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Global output feedback adaptive stabilization for systems with long uncertain input delay

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Dear editor,

The uncertain input delay is frequently encountered in engineering control systems. Adaptation is indispensable when the uncertain input delay is significant. In existing delay-adaptive controllers [1–6], the actuator state must be measured to achieve global stability. Recently, a logic-based switching delay-adaptive state-feedback controller [7] was proposed to realize global stability without measuring the actuator state. The idea of this approach is as follows: first, several controller candidates are designed offline by solving a set of linear matrix inequalities (LMIs) such that at least one controls the system satisfactorily; then, a logic-based switching mechanism is designed to identify the proper controller online.

This study further develops the approach of logic switching [7] to enable output feedback control. The challenges of this study are summarized as follows. First, the separation principle does not hold for linear systems with an unknown input delay; thus, stability analysis is difficult for output feedback control. Second, the Lyapunov-Krasovskii functional contains immeasurable states when only the output is measured; thus, the switching mechanism is difficult to design.

Problem formulation. Consider the following linear time-invariant system with an unknown input delay:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-d), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^q$ is the measured output, and A, B, and C are known constant matrices with compatible dimensions. Here (A, B) is stabilizable and (C, A) is detectable. In addition, d is the uncertain constant input delay, and there are known constants $0 \leqslant \underline{d} < \overline{d}$ such that $\underline{d} \leqslant d < \overline{d}$. Theoretically, d can be arbitrarily large because we do not impose restrictions on the value of $\bar{d}$.

The objective is to design an output feedback adaptive control law to asymptotically stabilize the system (1).

Controller structure. We define $N+1$ constant numbers $\hat{d}_0, \hat{d}_1, \ldots, \hat{d}_N$ to be determined, which serve as the candidate delay estimates. The interval $[\underline{d}, \overline{d}]$ is divided into disjoint subintervals of $[\hat{d}_0, \hat{d}_1), [\hat{d}_1, \hat{d}_2), \ldots, [\hat{d}_{N-1}, \hat{d}_N)$ with

$$\hat{d}_0 = \underline{d};\ \hat{d}_r = \hat{d}_{r-1} + \Delta d_{r-1}\,(r = 1, 2, \ldots, N);\ \overline{d} < \hat{d}_N. \tag{2}$$

Since our controller is a switching type controller, the switching moments are denoted as $0 = t_0 < t_1 < \cdots < t_k < \cdots$, where t_k is the k-th switching moment. The control law between any two consecutive switching moments is time-invariant. Assume there exist $i \in \mathbb{N}$ and $r \in \{0, 1, \ldots, N-1\}$ such that $k = iN + r$. Then, for $t \in [t_k, t_{k+1})$, we guess that $d \in [\hat{d}_r, \hat{d}_r + \Delta d_r)$, where $\hat{d}_r$ is the estimate. In addition, we use L_r and K_r to represent the observer and controller gain matrices, respectively. Specifically, we give the following delay-dependent observer:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t - \hat{d}_r) - L_r(y(t) - C\hat{x}(t)), \tag{3}$$

where $\hat{x} \in \mathbb{R}^n$ is the observer state. Then, the observation error $\bar{x} = x - \hat{x}$ is governed by

$$\dot{\bar{x}}(t) = (A + L_r C)\bar{x}(t) + B[u(t-d) - u(t - \hat{d}_r)]. \tag{4}$$

Next, we define the following predictor transformation:

$$\zeta(t, \hat{d}_r) = \hat{x}(t) + \mathrm{e}^{A(t-\hat{d}_r)} \int_{t-\hat{d}_r}^{t} \mathrm{e}^{-A\tau} Bu(\tau)\mathrm{d}\tau. \tag{5}$$

Finally, the following piecewise linear time-invariant controller is applied for every $t \in [t_k, t_{k+1})$:

$$u(t) = K_r \zeta(t, \hat{d}_r),\ t_k \leqslant t < t_{k+1}, \tag{6}$$

where K_r is designed to render $A + \mathrm{e}^{-A\hat{d}_r} BK_r$ Hurwitz. Note that if (A, B) is stabilizable, $(A, \mathrm{e}^{-A\hat{d}_r} B)$ is stabilizable [8]. By combining (4), (5), and (6) and utilizing the transformation $\zeta(t - \hat{d}_r, \hat{d}_r) - \zeta(t - d, \hat{d}_r) = \int_{t-d}^{t-\hat{d}_r} \dot{\zeta}(\tau, \hat{d}_r)\mathrm{d}\tau$,

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we can obtain the following closed-loop system for $t \in [t_k + d, t_{k+1})$:

$$\begin{cases} \dot{\bar{x}}(t) = (A + L_rC)\bar{x}(t) + BK_rL_rC \displaystyle\int_{t-d}^{t-\hat{d}_r} \bar{x}(\tau)\mathrm{d}\tau \\ \qquad - BK_r(A + \mathrm{e}^{-A\hat{d}_r}BK_r) \displaystyle\int_{t-d}^{t-\hat{d}_r} \zeta(\tau, \hat{d}_r)\mathrm{d}\tau, \\ \dot{\zeta}(t, \hat{d}_r) = (A + \mathrm{e}^{-A\hat{d}_r}BK_r)\zeta(t, \hat{d}_r) - L_rC\bar{x}(t). \end{cases} \tag{7}$$

Note that there exists $0 \leqslant s < N$ satisfying $d \in [\hat{d}_s, \hat{d}_s + \Delta d_s)$. We then assume that a switching is triggered at $t = t_s$, while our switching mechanism ensures that no more switchings will be triggered prior to $t = p_s$, where $p_s = t_s + (\hat{d}_s + \Delta d_s) \geqslant t_s + d$. Thus, the closed-loop system takes the form of (7) for $t \in [p_s, t_{s+1})$.

We define the Lyapunov-Krasovskii functional $V_s = V_{1s} + V_{2s}$ (Appendix A), where $P_s, Q_s \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices. We present the following result, which is the key to obtaining the main result.

Lemma 1. Suppose $d \in [\hat{d}_s, \hat{d}_s + \Delta d_s)$ and a switching is triggered at $t = t_s$, while our switching mechanism ensures that no more switchings will be triggered prior to $t = p_s$. Then, for $t \in [p_s, t_{s+1})$, the time derivative of V_s satisfies

$$\dot{V}_s \leqslant -\varepsilon_s \zeta^{\mathrm{T}} P_s^{-1} Q_s^{-1} P_s^{-1} \zeta \tag{8}$$

for some $\varepsilon_s > 0$, if the following LMIs:

$$\begin{gathered} \begin{bmatrix} \Theta_s & (AP_s + \mathrm{e}^{-A\hat{d}_s}BR_s)^{\mathrm{T}} \\ (AP_s + \mathrm{e}^{-A\hat{d}_s}BR_s) & \dfrac{1}{\Delta d_s}P_s \end{bmatrix} \geqslant 0, \\ (Q_sA + S_sC) + (Q_sA + S_sC)^{\mathrm{T}} + M_{1s} + 2M_{2s} \leqslant 0, \\ \begin{bmatrix} Q_s & I \\ I & P_s \end{bmatrix} \geqslant 0, \quad \begin{bmatrix} M_{1s} & (S_sC)^{\mathrm{T}} \\ S_sC & \left(\dfrac{1}{\varepsilon_s} + \Delta d_s\right)^{-1} Q_s \end{bmatrix} \geqslant 0, \\ \begin{bmatrix} M_{2s} & Q_s \\ Q_s & \dfrac{1}{\varepsilon_s}I \end{bmatrix} \geqslant 0, \quad \begin{bmatrix} \varepsilon_s I & BR_s \\ (BR_s)^{\mathrm{T}} & \dfrac{1}{\Delta d_s}P_s \end{bmatrix} \geqslant 0, \end{gathered} \tag{9}$$

where $\varepsilon_s > 0$, $\Delta d_s > 0$ are preselected constant numbers and $\Theta_s = -(AP_s + \mathrm{e}^{-A\hat{d}_s}BR_s) - (AP_s + \mathrm{e}^{-A\hat{d}_s}BR_s)^{\mathrm{T}} - 2\varepsilon_s P_s$, are solvable by symmetric positive definite matrices P_s, Q_s and matrices $M_{1s}, M_{2s} \in \mathbb{R}^{n\times n}$, $R_s \in \mathbb{R}^{m\times n}$, $S_s \in \mathbb{R}^{n\times q}$. In addition, the designed gain matrices are given by $K_s = R_sP_s^{-1}$ and $L_s = Q_s^{-1}S_s$. A detailed proof is provided in Appendix B.

Parameter selection. The controller and observer parameters include $\hat{d}_r$, Δd_r, ε_r, P_r, Q_r, K_r, L_r for $r = 0, 1, \ldots, N-1$. Algorithm 1 is an offline algorithm to find these parameters and ceases after finite iterations [7].

Algorithm 1 Parameter selection (offline)

Require: $r = 0, \hat{d}_0 = \underline{d}$;
1: **while** $\hat{d}_r \leqslant \bar{d}$ **do**
2: Preselect sufficiently small $\Delta d_r > 0$ and $\varepsilon_r > 0$ such that the quadruple (P_r, Q_r, R_r, S_r) is the solution to the LMIs (9) with s replaced by r;
3: Compute $K_r = R_rP_r^{-1}$ and $L_r = Q_r^{-1}S_r$;
4: $r \Leftarrow r + 1$;
5: $\hat{d}_r \Leftarrow \hat{d}_r + \Delta d_r$;
6: $N \Leftarrow r$;
7: **end while**

Tuning mechanism. The purpose of this mechanism is to tune the uncertain d with a logic-based switching rule. Here, the Lyapunov-Krasovskii functional $V_r(t)$ is divided into $V_r(t) = V_{r,\zeta}(t) + V_{r,\bar{x}}(t)$ ($V_{r,\zeta}(t)$ and $V_{r,\bar{x}}(t)$ are described in Appendix A). Note that $V_{r,\zeta}(t)$ is measurable while $V_{r,\bar{x}}(t)$ is not. The following tuning mechanism is given.

A sequence of strictly increasing numbers $\{\theta_k, k = 0, 1, \ldots\}$ is preselected such that $\lim_{t\to\infty} \theta_k = \infty$. Here, consider the general situation where $t \in [t_k, t_{k+1})$ with $k = iN + r$ and $p_k = t_k + (\hat{d}_r + \Delta d_r)$. The parameters are set to $(\hat{d}_r, \Delta d_r, \varepsilon_r, P_r, Q_r, K_r, L_r, V_r)$ generated by Algorithm 1. If the inequality

$$\begin{aligned} & V_{r,\zeta}(t) - V_{r,\zeta}(p_k) \\ & - \lambda_{\max}(Q_r)\left(\alpha_r^2 + \int_{-(\hat{d}_r+\Delta d_r)}^{-\hat{d}_r} \int_{p_k+\theta}^{p_k} \beta_r^2(\tau)\mathrm{d}\tau\mathrm{d}\theta\right) \\ & > -\varepsilon_r \int_{p_k}^{t} \zeta^{\mathrm{T}}(\tau)(P_rQ_rP_r)^{-1}\zeta(\tau)\mathrm{d}\tau \end{aligned} \tag{10}$$

holds at some time $t \geqslant p_k$ (α_r and $\beta_r(\tau)$ are described in Appendix A), then $t = t_{k+1}$ is set as the new switching moment.

Theorem 1. The closed-loop system corresponding to the system (1), observer (3), controller (6), and the tuning mechanism is globally and asymptotically stable. In other words, all closed-loop states are bounded on $[0, \infty)$ for any initial condition, and $\lim_{t\to\infty} x(t) = 0$ holds.

Proof. See Appendix C for the proof.

Simulations are discussed in Appendix D.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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