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Compensation of uncertain linear actuator dynamics for a class of cascaded PDE-ODE systems

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Dear editor,

Boundary control of the distributed parameter systems (DPSs) described by partial differential equations (PDEs) or cascaded PDE-ODEs (ordinary differential equations) has received considerable attention over the last two decades (see [1–4] and the references therein). But most studies just consider the actuation of PDE boundary conditions, regardless of the actuator dynamics. The actuator is a pivotal component in practice whose dynamics cannot be neglected and usually yields the ineffectiveness of the traditional methods on this topic. It motivates us to investigate such a nontrivial control problem to design a desirable feedback controller for DPSs with actuator dynamics.

In this study, we consider the stabilization of the following cascaded heat PDE-ODE system with arbitrary-order uncertain linear actuator dynamics:

$$\begin{cases} \dot{X}(t) = AX(t) + Bu(0,t), \\ \partial_t u(x,t) = \partial_x^2 u(x,t), \\ \partial_x u(0,t) = 0, \\ u(1,t) = Z_1(t), \\ \dot{Z}_i(t) = Z_{i+1}(t) + Z_{[i]}^{\mathrm{T}}(t)\nu_i,\ i = 1,\ldots,n-1, \\ \dot{Z}_n(t) = U(t) + Z^{\mathrm{T}}(t)\nu_n, \end{cases} \tag{1}$$

where $X \in \mathbb{R}^m$ and $u \in \mathbb{R}$ are system states, $Z \in \mathbb{R}^n$ with $Z_{[i]} = (Z_1,\ldots,Z_i)^{\mathrm{T}}$ and $Z = Z_{[n]}$ is the actuator state, U is the control input; A and B are known matrices with appropriate dimensions and the pair (A,B) is stabilizable; $\nu_i = (\nu_{i1},\ldots,\nu_{ii})^{\mathrm{T}}$, $i = 1,\ldots,n$ are unknown constant vectors.

Remarkably, system (1) has the following distinctive characteristics compared with related literature, which highlights the main contributions and novelty of the study. (1) The actuator dynamics is considered in this study but neglected in [1–4]. (2) Unknowns are contained in the actuator dynamics but not in [5–9]. (3) The PDE subsystem is parabolic in this study, different from the hyperbolic ones in [7–9]. Mainly owing to the essential differences mentioned above, the existing control design and performance analysis methods become incapable. Thus, a compensation mechanism for the actuator dynamics and the unknowns therein should be introduced to develop a novel control design method.

To solve the control problem, a novel control design framework is established by skillfully combining the adaptive technique with the backstepping method in both infinite and finite dimensions. First, an infinite-dimensional backstepping transformation is introduced to change the original system into a new one. Then, for the new system, a series of ODE backstepping transformations are adopted to derive an adaptive state-feedback controller which guarantees that all the states of the resulting closed-loop system are bounded while those of the original system converge to zero.

Adaptive control design. We first adopt the following infinite-dimensional backstepping transformation and its inverse ([4]):

$$\begin{cases} \omega(x) = u(x) - \int_0^x \kappa(x,y)u(y)\mathrm{d}y - \varphi(x)X, \\ u(x) = \omega(x) + \int_0^x \iota(x,y)\omega(y)\mathrm{d}y + \psi(x)X, \end{cases} \tag{2}$$

where κ, ι, φ and ψ are given as follows:

$$\begin{cases} \varphi(x) = [K \quad 0] \exp\left(\begin{bmatrix} 0 & A \\ I & 0 \end{bmatrix} x \right) \begin{bmatrix} I \\ 0 \end{bmatrix}, \\ \psi(x) = [K \quad 0] \exp\left(\begin{bmatrix} 0 & A+BK \\ I & 0 \end{bmatrix} x \right) \begin{bmatrix} I \\ 0 \end{bmatrix}, \\ \kappa(x,y) = \int_0^{x-y} \varphi(\xi)B\mathrm{d}\xi,\ \iota(x,y) = \int_0^{x-y} \psi(\xi)B\mathrm{d}\xi. \end{cases} \tag{3}$$

Note that all the functions given above are bounded and smooth. Such property is useful to guarantee the implementation of the designed controller (see Appendix C).

By the above transformations, system (1) is changed into

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the following new one (called the target system):

$$\begin{cases} \dot{X} = (A + BK)\, X + B\omega(0), \\ \partial_t \omega = \partial_x^2 \omega, \\ \partial_x \omega(0) = 0, \\ \omega(1) = Z_1 - \chi(u, X), \\ \dot{Z}_i = Z_{i+1} + Z_{[i]}^{\mathrm{T}} \nu_i,\ i = 1, \ldots, n-1, \\ \dot{Z}_n = U + Z^{\mathrm{T}} \nu_n, \end{cases} \tag{4}$$

where K is chosen to make $A + BK$ be Hurwitz and $\chi = \int_0^1 \kappa(1, y)u(y)\mathrm{d}y + \varphi(1)X$. The detailed derivation of the above system is quite similar to that of [4] (see Proposition 1 therein) and hence omitted here.

For system (4), an adaptive state-feedback controller is designed as follows:

$$\begin{cases} U = -c_n\zeta_n - \zeta_{n-1} - h_n^{\mathrm{T}}\hat{\delta}_n + \sum_{i=0}^{n-1} \frac{\partial \tau_{n-1}}{\partial \chi^{(i)}} \chi^{(i+1)} \\ \quad + \sum_{i=1}^{n-1} \Big(\frac{\partial \tau_{n-1}}{\partial Z_i} Z_{i+1} + \frac{\partial \tau_{n-1}}{\partial \hat{\delta}_i} \zeta_i h_i \\ \quad + \frac{\partial \tau_{n-1}}{\partial (\partial_x^{2i-1}\omega(1))} \partial_x^{2i+1}\omega(1) \Big), \\ \dot{\hat{\delta}}_k = \zeta_k h_k,\ k = 1, \ldots, n, \end{cases} \tag{5}$$

where $\tau_0 = \chi$, ζ_k and τ_k are defined as

$$\begin{aligned} \zeta_k \ = \ & Z_k - \tau_{k-1}(Z_1, \ldots, Z_{k-1}, \hat{\delta}_1, \ldots, \hat{\delta}_{k-1}, \chi, \dot{\chi}, \ldots, \\ & \chi^{(k-1)}, \partial_x^{2k-3}\omega(1), \partial_x^{2k-5}\omega(1), \ldots, \partial_x \omega(1)), \end{aligned} \tag{6}$$

$$\begin{cases} \tau_1 = -\rho \partial_x \omega(1) - \Big(c_1 + \frac{3|PB|^2}{\underline{\lambda}_Q} \Big)(Z_1 - \chi) - Z_{[1]}^{\mathrm{T}} \hat{\delta}_1 + \dot{\chi}, \\ \tau_k = -c_k\zeta_k - \zeta_{k-1} - h_k^{\mathrm{T}}\hat{\delta}_k + \sum_{i=0}^{k-1} \frac{\partial \tau_{k-1}}{\partial \chi^{(i)}} \chi^{(i+1)} \\ \quad + \sum_{i=1}^{k-1} \Big(\frac{\partial \tau_{k-1}}{\partial Z_i} Z_{i+1} + \frac{\partial \tau_{k-1}}{\partial \hat{\delta}_i} \zeta_i h_i \\ \quad + \frac{\partial \tau_{k-1}}{\partial (\partial_x^{2i-1}\omega(1))} \partial_x^{2i+1}\omega(1) \Big), \end{cases} \tag{7}$$

$h_1 = Z_{[1]}$, $h_k^{\mathrm{T}} = -(\frac{\partial \tau_{k-1}}{\partial Z_1} Z_{[1]}^{\mathrm{T}}, \ldots, \frac{\partial \tau_{k-1}}{\partial Z_{k-1}} Z_{[k-1]}^{\mathrm{T}}, -Z_{[k]}^{\mathrm{T}})$, $c_k > 0$, $P = P^{\mathrm{T}} > 0$ is the solutions of the following Lyapunov equation:

$$(A + BK)^{\mathrm{T}} P + P(A + BK) = -Q, \tag{8}$$

for any $Q = Q^{\mathrm{T}} > 0$ and $\rho > \frac{5|PB|^2}{\underline{\lambda}_Q}$ with $\underline{\lambda}_Q$ being the minimal eigenvalue of Q.

Stability analysis. The following proposition is given to show the boundedness and integrability of closed-loop system signals. The proofs of the following claims (9) and (10) are given in Appendixes A and B, respectively.

Proposition 1. The designed controller (5) with (6) and (7) guarantees that

$$\begin{cases} |X|, \|\omega\|, \zeta_k, \|\partial_x \omega\| \in \mathcal{L}_2 \cap \mathcal{L}_\infty, \\ |\hat{\delta}_k|, \sup_{x\in[0,1]} |\omega(x,t)| \in \mathcal{L}_\infty, \\ \lim_{t\to+\infty} |X(t)| = 0, \\ \lim_{t\to+\infty} \sup_{x\in[0,1]} |\omega(x,t)| = 0, \end{cases} \tag{9}$$

$$\begin{cases} \|\partial_x^{2k}\omega\|, \|\partial_x^{2k+1}\omega\|, \partial_x^{2k-1}\omega(1) \in \mathcal{L}_2 \cap \mathcal{L}_\infty, \\ \dot{\zeta}_k, \tau_{k-1}, \dot{\tau}_{k-1}, Z_k, \dot{Z}_k \in \mathcal{L}_2 \cap \mathcal{L}_\infty, \end{cases} \tag{10}$$

where $\|f\| = \sqrt{\int_0^1 f(x)^2 \mathrm{d}x}$ for the function f defined on $[0, 1]$ and $k = 1, ..., n$.

By the above proposition and using the well-known Barbalat's Lemma, we obtain that $\lim_{t\to+\infty} |Z(t)| = 0$. Moreover, because of the converge of w and X, transformation (2) gives that u is uniformly bounded and converges to zero ultimately. The main results are summarized by the following theorem.

Theorem 1. Consider system (1) with unknown parameters therein. The controller (5) with (6) and (7) guarantees that all the states of the closed-loop system are bounded, and all the states of the original system, i.e., X, u and Z, converge to zero, i.e.,

$$\lim_{t\to+\infty} |X(t)| = 0, \lim_{t\to+\infty} \sup_{x\in[0,1]} |u(x,t)| = 0, \lim_{t\to+\infty} |Z(t)| = 0. \tag{11}$$

The implementation of the designed controller is discussed in Appendix C. A simulation example is provided in Appendix D to validate the effectiveness of the proposed controller.

Concluding remarks. In this study, the stabilization problem has been solved for a class of cascaded PDE-ODE systems with uncertain linear actuator dynamics. An adaptive state-feedback controller is explicitly designed by skillfully incorporating the adaptive technique into the backstepping schemes in both infinite and finite dimensions, which guarantees the desirable stability of the resulting closed-loop system. It is worth pointing out that the proposed control scheme is applicable for the stabilization of other systems. For example, the actuator dynamics may be nonlinear which can be stabilized by the ODE backstepping scheme, and the PDE subsystem may be other linear ones, such as the wave equation, which are suitable for using an infinite-dimensional backstepping scheme. The future research direction will focus on the output feedback stabilization of such a system, which is rather challenging since the unmeasurable states should be reconstructed while the parametric unknowns should be compensated.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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