $X \in \mathbb{R}^{\gamma r}$ and $t \geqslant 0$.

Condition C2. $\det B_2(Y,t) \neq 0$ or ∞, for all $Y \in \mathbb{R}^{2\gamma r}$ and $t \geqslant 0$.

While all the other variables are as stated in [44].

Proposition 2. Let Conditions C1 and C2 be met, then the state-space system (6) is equivalent to the following second-order system:

$$\ddot{x}_1(t) = f(t) + B\left(x_k(t-\sigma_j(t))|_{j=1\sim\gamma,k=1,2}, t\right)u(t-\delta(t)-h), \tag{9}$$

where

$$\begin{aligned} B\left(x_k(t-\sigma_j(t))|_{j=1\sim\gamma,k=1,2}, t\right) = {} & B_1\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right) \\ & \times B_2\left(x_k(t-\delta(t)-\sigma_j(t-\delta(t)))|_{j=1\sim\gamma,k=1,2}, t-\delta(t)\right), \end{aligned} \tag{10}$$

and

$$\begin{aligned} f(t) \triangleq {} & \dot{\alpha}\left(x_1(t-\tau_j(t))|_{j=1\sim\zeta}, t\right) + \dot{B}_1\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right)x_2(t-\delta(t)) \\ & + B_1\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right)\beta\left(x_k(t-\delta(t)-\tau_j(t-\delta(t)))|_{j=1\sim\zeta,k=1,2}, t-\delta(t)\right), \end{aligned} \tag{11}$$

with the x_2 related terms determined by the following relation:

$$x_2(t-\delta(t)) = B_1^{-1}\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right)\left[\dot{x}_1(t) - \alpha\left(x_1(t-\tau_j(t))|_{j=1\sim\zeta}, t\right)\right]. \tag{12}$$

Proof. Due to Condition C1, we can obtain (12) from the first equation in (6). Further, taking the derivatives of both sides of the first equation in (6) gives

$$\begin{aligned}\ddot{x}_1(t) =\ & \dot{\alpha}\left(x_1(t-\tau_j(t))|_{j=1\sim\zeta}, t\right) + \dot{B}_1\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right) x_2(t-\delta(t)) \\ & + B_1\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right) \dot{x}_2(t-\delta(t)).\end{aligned} \tag{13}$$

It follows from the second one in (6) that

$$\begin{aligned}\dot{x}_2(t-\delta(t)) =\ & \beta\left(x_k(t-\delta(t)-\tau_j(t-\delta(t)))|_{j=1\sim\zeta,k=1,2}, t-\delta(t)\right) \\ & + B_2\left(x_k(t-\delta(t)-\sigma_j(t-\delta(t)))|_{j=1\sim\gamma,k=1,2}, t-\delta(t)\right) u(t-\delta(t)-h).\end{aligned} \tag{14}$$

Substituting the above equation into (13) yields

$$\begin{aligned}\ddot{x}_1(t) =\ & \dot{\alpha}\left(x_1(t-\tau_j(t))|_{j=1\sim\zeta}, t\right) + \dot{B}_1\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right) x_2(t-\delta(t)) \\ & + B_1\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right) \beta\left(x_k(t-\delta(t)-\tau_j(t-\delta(t)))|_{j=1\sim\zeta,k=1,2}, t-\delta(t)\right) \\ & + B_1\left(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right) \\ & \times B_2\left(x_k(t-\delta(t)-\sigma_j(t-\delta(t)))|_{j=1\sim\gamma,k=1,2}, t-\delta(t)\right) u(t-\delta(t)-h).\end{aligned} \tag{15}$$

This can clearly be written into the form of (9)-(11), which turns out to be a system with only the variable x_1 when the x_2 related terms therein are converted into x_1 related ones by using (12).

Due to Conditions C1 and C2, we obviously have

$$\det B(Y,t) \neq 0 \text{ or } \infty \text{ for all } Y \in \mathbb{R}^{2\gamma r} \text{ and } t \geqslant 0. \tag{16}$$

Example 1. Consider the following system [45]:

$$\begin{cases}\dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -x_1(t) + \left(1 - x_1^2(t)\right) x_2(t) + u(t-1),\end{cases} \tag{17}$$

where x_1 and x_2 are two scalar variables. Corresponding to system (6), we have

$$\begin{cases}\alpha(x_1(t-\tau_j(t))|_{j=1\sim\zeta}, t) = 0, \\ \tau_j(t)|_{j=1\sim\zeta} = 0, \\ B_1(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t) = B_2(x_1(t-\sigma_j(t))|_{j=1\sim\gamma}, t) = 1, \\ \sigma_j(t)|_{j=1\sim\gamma} = 0, \\ \delta(t) = 0, \\ \beta(x_k(t-\tau_j(t))|_{j=1\sim\zeta,k=1,2}, t) = -x_1(t) + \left(1 - x_1^2(t)\right) x_2(t), \\ h = 1.\end{cases}$$

Thus it follows from Proposition 2 that

$$f\left(x_1^{(0\sim1)}\right) = -x_1(t) + \left(1 - x_1^2(t)\right) \dot{x}_1(t). \tag{18}$$

Hence, the system (17) is equivalent to the following second-order FAS:

$$\ddot{x}_1(t) = -x_1(t) + \left(1 - x_1^2(t)\right) \dot{x}_1(t) + u(t-1). \tag{19}$$

The above Proposition 2 clearly indicates that a time-delay system in a state-space form may be converted equivalently into a high-order FAS. As a matter of fact, this conclusion can be generalized to a type of general strict-feedback systems with time delays.

In fact, based on the usual continuous-time strict-feedback system [32], we can further propose the following strict-feedback system with time delays:

$$\begin{cases} \dot{x}_1(t) = \alpha_1\left(x_1(t-\tau_{1j}(t))|_{j=1\sim\zeta_1}, t\right) + B_1\left(x_1(t-\sigma_{1j}(t))|_{j=1\sim\gamma_1}, t\right) x_2(t-\delta_1(t)), \\ \dot{x}_2(t) = \alpha_2\left(x_{1\sim 2}(t-\tau_{2j}(t))|_{j=1\sim\zeta_2}, t\right) + B_2\left(x_{1\sim 2}(t-\sigma_{2j}(t))|_{j=1\sim\gamma_2}, t\right) x_3(t-\delta_2(t)), \\ \vdots \\ \dot{x}_{n-1}(t) = \alpha_{n-1}\left(x_{1\sim n-1}(t-\tau_{n-1,j}(t))|_{j=1\sim\zeta_{n-1}}, t\right) \\ \qquad\qquad + B_{n-1}\left(x_{1\sim n-1}(t-\sigma_{n-1,j}(t))|_{j=1\sim\gamma_{n-1}}, t\right) x_n(t-\delta_{n-1}(t)), \\ \dot{x}_n(t) = \alpha_n\left(x_{1\sim n}(t-\tau_{nj}(t))|_{j=1\sim\zeta_n}, t\right) + B_n\left(x_{1\sim n}(t-\sigma_{nj}(t))|_{j=1\sim\gamma_n}, t\right) u(t-h), \end{cases} \tag{20}$$

where $x_i \in \mathbb{R}^r, i = 1, 2, \ldots, n$ are the state vectors, $u \in \mathbb{R}^r$ is the control vector, $\tau_{ij}(t), j = 1, 2, \ldots, \zeta_i$; $\sigma_{ij}(t), j = 1, 2, \ldots, \gamma_i$, $i = 1, 2, \ldots, n$, and $\delta_i(t), i = 1, 2, \ldots, n-1$ are all nonnegative scalar functions representing the time delays in the system. Furthermore, $\alpha_i(\cdot, t), i = 1, 2, \ldots, n$ are a set of nonlinear vector functions, and $B_i(\cdot, t), i = 1, 2, \ldots, n$ are a set of matrix functions satisfying the following full-actuation condition:

$$\det B_i\left(x_{1\sim i}(t-\sigma_{ij}(t))|_{j=1\sim\gamma_i}, t\right) \neq 0 \text{ or } \infty,\ i = 1, 2, \ldots, n$$

is met for all possible values of $x_{1\sim i}(t-\sigma_{ij}(t))|_{j=1\sim\gamma_i}$, $i = 1, 2, \ldots, n$, and $t \geqslant 0$. Under this condition, the general time-delay strict-feedback system (20) can also be converted into a high-order FAS.

3 Time-delay FASs

Motivated by our former results and Proposition 2, we now present the single-order FASs with input delays.

3.1 Single-order FASs

Motivated by the above fact, we can give the following single-order affine FAS with both state and input delays:

$$x^{(n)}(t) = f\left(x^{(0\sim n-1)}(t-\tau_j(t))|_{j=1\sim\zeta}, t\right) + B\left(x^{(0\sim n-1)}(t-\sigma_j(t))|_{j=1\sim\gamma}, t\right) u(t-h), \tag{21}$$

where h is some nonnegative scalar representing the time delay in the system input, while all the other variables are the same as introduced in the Subsection 3.1.1 of [44]. Also, as in the Subsection 3.1.1 of [44], the function $B(\cdot, t)$ is required to satisfy the following full-actuation assumption.

Assumption A1. $\det B(X, t) \neq 0$ or ∞, $\forall X \in \mathbb{R}^{\gamma nr}$ and $t \geqslant 0$.

Clearly, a constant linear single-order FAS with state time-varying delays and a constant input delay is of the following form:

$$x^{(n)}(t) = \sum_{i=0}^{m} C_i x^{(0\sim n-1)}(t-\tau_i(t)) + Bu(t-h), \tag{22}$$

where B is a constant square nonsingular matrix, and $C_i \in \mathbb{R}^{r\times nr}, i = 0, 1, \ldots, m$ are the coefficient matrices.

Parallel to the above single-order affine FAS (21), we can also define the following non-affine one:

$$x^{(n)}(t) = f\left(x^{(0\sim n-1)}(t-\tau_j(t))|_{j=1\sim\zeta}, t\right) + g\left(x^{(0\sim n-1)}(t-\sigma_j(t))|_{j=1\sim\gamma}, t, u(t-h)\right), \tag{23}$$

where $g(\cdot) \in \mathbb{R}^r$ is a nonlinear function, and the other variables are as stated before. Particularly, we call the system (23) a single-order non-affine (global) FAS if the function $g(\cdot)$ satisfies the following assumption.

Assumption A1′. The following mapping:

$$\tilde{u}(t) = g(X, t, u(t)) \tag{24}$$

forms a differential homeomorphism from u to $\tilde{u}$ for all $X \in \mathbb{R}^{\gamma nr}$ and $t \geqslant 0$.

Remark 1. For an FAS with state delays only, that is, the case of $h = 0$, we can define the new control under Assumption A1:

$$\tilde{u}\left(t-h\right)=B\left(x^{(0\sim n-1)}\left(t-\sigma_j\left(t\right)\right)|_{j=1\sim\gamma},t\right)u\left(t-h\right), \tag{25}$$

or the following new one under Assumption A1′:

$$\tilde{u}\left(t-h\right)=g\left(x^{(0\sim n-1)}\left(t-\sigma_j\left(t\right)\right)|_{j=1\sim\gamma},t,u\left(t-h\right)\right), \tag{26}$$

such that the affine FAS (21) or the non-affine one (23) is converted equivalently into the following standard form:

$$x^{(n)}\left(t\right)=f\left(x^{(0\sim n-1)}\left(t-\tau_j\left(t\right)\right)|_{j=1\sim\zeta},t\right)+\tilde{u}\left(t-h\right). \tag{27}$$

However, we point out that this may be not realizable when $h > 0$ since future state information may have been used in the transformations (25) and (26).

3.2 Multi-order FASs

In this subsection, let us generalize the above single-order time-delay FASs into the case of multi-order ones.

Stimulated by the models of multi-order FASs without delays [37–39] and with state delays only [44], we can give the following general form of a multi-order affine FAS model with time delays:

$$\begin{aligned}x_k^{(\mu_k)}|_{k=1\sim\eta}\left(t\right) = {} & f\left(x_k^{(0\sim\mu_k-1)}\left(t-\tau_j\left(t\right)\right)|_{k=1\sim\eta,\ j=1\sim\zeta},t\right)\\ & +B\left(x_k^{(0\sim\mu_k-1)}\left(t-\sigma_j\left(t\right)\right)|_{k=1\sim\eta,\ j=1\sim\gamma},t\right)u\left(t-h\right),\end{aligned} \tag{28}$$

where, again, h is a nonnegative scalar representing the delay in the control input, and all the other variables are as introduced in the Subsection 3.2.1 of [44].

Recall that

$$\varkappa_0=\sum_{k=1}^{\eta}\mu_k r_k, \tag{29}$$

and

$$x_k^{(0\sim\mu_k-1)}\left(t\right)|_{k=1\sim\eta}\in\mathbb{R}^{\varkappa_0},\ x_k^{(0\sim\mu_k-1)}\left(t-\sigma_j\left(t\right)\right)|_{k=1\sim\eta,\ j=1\sim\gamma}\in\mathbb{R}^{\gamma\varkappa_0}. \tag{30}$$

The above system (28) is called a (global) multi-order FAS with delays if the following assumption is met.

Assumption A2. $\det B\left(X,t\right)\neq 0$ or ∞, $\forall X\in\mathbb{R}^{\gamma\varkappa_0}$ and $t\geqslant 0$.

Obviously, in the case of $\eta = 1$, the above FAS model (28) reduces to the form of the single-order FAS (21).

As a special case of the multi-order nonlinear time-delay FAS (28), a constant linear multi-order FAS with time-varying state delays and a constant input delay is in the form of

$$x_k^{(\mu_k)}|_{k=1\sim\eta}\left(t\right)=\sum_{i=0}^{m}D_i x_k^{(0\sim\mu_k-1)}\left(t-\tau_i\left(t\right)\right)|_{k=1\sim\eta}+Bu\left(t-h\right), \tag{31}$$

where B is a constant square nonsingular matrix, and $D_i\in\mathbb{R}^{r\times\varkappa_0}, i=0,1,\ldots,m$ are the coefficient matrices.

Parallel to the above multi-order affine FAS (28), we can also define the following multi-order non-affine one:

$$\begin{aligned}x_k^{(\mu_k)}|_{k=1\sim\eta}\left(t\right) = {} & f\left(x_k^{(0\sim\mu_k-1)}\left(t-\tau_j\left(t\right)\right)|_{k=1\sim\eta,\ j=1\sim\zeta},t\right)\\ & +g\left(x_k^{(0\sim\mu_k-1)}\left(t-\sigma_j\left(t\right)\right)|_{k=1\sim\eta,\ j=1\sim\gamma},t,u\left(t-h\right)\right),\end{aligned} \tag{32}$$

where $g\left(\cdot\right)\in\mathbb{R}^r$ is a nonlinear function, and the other variables are as stated before. We call the system (32) a non-affine (global) multi-order FAS if the function $g\left(\cdot\right)$ satisfies the following assumption:

Assumption A2′. The following mapping:

$$\tilde{u}(t)=g(X,t,u(t)) \tag{33}$$

forms a differential homeomorphism from u to $\tilde{u}$ for all $X\in\mathbb{R}^{\gamma\varkappa_0}$ and $t\geqslant 0$.

Remark 2. In the case of $h=0$, we can introduce the following new control vector for FAS (28) under Assumption A2:

$$\tilde{u}(t-h)=B\left(x_k^{(0\sim\mu_k-1)}(t-\sigma_j(t))\,|_{k=1\sim\eta,\ j=1\sim\gamma},t\right)u(t-h), \tag{34}$$

or the following new one for the non-affine FAS (32) under Assumption A2′:

$$\tilde{u}(t-h)=g\left(x_k^{(0\sim\mu_k-1)}(t-\sigma_j(t))\,|_{k=1\sim\eta,\ j=1\sim\gamma},t,u(t-h)\right). \tag{35}$$

Consequently, the (global) affine FAS (28) or the non-affine FAS (32) can be converted into the following standard form:

$$x_k^{(\mu_k)}|_{k=1\sim\eta}(t)=f\left(x_k^{(0\sim\mu_k-1)}(t-\tau_j(t))\,|_{k=1\sim\eta,\ j=1\sim\zeta},t\right)+\tilde{u}(t-h), \tag{36}$$

which can be written separately as

$$x_i^{(\mu_i)}(t)=f_i\left(x_k^{(0\sim\mu_k-1)}(t-\tau_j(t))\,|_{k=1\sim\eta,\ j=1\sim\zeta},t\right)+\tilde{u}_i(t-h),\ \ i=1,2,\ldots,\eta, \tag{37}$$

where $\tilde{u}_i, i=1,2,\ldots,\eta$ are defined by

$$\tilde{u}=\begin{bmatrix}\tilde{u}_1\\ \tilde{u}_2\\ \vdots\\ \tilde{u}_\eta\end{bmatrix},\quad \tilde{u}_k\in\mathbb{R}^{r_i}. \tag{38}$$

However, the transformations (34) and (35) may become unrealizable if $h>0$ since future states may be involved in these transformations.

4 Control of single-order FASs

For preparation, let us first provide a prediction scheme to be used in the controller design.

4.1 Prediction scheme

In order to design the controller for the single-order FAS (21), the following state prediction problem needs to be solved first.

Suppose that the full state of the FAS (21), namely, $x^{(0\sim n-1)}(s)$, is known for $s\leqslant t$. Find $x^{(0\sim n-1)}(s)$, $s\in[t,t+h]$ satisfying the FAS (21).

Introduce the following symbols:

$$\Phi\left(0_{0\sim n-1}\right)=\begin{bmatrix}0 & I_r & & \\ & & \ddots & \\ & & & I_r\\ 0 & 0 & \cdots & 0\end{bmatrix},\quad B_c=\begin{bmatrix}0\\ \vdots\\ 0\\ I_r\end{bmatrix}.$$

Then the single-order FAS (21) can be converted equivalently into the following state-space one:

$$\begin{aligned}\dot{x}^{(0\sim n-1)}(t)\ =\ &\Phi\left(0_{0\sim n-1}\right)x^{(0\sim n-1)}(t)+B_cf\left(x^{(0\sim n-1)}(t-\tau_j(t))\,|_{j=1\sim\zeta},t\right)\\ &+B_cB\left(x^{(0\sim n-1)}(t-\sigma_j(t))\,|_{j=1\sim\gamma},t\right)u(t-h).\end{aligned} \tag{39}$$

Based on the above system equation (39), we can construct the following prediction system:

$$\begin{aligned}\dot{Y}(s+t) = {} & \Phi\left(0_{0\sim n-1}\right) Y\left(s+t\right) + B_c f\left(Y\left(s+t-\tau_j\left(s+t\right)\right)|_{j=1\sim\zeta}, s+t\right) \\ & + B_c B\left(Y\left(s+t-\sigma_j\left(s+t\right)\right)|_{j=1\sim\gamma}, s+t\right) u\left(s+t-h\right),\ s\in[0,h],\end{aligned} \tag{40}$$

where Y is a vector of the same dimension as $x^{(0\sim n-1)}\left(t\right)$. Let

$$\tau\left(t\right) = \sup_{s\in[0,h]}\left\{\tau_i\left(s+t\right), i=1,2,\ldots,\zeta; \sigma_j\left(s+t\right), j=1,2,\ldots,\gamma\right\}. \tag{41}$$

Then it can be easily observed that the system (40) gives an accurate estimation of $x^{(0\sim n-1)}\left(t+s\right)$, that is,

$$Y\left(t+s\right) = x^{(0\sim n-1)}\left(t+s\right), \tag{42}$$

when

(1) the system (40) has a unique solution with respect to arbitrarily given initial values and external input, and

(2) the initial value of the predictor system (40) is chosen to be

$$Y\left(\theta+t\right) = x^{(0\sim n-1)}(\theta+t),\ \theta\in\left[-\tau\left(t\right),0\right]. \tag{43}$$

Further, let us introduce the following variable transformation:

$$Z\left(s\right) = Y\left(s+t\right). \tag{44}$$

Then we obviously have

$$Z\left(s-\tau_j\left(s+t\right)\right) = Y\left(s+t-\tau_j\left(s+t\right)\right),\ s\in[0,h]. \tag{45}$$

Therefore, with the help of the above relations (44) and (45), the above predictor system (40) is equivalently converted into the following form:

$$\begin{aligned}\dot{Z}(s) = {} & \Phi\left(0_{0\sim n-1}\right) Z\left(s\right) + B_c f\left(Z\left(s-\tau_j\left(s+t\right)\right)|_{j=1\sim\zeta}, s+t\right) \\ & + B_c B\left(Z\left(s-\sigma_j\left(s+t\right)\right)|_{j=1\sim\gamma}, s+t\right) u\left(s+t-h\right),\ s\in[0,h],\end{aligned} \tag{46}$$

and the initial value condition (43) becomes

$$Z\left(\theta\right) = Y(\theta+t) = x^{(0\sim n-1)}(\theta+t),\ \theta\in\left[-\tau\left(t\right),0\right]. \tag{47}$$

Introduce the following assumption.

Assumption A3. The system (46) has a unique solution with respect to arbitrarily given initial values and external input $u\left(t\right)$.

Then, note that

$$Z\left(s\right) = Y(t+s) = x^{(0\sim n-1)}(t+s).$$

We immediately have the following result.

Lemma 1. Let Assumptions A1 and A3 be met, $\tau\left(t\right)$ be defined in (41), and $x^{(0\sim n-1)}\left(s\right),\ s\leqslant t$ be available and satisfy the FAS (21). Then there holds

$$x^{(0\sim n-1)}\left(t+s\right) = Z\left(s\right),\ s\in[0,h], \tag{48}$$

where $Z\left(s\right)$ is given by the predictor (46) with the following initial value condition:

$$Z\left(\theta\right) = x^{(0\sim n-1)}(\theta+t),\ \theta\in\left[-\tau\left(t\right),0\right]. \tag{49}$$

It clearly follows from the above lemma that

$$x^{(0\sim n-1)}\left(t+h\right) = Z\left(h\right). \tag{50}$$

Example 2. Let us continue with the system in Example 1.

Suppose that the full state of the FAS (19), namely, $x_1^{(0\sim1)}(s)$, is known for $s \leqslant t$. We will find $x_1^{(0\sim1)}(s), s \in [t, t+1]$ satisfying the FAS (19) following Lemma 1.

Note that

$$\Phi(0_{0\sim1}) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

We can convert the single-order FAS (19) equivalently into the following state-space one:

$$\dot{x}_1^{(0\sim1)}(t) = \Phi(0_{0\sim1}) x_1^{(0\sim1)}(t) + B_c\left[-x_1(t) + \left(1 - x_1^2(t)\right)\dot{x}_1(t)\right] + B_c u(t-1). \tag{51}$$

Since $\tau_j(t)|_{j=1\sim\zeta} = 0$ and $\sigma_j(t)|_{j=1\sim\gamma} = 0$, it follows from (41) that $\tau(t) = 0$.

Let

$$Z(s) = \begin{bmatrix} z_1(s) & z_2(s) \end{bmatrix}^{\mathrm{T}}.$$

Then, it follows from Lemma 1 that the predictor of FAS (19) is given by

$$\dot{Z}(s) = \Phi(0_{0\sim1}) Z(s) + B_c\left[-z_1(s) + \left(1 - z_1^2(s)\right)z_2(s)\right] + B_c u(s+t-1),\ s \in [0,1], \tag{52}$$

with the following initial value condition:

$$Z(0) = x_1^{(0\sim1)}(t). \tag{53}$$

As a consequence, there holds the following prediction relation:

$$x_1^{(0\sim1)}(t+s) = Z(s),\ s \in [0,1]. \tag{54}$$

Remark 3. For the single-order non-affine FAS (23), a result similar to Lemma 1 still holds. As a matter of fact, for the FAS (23), the predictor (46) is clearly replaced with the following one:

$$\begin{aligned}\dot{Z}(s) = {} & \Phi(0_{0\sim n-1}) Z(s) + B_c f\left(Z(s - \tau_j(s+t))|_{j=1\sim\zeta}, s+t\right) \\ & + B_c g\left(Z(s - \sigma_j(s+t))|_{j=1\sim\gamma}, s+t, u(s+t-h)\right),\ s \in [0,h],\end{aligned} \tag{55}$$

with the same initial value condition (49).

Remark 4. Prediction of future states is a core procedure in the control of a system with input delays. The approach provided in this subsection is a general one but may be subject to certain problems:

(1) in complicated cases the predictor equation (46) may not be solvable both analytically and numerically;

(2) when the numerical solution is adopted, computation errors inevitably occur, which on one side may reduce the control precision, and on the other side may even cause instability of the system; and

(3) for systems with a very long input delay h, the open-loop stability is also needed in order that the designed predictor produces reasonable large prediction values.

4.2 Controller design

Due to the full-actuation feature of the single-order FAS (21), we can construct the following controller:

$$\begin{cases} u(t-h) = -B^{-1}\left(x^{(0\sim n-1)}(t - \sigma_j(t))|_{j=1\sim\gamma}, t\right)\left[f\left(x^{(0\sim n-1)}(t - \tau_j(t))|_{j=1\sim\zeta}, t\right) + u^*(t-h)\right], \\ u^*(t-h) = A_{0\sim n-1} x^{(0\sim n-1)}(t) - v(t),\ t \geqslant h, \end{cases} \tag{56}$$

where $v(t)$ is an external input vector. It is clearly seen that this controller gives the following constant linear closed-loop system:

$$x^{(n)}(t) + A_{0\sim n-1} x^{(0\sim n-1)}(t) = v(t),\ t \geqslant h. \tag{57}$$

Rewrite the above controller as

$$\begin{cases} u(t) = -B^{-1}\left(x^{(0\sim n-1)}(t+h - \sigma_j(t+h))|_{j=1\sim\gamma}, t+h\right) \\ \qquad \times \left[f\left(x^{(0\sim n-1)}(t+h - \tau_j(t+h))|_{j=1\sim\zeta}, t+h\right) + u^*(t)\right], \\ u^*(t) = A_{0\sim n-1} x^{(0\sim n-1)}(t+h) - v(t+h),\ t \geqslant 0. \end{cases} \tag{58}$$

Clearly, the controller (58) uses future values of the states and is not realizable. Fortunately, the above Lemma 1 has given these values of the future states as

$$x^{(0\sim n-1)}(t+h) = Z(h)$$

and

$$\begin{aligned} x^{(0\sim n-1)}(t+h-\sigma_j(t+h)) &= Z(h-\sigma_j(t+h)), \ j=1,2,\ldots,\gamma, \\ x^{(0\sim n-1)}(t+h-\tau_j(t+h)) &= Z(h-\tau_j(t+h)), \ j=1,2,\ldots,\zeta. \end{aligned}$$

Based on the above analysis, we can immediately give the following result.

Theorem 1. Let Assumptions A1 and A3 be satisfied, and $A_{0\sim n-1} \in \mathbb{R}^{r\times nr}$ be an arbitrarily given matrix. Then the following controller:

$$\begin{cases} u(t) = -B^{-1}\left(Z(h-\sigma_j(t+h))|_{j=1\sim\gamma}, t+h\right)\left[f\left(Z(h-\tau_j(t+h))|_{j=1\sim\zeta}, t+h\right) + u^*(t)\right], \\ u^*(t) = A_{0\sim n-1}Z(h) - v(t+h), \ t \geqslant 0, \\ \dot{Z}(s) = \Phi(0_{0\sim n-1})Z(s) + B_c f\left(Z(s-\tau_j(s+t))|_{j=1\sim\zeta}, s+t\right) \\ \qquad +B_c B\left(Z(s-\sigma_j(s+t))|_{j=1\sim\gamma}, s+t\right)u(s+t-h), \ s\in[0,h], \\ Z(\theta) = x^{(0\sim n-1)}(\theta+t), \ \theta \in [-\tau(t), 0], \end{cases} \tag{59}$$

for the single-order affine system (21) produces the constant linear closed-loop system (57), where $v(t)$ is an external input vector.

Example 3. Let us continue with Example 2.

Based on the above Theorem 1, the controller for the system (19) can be designed as follows:

$$\begin{cases} u(t) = z_1(1) - \left(1 - z_1^2(1)\right)z_2(1) - u^*(t), \\ u^*(t) = [a_0 \ \ a_1]\, Z(1) - v(t+1), \ t \geqslant 0, \\ \dot{Z}(s) = \Phi(0_{0\sim 1})Z(s) + B_c\left[-z_1(s) + \left(1 - z_1^2(s)\right)z_2(s)\right] + B_c u(s+t-1), \ s \in [0,1], \\ Z(0) = x^{(0\sim 1)}(t), \end{cases} \tag{60}$$

which produces the constant linear closed-loop system

$$\ddot{x} + a_1\dot{x} + a_0 x = v, \tag{61}$$

where a_0, a_1 are two real positive scalars, and $v(t)$ is an external input.

Remark 5. For the single-order non-affine FAS (23), a result similar to Theorem 1 still holds. In this case, the controller can be obtained using the predictor (46) and the differential homeomorphism property of the mapping (26) as

$$\begin{cases} u(t) = -g^{-1}\left(Z(h-\sigma_j(t+h))|_{j=1\sim\gamma}, t+h, \tilde{u}(t)\right), \\ \tilde{u}(t) = f\left(Z(h-\tau_j(t+h))|_{j=1\sim\zeta}, t+h\right) + u^*(t), \\ u^*(t) = A_{0\sim n-1}Z(h) - v(t+h), \ t \geqslant 0, \\ \dot{Z}(s) = \Phi(0_{0\sim n-1})Z(s) + B_c f\left(Z(s-\tau_j(s+t))|_{j=1\sim\zeta}, s+t\right) \\ \qquad +B_c B\left(Z(s-\sigma_j(s+t))|_{j=1\sim\gamma}, s+t, u(s+t-h)\right), \ s\in[0,h], \\ Z(\theta) = x^{(0\sim n-1)}(\theta+t), \ \theta \in [-\tau(t), 0], \end{cases} \tag{62}$$

and eventually, the same closed-loop linear system as (57) is resulted in.

Remark 6. Let

$$\Phi(A_{0\sim n-1}) = \begin{bmatrix} 0 & I & & \\ & & \ddots & \\ & & & I \\ -A_0 & -A_1 & \cdots & -A_{n-1} \end{bmatrix}.$$

Then it can be easily checked that the closed-loop system (57) can be expressed by the following state-space form:

$$\dot{x}^{(0\sim n-1)} = \Phi(A_{0\sim n-1})x^{(0\sim n-1)} + B_c v.$$

Further note that

$$\Phi(A_{0\sim n-1}) = \Phi(0_{0\sim n-1}) - B_c A_{0\sim n-1}.$$

The solution to the gain matrix $A_{0\sim n-1} \in \mathbb{R}^{r\times nr}$ in the controllers (59) and (62) can then be easily carried out by conducting the eigenstructure assignment in the matrix pair $(\Phi(0_{0\sim n-1}), -B_c)$ (see [46]). We point out that specific parametric eigenstructure assignment designs have been proposed in [37], which give the general expressions of the gain matrix, and provide all the degrees of freedom in the designs. The full degrees of freedom can be further utilized to achieve additional system performance.

5 Control of multi-order FASs

5.1 Prediction scheme

Similarly, in order to design the controller for the multi-order FAS (28), the following state prediction problem needs to be solved first.

Suppose that the full state of the FAS (28), namely, $x_k^{(0\sim\mu_k-1)}(s)|_{k=1\sim\eta}$, is known for $s \leqslant t$. Find $x_k^{(0\sim\mu_k-1)}|_{k=1\sim\eta}(s), s \in [t, t+h]$ satisfying FAS (28).

Introduce the following symbols:

$$\Phi_i(0_{0\sim\mu_i-1}) = \begin{bmatrix} 0 & I_{r_i} & & \\ & & \ddots & \\ & & & I_{r_i} \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B_{ci} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_{r_i} \end{bmatrix}, \ i = 1, 2, \ldots, \eta, \tag{63}$$

and further define

$$\Phi_E = \operatorname{blockdiag}(\Phi_i(0_{0\sim\mu_i-1}), \ i = 1, 2, \ldots, \eta), \tag{64}$$
$$B_{cE} = \operatorname{blockdiag}(B_{ci}, \ i = 1, 2, \ldots, \eta), \tag{65}$$

and

$$X_k(t) = x_k^{(0\sim\mu_k-1)}(t), \ k = 1, 2, \ldots, \eta,$$
$$X(t) = x_k^{(0\sim\mu_k-1)}(t)|_{k=1\sim\eta}.$$

Then it is easily proven that the multi-order FAS (28) can be converted equivalently into the following state-space one:

$$\dot{X}(t) = \Phi_E X(t) + B_{cE} f(X_k(t-\tau_j(t))|_{j=1\sim\zeta,k=1\sim\eta}, t) + B_{cE} B(X_k(t-\sigma_j(t))|_{j=1\sim\gamma,k=1\sim\eta}, t)\, u(t-h), \tag{66}$$

which can also be written as

$$\dot{X}(t) = \Phi_E X(t) + B_{cE} f(X(t-\tau_j(t))|_{j=1\sim\zeta}, t) + B_{cE} B(X(t-\sigma_j(t))|_{j=1\sim\gamma}, t)\, u(t-h). \tag{67}$$

Define $\tau(t)$ as in (41), that is,

$$\tau(t) = \sup_{s\in[0,h]} \{\tau_i(s+t), i = 1, 2, \ldots, \zeta; \sigma_j(s+t), j = 1, 2, \ldots, \gamma\}, \tag{68}$$

and introduce the following assumption.

Assumption A4. The system (67) has a unique solution with respect to arbitrarily given initial values and external input $u(t)$.

Then following the same procedure as in the case of single-order FAS, we can obtain the following result.

Lemma 2. Let Assumption A4 be met, and $\tau(t)$ be defined in (68), and $x_k^{(0\sim\mu_k-1)}(s)|_{k=1\sim\eta}$, $s\leqslant t$ be available and satisfy the FAS (28). Construct the system

$$\begin{aligned}\dot{Z}(s) &= \Phi_E Z(s) + B_{cE} f\left(Z\left(s-\tau_j(s+t)\right)|_{j=1\sim\zeta}, s+t\right)\\&\quad + B_{cE} B\left(Z\left(s-\sigma_j(s+t)\right)|_{j=1\sim\gamma}, s+t\right) u(s+t-h),\ s\in[0,h],\end{aligned} \tag{69}$$

where the coefficients Φ_E and B_{cE} are defined in (63)–(65). Then there holds

$$x_k^{(0\sim\mu_k-1)}(t+s)|_{k=1\sim\eta} = Z(s),\ s\in[0,h], \tag{70}$$

if the following initial value condition is taken:

$$Z(\theta) = x_k^{(0\sim\mu_k-1)}(\theta+t)|_{k=1\sim\eta},\ \theta\in[-\tau(t),0]. \tag{71}$$

It clearly follows from the above lemma that

$$x_k^{(0\sim\mu_k-1)}(t+h)|_{k=1\sim\eta} = Z(h). \tag{72}$$

For the multi-order non-affine FAS (32), a result similar to Lemma 2 still holds. In fact, for the FAS (32), the predictor (69) is clearly replaced by the following one:

$$\begin{aligned}\dot{Z}(s) &= \Phi_E Z(s) + B_{cE} f\left(Z\left(s-\tau_j(s+t)\right)|_{j=1\sim\zeta}, s+t\right)\\&\quad + B_{cE} g\left(Z\left(s-\sigma_j(s+t)\right)|_{j=1\sim\gamma}, s+t, u(s+t-h)\right),\ s\in[0,h],\end{aligned} \tag{73}$$

with the same initial value condition (71).

5.2 Controller design

Due to the full-actuation assumption, namely, Assumption A2, we can design the following controller for the FAS (28):

$$\begin{cases} u(t-h) = -B^{-1}\left(x_k^{(0\sim\mu_k-1)}(t-\sigma_j(t))|_{k=1\sim\eta,\ j=1\sim\gamma}, t\right)\\ \qquad\qquad \times\left[f\left(x_k^{(0\sim\mu_k-1)}(t-\tau_j(t))|_{k=1\sim\eta,\ j=1\sim\zeta}, t\right) - u^*(t-h)\right],\\ u^*(t-h) = A x_k^{(0\sim\mu_k-1)}(t)|_{k=1\sim\eta} + v(t),\end{cases} \tag{74}$$

where $A\in\mathbb{R}^{r\times\eta\varkappa_0}$ is a parameter matrix. Under the above controller, it is easily seen that the following constant linear closed-loop system is obtained:

$$x_k^{(\mu_k)}|_{k=1\sim\eta}(t) = A x_k^{(0\sim\mu_k-1)}(t)|_{k=1\sim\eta} + v(t). \tag{75}$$

Rewrite the above controller as

$$\begin{cases} u(t) = -B^{-1}\left(x_k^{(0\sim\mu_k-1)}(t+h-\sigma_j(t+h))|_{k=1\sim\eta,\ j=1\sim\gamma}, t+h\right)\\ \qquad \times\left[f\left(x_k^{(0\sim\mu_k-1)}(t+h-\tau_j(t+h))|_{k=1\sim\eta,\ j=1\sim\zeta}, t+h\right) - u^*(t)\right],\\ u^*(t) = A x_k^{(0\sim\mu_k-1)}(t+h)|_{k=1\sim\eta} + v(t+h),\end{cases} \tag{76}$$

which is clearly seen to use future state values and hence is not realizable directly. However, by combining the above result with Lemma 2, we can immediately obtain the following result.

Theorem 2. Let Assumptions A2 and A4 be satisfied, and $\tau(t)$ be defined in (68). Further, let $A\in\mathbb{R}^{r\times\eta\varkappa_0}$ be an arbitrarily given matrix. Then the following controller:

$$\begin{cases} u(t) = -B^{-1}\left(Z\left(h-\sigma_j(t+h)\right)|_{j=1\sim\gamma}, t+h\right)\\ \qquad \times\left[f\left(Z\left(h-\tau_j(t+h)\right)|_{j=1\sim\zeta}, t+h\right) - u^*(t)\right],\\ u^*(t) = AZ(h) + v(t+h),\ t\geqslant 0,\\ \dot{Z}(s) = \Phi_E Z(s) + B_{cE} f\left(Z\left(s-\tau_j(s+t)\right)|_{j=1\sim\zeta}, s+t\right)\\ \qquad + B_{cE} B\left(Z\left(s-\sigma_j(s+t)\right)|_{j=1\sim\gamma}, s+t\right) u(s+t-h),\ s\in[0,h],\\ Z(\theta) = x_k^{(0\sim\mu_k-1)}(\theta+t)|_{k=1\sim\eta},\ \theta\in[-\tau(t),0],\end{cases} \tag{77}$$

with the coefficients Φ_E and B_{cE} defined in (63)–(65), produces the constant linear system (75).

For the multi-order non-affine FAS (32), a result similar to Theorem 2 still holds. In this case, the controller can be obtained using the differential homeomorphism property of the mapping (35) as

$$\begin{cases} u\left(t\right)=-g^{-1}\left(Z\left(h-\sigma_{j}\left(t+h\right)\right)|_{j=1\sim\gamma},t+h,\tilde{u}\left(t\right)\right), \\ \tilde{u}\left(t\right)=f\left(Z\left(h-\tau_{j}\left(t+h\right)\right)|_{j=1\sim\zeta},t+h\right)-u^{*}\left(t\right), \\ u^{*}\left(t\right)=AZ\left(h\right)+v\left(t+h\right),\ t\geqslant 0, \\ \dot{Z}(s)=\Phi_{E}Z\left(s\right)+B_{cE}f\left(Z\left(s-\tau_{j}\left(s+t\right)\right)|_{j=1\sim\zeta},s+t\right) \\ \qquad\qquad +B_{cE}B\left(Z\left(s-\sigma_{j}\left(s+t\right)\right)|_{j=1\sim\gamma},s+t,u\left(s+t-h\right)\right),\ s\in\left[0,h\right], \\ Z\left(\theta\right)=x_{k}^{(0\sim\mu_{k}-1)}(\theta+t)|_{k=1\sim\eta},\ \theta\in\left[-\tau\left(t\right),0\right], \end{cases} \tag{78}$$

and the same closed-loop system as (75) is achieved.

Particularly, if we choose in the above controllers (77) and (78)

$$A=\mathrm{blockdiag}\left(\left[A_{i}\right]_{0\sim\mu_{i}-1},i=1,2,\ldots,\eta\right),$$

where $[A_i]_{0\sim\mu_i-1}\in\mathbb{R}^{r_i\times\mu_i r_i},\ i=1,2,\ldots,\eta$. Then the following set of decoupled constant linear closed-loop systems are obtained:

$$x_{i}^{(\mu_{i})}=\left[A_{i}\right]_{0\sim\mu_{i}-1}x_{i}^{(0\sim\mu_{i}-1)}+v_{i},\quad i=1,2,\ldots,\eta, \tag{79}$$

with v being an external input vector given by

$$v=\begin{bmatrix} v_{1} \\ v_{2} \\ \vdots \\ v_{\eta} \end{bmatrix},\quad v_{k}\in\mathbb{R}^{r_{i}}. \tag{80}$$

For a systematic parametric method to solve the controller parameter matrix A such that the closed-loop linear system possesses a desired eigenstructure, please refer to [39].

6 Illustrative example

Consider the following dynamical system with both state and input delays:

$$\begin{cases} \dot{x}_{1}(t)=x_{10}(t)+x_{2}^{2}\left(t-\tau_{1}\left(t\right)\right), \\ \dot{x}_{10}(t)=-x_{1}\left(t\right)+x_{20}\left(t-\tau_{2}\left(t\right)\right)+x_{20}^{2}\left(t-\tau_{2}\left(t\right)\right)+u_{1}\left(t-h\right), \\ \dot{x}_{2}\left(t\right)=x_{20}\left(t\right)+x_{1}\left(t\right), \\ \dot{x}_{20}\left(t\right)=-x_{1}^{3}\left(t-\tau_{3}\left(t\right)\right)+x_{21}\left(t\right), \\ \dot{x}_{21}\left(t\right)=-x_{2}^{2}\left(t-\tau_{1}\left(t\right)\right)+\left(1+x_{1}^{2}\left(t-\tau_{3}\left(t\right)\right)\right)u_{2}\left(t-h\right), \end{cases} \tag{81}$$

where $x_1(t)$, $x_2(t)$, $x_{10}(t)$, $x_{20}(t)$ and $x_{21}(t)$ are the states of the system, $u_1(t)$ and $u_2(t)$ are the control inputs, $\tau_i(t)$, $i=1,2,3$ are the time-varying delays in the states, and h is a constant time delay in the control inputs.

6.1 The FAS

From the first two equations in (81), we can obtain

$$\ddot{x}_{1}(t)=f_{1}\left(x_{1}\left(t\right),x_{20}\left(t-\tau_{2}\right),x_{2}^{(0\sim 1)}\left(t-\tau_{1}\right),\dot{\tau}_{1}\right)+B_{1}u\left(t-h\right), \tag{82}$$

where

$$B_{1}=\left[1\ \ 0\right],\quad u=\begin{bmatrix} u_{1} \\ u_{2} \end{bmatrix}, \tag{83}$$

$$f_1(\cdot) = -x_1(t) + x_{20}(t-\tau_2) + x_{20}^2(t-\tau_2) + 2x_2(t-\tau_1)\left(\dot{x}_2(t-\tau_1) + \frac{\partial x_2(t-\tau_1)}{\partial \tau_1}\dot{\tau}_1\right). \tag{84}$$

From the last three equations in (81), we can obtain

$$\dddot{x}_2(t) = f_2\left(x_1(t), x_1^{(0\sim1)}(t-\tau_3), x_{20}(t-\tau_2), x_2^{(0\sim1)}(t-\tau_1), \dot{\tau}_1, \dot{\tau}_3\right) + B_2(x_1(t-\tau_3))\,u(t-h), \tag{85}$$

where

$$B_2(x_1(t-\tau_3)) = \begin{bmatrix} 1 & \left(1 + x_1^2(t-\tau_3)\right) \end{bmatrix},$$

$$\begin{aligned} f_2(\cdot) \;=\; & -3x_1^2(t-\tau_3)\left[\dot{x}_1(t-\tau_3) + \frac{\partial x_1(t-\tau_3)}{\partial \tau_3}\dot{\tau}_3\right] - x_2^2(t-\tau_1) \\ & + f_1\left(x_1(t), x_{20}(t-\tau_2), x_2^{(0\sim1)}(t-\tau_1), \dot{\tau}_1\right). \end{aligned} \tag{86}$$

Combining (82)-(84) with (85)-(86) gives the following FAS of the system (81):

$$\begin{bmatrix} \ddot{x}_1(t) \\ \dddot{x}_2(t) \end{bmatrix} = f\left(x_1(t), x_1^{(0\sim1)}(t-\tau_3), x_{20}(t-\tau_2), x_2^{(0\sim1)}(t-\tau_1), \dot{\tau}_1, \dot{\tau}_3\right) + B(x_1(t-\tau_3))\,u(t-h), \tag{87}$$

where

$$f = \begin{bmatrix} f_1\left(x_1(t), x_{20}(t-\tau_2), x_2^{(0\sim1)}(t-\tau_1), \dot{\tau}_1\right) \\ f_2\left(x_1(t), x_1^{(0\sim1)}(t-\tau_3), x_{20}(t-\tau_2), x_2^{(0\sim1)}(t-\tau_1), \dot{\tau}_1, \dot{\tau}_3\right) \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2(x_1(t-\tau_3)) \end{bmatrix}. \tag{88}$$

Therefore,

$$\mu_1 = 2,\ \mu_2 = 3,\ \zeta = 3,\ \gamma = 1,\ \text{and } \sigma_1 = \tau_3.$$

Once $x_1^{(0\sim1)}(t)$ and $x_2^{(0\sim2)}(t)$ are obtained via the FAS (87)-(88), it can be easily observed that the original states $x_{10}(t)$, $x_{20}(t)$ and $x_{21}(t)$ can be immediately given by

$$\begin{cases} x_{10}(t) = \dot{x}_1(t) - x_2^2(t-\tau_1), \\ x_{20}(t) = \dot{x}_2(t) - x_1(t), \\ x_{21}(t) = \dot{x}_{20}(t) + x_1^3(t-\tau_3). \end{cases}$$

6.2 The predictor

Introduce the following symbols:

$$\Phi_1(0_{0\sim1}) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_{c1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{89}$$

$$\Phi_2(0_{0\sim2}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_{c2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{90}$$

Hence,

$$\Phi_E = \begin{bmatrix} 0 & 1 & & & \\ 0 & 0 & & & \\ & & 0 & 1 & 0 \\ & & 0 & 0 & 1 \\ & & 0 & 0 & 0 \end{bmatrix}, \quad B_{cE} = \begin{bmatrix} 0 & \\ 1 & \\ & 0 \\ & 0 \\ & 1 \end{bmatrix}. \tag{91}$$

According to Lemma 2, the predictor of the system is given by

$$\dot{Z}(s) \;=\; \Phi_E Z(s) + B_{cE} f\left(Z(s-\tau_j(s+t))\right|_{j=1\sim3}, s+t)$$

$$+B_{cE}B\left(Z\left(s-\tau_3\left(s+t\right)\right),s+t\right)u\left(s+t-h\right),\ s\in\left[0,h\right], \tag{92}$$

with the following initial value condition:

$$Z\left(\theta\right)=x_k^{(0\sim\mu_k-1)}(\theta+t)|_{k=1\sim2},\ \theta\in\left[-\tau\left(t\right),0\right]. \tag{93}$$

As a consequence, there holds

$$x_k^{(0\sim\mu_k-1)}\left(t+s\right)|_{k=1\sim2}=\begin{bmatrix}x_1\left(t+s\right)\\ \dot{x}_1\left(t+s\right)\\ x_2\left(t+s\right)\\ \dot{x}_2\left(t+s\right)\\ \ddot{x}_2\left(t+s\right)\end{bmatrix}=Z\left(s\right),\ s\in\left[0,h\right]. \tag{94}$$

6.3 The controller

It follows from Theorem 2 that the controller for the system is then given by

$$\begin{cases}u\left(t\right)=-B^{-1}\left(Z\left(h-\tau_3\left(t+h\right)\right),t+h\right)\\ \qquad\times\left[f\left(Z\left(h-\tau_j\left(t+h\right)\right)|_{j=1\sim3},t+h\right)-u^*\left(t\right)\right],\\ u^*\left(t\right)=AZ\left(h\right)+v\left(t+h\right),\ t\geqslant0,\\ \dot{Z}(s)=\Phi_EZ\left(s\right)+B_{cE}f\left(Z\left(s-\tau_j\left(s+t\right)\right)|_{j=1\sim3},s+t\right)\\ \qquad+B_{cE}B\left(Z\left(s-\tau_3\left(s+t\right)\right),s+t\right)u\left(s+t-h\right),\ s\in\left[0,h\right],\\ Z\left(\theta\right)=x_k^{(0\sim\mu_k-1)}(\theta+t)|_{k=1\sim2},\ \theta\in\left[-\tau\left(t\right),0\right],\end{cases} \tag{95}$$

where $A\in\mathbb{R}^{2\times5}$ is a parameter matrix. The closed-loop system is given by

$$x_k^{(\mu_k)}|_{k=1\sim2}=Ax_k^{(0\sim\mu_k-1)}|_{k=1\sim2}+v,$$

which can be rewritten into the following state-space form:

$$\dot{x}_k^{(0\sim\mu_k-1)}|_{k=1\sim2}=\left(\Phi_E+B_{cE}A\right)x_k^{(0\sim\mu_k-1)}|_{k=1\sim2}+B_{cE}v. \tag{96}$$

Select the desired poles as $\{-1,-2,-3,-4,-5\}$. Then, applying the eigenstructure assignment method in [46] to the matrix pair $[\Phi_E,B_{cE}]$ gives

$$A=\begin{bmatrix}-13.4929 & -7.5428 & 2.1753 & 2.7877 & 0.5713\\ 4.9263 & 1.5856 & -9.6878 & -16.1647 & -7.4572\end{bmatrix}.$$

Please note that the initial values of the closed-loop system, namely, $x_1^{(0\sim1)}\left(0\right)$ and $x_2^{(0\sim2)}\left(0\right)$ are not arbitrarily chosen. They are generated by the system initial values

$$x_1\left(s\right)=\phi_1\left(s\right),\ \dot{x}_1\left(s\right)=\phi_2\left(s\right),\ s\in(-c_1,0],$$

$$x_2\left(s\right)=\phi_3\left(s\right),\ \dot{x}_2\left(s\right)=\phi_4\left(s\right),\ \ddot{x}_2\left(s\right)=\phi_5\left(s\right),\ s\in(-c_2,0],$$

where $\phi_i\left(t\right),\ i=1,2,\ldots,5$ are the properly selected initial functions, and

$$\begin{aligned}c_1&=-\inf\{t-\tau_i\left(t\right),\ i=1,2,\ t\geqslant0\},\\ c_2&=-\inf\{t-\tau_i\left(t\right),\ i=1,2,3,\ t\geqslant0\}.\end{aligned}$$

7 Concluding remarks

Control of a nonlinear system has already been a challenging problem in state-space theories, while control of a nonlinear system with time delays is even more challenging, especially that of a nonlinear system with both state and input delays.

Very recently, a new simple and effective approach, different from the state-space approach, is proposed for the problem of nonlinear control, which is termed as the FAS approach (see [28–30] and [31–40]). It is shown in the separate first part of [44] that the FAS approach can also be well-extended to effectively solve the control of a nonlinear system with state delays only. Meanwhile, in this paper, it is further shown that the control of a nonlinear system with both state and input delays can also be addressed by the proposed FAS approaches.

The general continuous-time single-order and multi-order FAS models with state delays only, introduced in the first part of the paper, are generalized into corresponding FAS models with both state and input delays. With the help of the proposed prediction schemes for both the single-order and multi-order FASs with input delays, controllers of both types of systems are designed using the full-actuation feature. As a result, the closed-loop systems can also be turned into constant and linear ones with arbitrarily assignable eigenstructures.

The results in this paper can be generalized into the case that the considered FAS also possesses multi-input delays and/or time-varying delays. Furthermore, for simplicity, this paper concentrates on only the case of (global) FASs, whereas the first part of the paper has also proposed the continuous-time nonlinear sub-FASs with time-varying delays. Control of continuous-time sub-FASs with both state and input delays will be addressed in the third part of this paper. Finally, the robust control approaches developed in [33, 35] and the disturbance attenuation technique presented in [36] can also be generalized to time-delay systems.

Acknowledgements This work has been partially supported by Major Program of National Natural Science Foundation of China (Grant Nos. 61690210, 61690212), National Natural Science Foundation of China (Grant No. 61333003), and Science Center Program of the National Natural Science Foundation of China (Grant No. 62188101). The author is grateful to his Ph.D. students Guangtai TIAN, Qin ZHAO, Weizhen LIU, Hong JIANG, Xiubo WANG, Liyao HU, Kaixin CUI, etc., for helping him with reference selection and proofreading. His particular thanks go to Prof. Bin ZHOU for his helpful comments, and his student Tianyi ZHAO for help with the example simulations.

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