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Static output feedback control for uncertain Roesser-type continuous-time two-dimensional piecewise affine systems

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Dear editor,

Many physical processes are of two-dimensional (2-D) characteristics and can be modeled by the celebrated Roesser-type 2-D systems [1,2]. Due to the approximating capability of piecewise affine (PWA) dynamic modeling [3], it is more interesting to study the 2-D PWA systems [4]. However, to the best of our knowledge, there are few results addressing the continuous-time 2-D PWA systems, especially based on piecewise Lyapunov functions. Due to substantial differences in their structures, attempts to analyze 2-D processes modeled by 2-D piecewise affine systems using conventional 1-D systems theory generally fail because such an approach ignores their inherent 2-D systems structure. The structural complexity of 2-D piecewise affine systems makes the construction of the corresponding piecewise Lyapunov function more challenging than 1-D systems.

The contributions can be summarized as follows. First, our work, to our best knowledge, is the first attempt at studying the piecewise Lypuanov function-based stability analysis and synthesis problem of Roesser-type continuous-time 2-D PWA systems. Second, an augmentation approach is developed such that the closed loop system can be rewritten in a descriptor form, which makes it more convenient for PWA static output feedback (SOF) controller synthesis. Third, a very general control approach is developed for continuous-time 2-D PWA systems, which can be easily extended to the cases of dynamic output feedback and/or piecewise linear control.

Problem description. Consider an uncertain Roesser-type continuous-time 2-D PWA system:

$$\begin{aligned} x^+(t_1,t_2) &= (A_i+\Delta A_i)x(t_1,t_2)+a_i+\Delta a_i \\ &\quad +(B_i+\Delta B_i)u(t_1,t_2)+D_{i1}w(t_1,t_2), \\ y(t_1,t_2) &= (C_i+\Delta C_i)x(t_1,t_2)+D_{i2}w(t_1,t_2), \\ z(t_1,t_2) &= L_i x(t_1,t_2),\ x(t_1,t_1)\in\mathcal{S}_i,\ \ i\in\mathcal{I}, \end{aligned} \tag{1}$$

where $\{\mathcal{S}_i\}_{i\in\mathcal{I}} \subseteq \mathbb{R}^{n_x}$ denotes the partitioned polyhedral subspaces of the system state space, $\mathcal{I} = \{1,2,\ldots,r\}$ is an index set of the subspaces; $x(t_1,t_2) = [\,x^{\rm h}(t_1,t_2)\quad x^{\rm v}(t_1,t_2)\,]$ is the system state, $x^+(t_1,t_2) = [\,\frac{\partial x^{\rm h}(t_1,t_2)}{\partial t_1}\quad \frac{\partial x^{\rm v}(t_1,t_2)}{\partial t_2}\,]^{\rm T}$, $x^{\rm h}(t_1,t_2)\in\mathbb{R}^{n_x^{\rm h}}$ is the horizontal state; $x^{\rm v}(t_1,t_2)\in\mathbb{R}^{n_x^{\rm v}}$ is the vertical state; $u(t_1,t_2)\in\mathbb{R}^{n_u}$ and $w(t_1,t_2)\in\mathbb{R}^{n_w}$ denote the control input and disturbance input, respectively; $y(t_1,t_2)\in\mathbb{R}^{n_y}$ and $z(t_1,t_2)\in\mathbb{R}^{n_z}$ denote the measured and regulated outputs, respectively; the matrices A_i, a_i, B_i, D_{i1}, C_i, D_{i2}, and L_i are known real constant matrices. The uncertain matrices can be written as $[\,\Delta A_i\ \ \Delta a_i\ \ \Delta B_i\,] = U_{i1}\Delta_i(t_1,t_2)[\,W_{i1}\ \ W_{i2}\ \ W_{i3}\,]$, $\Delta C_i = U_{i2}\Delta_i(t_1,t_2)W_{i1}$, $i\in\mathcal{I}$ with U_{i1}, U_{i2}, W_{i1}, W_{i2}, and W_{i3} being known real constant matrices, and $\Delta_i^{\rm T}(t_1,t_2)\Delta_i(t_1,t_2)\leqslant \mathbf{I}_{p_2}$, $i\in\mathcal{I}$. Divide the state subspaces $\{\mathcal{S}_i\}_{i\in\mathcal{I}}$ into the subspaces covering the origin $\{\mathcal{S}_i\}_{i\in\mathcal{I}_0}$ and the subspaces not covering the origin $\{\mathcal{S}_i\}_{i\in\mathcal{I}_1}$, where $\mathcal{I}_0$ and $\mathcal{I}_1$ represent their corresponding subspace index sets and satisfy $\mathcal{I}=\mathcal{I}_0\cup\mathcal{I}_1$. Propose a PWA SOF controller

$$u(t_1,t_2) = K_i y(t_1,t_2)+k_i,\ i\in\mathcal{I}, \tag{2}$$

where $k_i = 0$, $i\in\mathcal{I}_0$, $k_i\in\mathbb{R}^{n_u\times 1}$, $i\in\mathcal{I}_1$, and $K_i\in\mathbb{R}^{n_u\times n_y}$, $i\in\mathcal{I}$ are controller gains. Rewrite (2) as $0\times\frac{\partial u(t_1,t_2)}{\partial t_2} = K_i y(t_1,t_2)+k_i-u(t_1,t_2)$, $i\in\mathcal{I}$. Let

$$\left[\,\bar{x}^+(t_1,t_2)\,\middle|\,\bar{x}(t_1,t_2)\,\right] = \begin{cases} \left[\begin{array}{c|c} x^+(t_1,t_2) & x(t_1,t_2) \\ \frac{\partial u(t_1,t_2)}{\partial t_2} & u(t_1,t_2) \end{array}\right], & i\in\mathcal{I}_0, \\ \left[\begin{array}{c|c} 0 & 1 \\ x^+(t_1,t_2) & x(t_1,t_2) \\ \frac{\partial u(t_1,t_2)}{\partial t_2} & u(t_1,t_2) \end{array}\right], & i\in\mathcal{I}_1. \end{cases}$$

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One has the closed loop system

$$\begin{cases} E_i\bar{x}^{+}(t_1,t_2)=\bar{A}_i\bar{x}(t_1,t_2)+\bar{D}_iw(t_1,t_2), \\ z(t_1,t_2)=\bar{L}_i\bar{x}(t_1,t_2), \end{cases} \tag{3}$$

where $E_i=\mathsf{diag}\left\{\mathbf{I}_{n_{\bar{x}}},\ \mathbf{0}_{n_u\times n_u}\right\}$, $n_{\bar{x}}=n_x^{\mathrm{h}}+n_x^{\mathrm{v}}$, if $i\in\mathcal{I}_0$, $n_{\bar{x}}=1+n_x^{\mathrm{h}}+n_x^{\mathrm{v}}$, if $i\in\mathcal{I}_1$, and

$$\begin{cases} \begin{cases} \bar{A}_i=\begin{bmatrix} A_i+\Delta A_i & B_i+\Delta B_i \\ K_i(C_i+\Delta C_i) & -\mathbf{I}_{n_u} \end{bmatrix}, \\ \bar{D}_i=\begin{bmatrix} D_{i1}^{\mathrm{T}} & D_{i2}^{\mathrm{T}}K_i^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ \bar{L}_i=\begin{bmatrix} L_i & \mathbf{0}_{n_z\times n_u} \end{bmatrix},\ i\in\mathcal{I}_0, \end{cases} \\ \begin{cases} \bar{A}_i=\begin{bmatrix} 0 & \mathbf{0}_{1\times n_x} & \mathbf{0}_{1\times n_u} \\ a_i+\Delta a_i & A_i+\Delta A_i & B_i+\Delta B_i \\ k_i & K_i(C_i+\Delta C_i) & -\mathbf{I}_{n_u} \end{bmatrix}, \\ \bar{D}_i=\begin{bmatrix} \mathbf{0}_{n_w\times 1} & D_{i1}^{\mathrm{T}} & D_{i2}^{\mathrm{T}}K_i^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ \bar{L}_i=\begin{bmatrix} \mathbf{0}_{n_z\times 1} & L_i & \mathbf{0}_{n_z\times n_u} \end{bmatrix},\ i\in\mathcal{I}_1. \end{cases} \end{cases} \tag{4}$$

The closed loop system (3) is in a descriptor form and its regularity and non-impulsiveness can be guaranteed. More details are shown in Appendix A.

The aim is to design a controller (2) for the system (1) such that: (i) with zero disturbance input, the system (3) is asymptotically stable; (ii) under zero boundary conditions, i.e., $x^{\mathrm{h}}(0,t_2)=0$, $x^{\mathrm{v}}(t_1,0)=0$, $t_1,t_2\in[0,\infty)$, the prescribed $\mathcal{H}_\infty$ performance γ is ensured, i.e., $\|z\|_2\leqslant\gamma\|w\|_2$.

Main results. For the purpose of ensuring the continuity of the piecewise Lyapunov function across the subspace boundaries, the matrices $\mathcal{F}_i=\mathsf{diag}\left\{\bar{F}_i,\ \mathbf{I}_{n_u}\right\}$, $i\in\mathcal{I}$ are introduced, where

$$\bar{F}_i=\begin{cases} F_i=\begin{bmatrix} F_i^{\mathrm{h}} & \mathbf{0}_{n_f^{\mathrm{h}}\times n_x^{\mathrm{v}}} \\ \mathbf{0}_{n_f^{\mathrm{v}}\times n_x^{\mathrm{h}}} & F_i^{\mathrm{v}} \end{bmatrix}, & i\in\mathcal{I}_0, \\ \left[\, f_i \,\middle|\, F_i \,\right]=\left[\begin{array}{c|cc} f_i^{\mathrm{h}} & F_i^{\mathrm{h}} & \mathbf{0}_{n_x^{\mathrm{h}}\times n_f^{\mathrm{v}}} \\ f_i^{\mathrm{v}} & \mathbf{0}_{n_x^{\mathrm{v}}\times n_f^{\mathrm{h}}} & F_i^{\mathrm{v}} \end{array}\right], & i\in\mathcal{I}_1, \end{cases} \tag{5}$$

such that $\mathcal{F}_i\bar{x}(t_1,t_2)=\mathcal{F}_j\bar{x}(t_1,t_2)$, $x(t_1,t_2)\in\mathcal{S}_i\cap\mathcal{S}_j$, $i,j\in\mathcal{I}$. Note that in (5), $F_i^{\mathrm{h}}\in\mathbb{R}^{n_f^{\mathrm{h}}\times n_x^{\mathrm{h}}}$, $F_i^{\mathrm{v}}\in\mathbb{R}^{n_f^{\mathrm{h}}\times n_x^{\mathrm{v}}}$, $f_i^{\mathrm{h}}\in\mathbb{R}^{n_f^{\mathrm{h}}\times 1}$, and $f_i^{\mathrm{v}}\in\mathbb{R}^{n_f^{\mathrm{v}}\times 1}$ are matrices to be constructed. S-procedure will be utilized, and the S-procedure related matrices are introduced as $\mathcal{G}_i=\left[\, G_i\ \ \mathbf{0}_{n_g\times n_u} \,\right]$, for $i\in\mathcal{I}_0$, and $\mathcal{G}_i=\left[\, g_i\ \ G_i\ \ \mathbf{0}_{n_g\times n_u} \,\right]$, for $i\in\mathcal{I}_1$, such that $\mathcal{G}_i\bar{x}(t_1,t_2)\succeq 0$, $i\in\mathcal{I}$, where $g_i\in\mathbb{R}^{n_g\times 1}$, $G_i\in\mathbb{R}^{n_g\times n_x}$ are matrices to be constructed, and $\succeq$ means that the elements of the vector are all non-negative.

Theorem 1. The closed loop system (3) is asymptotically stable with $\mathcal{H}_\infty$ performance $\gamma>0$, if there exist positive definite matrices $P_{\mathrm{h}}\in\mathbb{S}^{n_f^{\mathrm{h}}}$, $P_{\mathrm{v}}\in\mathbb{S}^{n_f^{\mathrm{v}}}$, and matrices $0\preceq N_i\in\mathbb{S}^{n_g}$, $R_{i1}\in\mathbb{R}^{n_u\times(n_f^{\mathrm{h}}+n_f^{\mathrm{v}})}$, $R_{i2}\in\mathbb{R}^{n_u\times n_u}$, $i\in\mathcal{I}$, such that

$$\begin{bmatrix} \mathsf{Sym}\left\{\bar{A}_i^{\mathrm{T}}\mathcal{F}_i^{\mathrm{T}}\mathcal{P}_i\mathcal{F}_i\right\}+\bar{L}_i^{\mathrm{T}}\bar{L}_i+\mathcal{G}_i^{\mathrm{T}}N_i\mathcal{G}_i & \mathcal{F}_i^{\mathrm{T}}\mathcal{P}_i^{\mathrm{T}}\mathcal{F}_i\bar{D}_i \\ \bar{D}_i^{\mathrm{T}}\mathcal{F}_i^{\mathrm{T}}\mathcal{P}_i\mathcal{F}_i & -\gamma^2\mathbf{I}_{n_w} \end{bmatrix}<0, \tag{6}$$

where $\bar{A}_i$, $\bar{D}_i$, and $\bar{L}_i$ are defined in (4) and

$$\mathcal{P}_i=\begin{bmatrix} P & \mathbf{0} \\ R_{i1} & R_{i2} \end{bmatrix},\ P=\begin{bmatrix} P_{\mathrm{h}} & \mathbf{0} \\ \mathbf{0} & P_{\mathrm{v}} \end{bmatrix}. \tag{7}$$

The proof of Theorem 1 is shown in Appendix B.

Theorem 2. The closed loop system (3) is asymptotically stable with $\mathcal{H}_\infty$ performance $\gamma>0$, if there exist positive definite matrices $P_{\mathrm{h}}\in\mathbb{S}^{n_f^{\mathrm{h}}}$, $P_{\mathrm{v}}\in\mathbb{S}^{n_f^{\mathrm{v}}}$, and matrices $0\preceq N_i\in\mathbb{S}^{n_g}$, $R_{i2}\in\mathbb{R}^{n_u\times n_u}$, $\bar{K}_i\in\mathbb{R}^{n_u\times n_y}$, $i\in\mathcal{I}$, $\bar{k}_i\in\mathbb{R}^{n_u\times 1}$, $i\in\mathcal{I}_1$, and scalars $\epsilon_i>0$, $i\in\mathcal{I}$, such that

$$\begin{bmatrix} \Phi_i & \star & \star \\ D_{i1}^{\mathrm{T}}\Pi_{i1}+D_{i2}^{\mathrm{T}}\bar{K}_i\Pi_{i2} & -\gamma^2\mathbf{I}_{n_w} & \star \\ \mathcal{U}_i^{\mathrm{T}}\bar{\Pi}_i & \mathbf{0}_{2p_1\times n_w} & -\epsilon_i\mathbf{I}_{2p_1} \end{bmatrix}<0, \tag{8}$$

where $\Phi_i=\mathsf{Sym}\left\{\mathcal{A}_i^{\mathrm{T}}\Pi_{i1}+\mathcal{K}_i^{\mathrm{T}}\Pi_{i2}\right\}+\bar{L}_i^{\mathrm{T}}\bar{L}_i+\mathcal{G}_i^{\mathrm{T}}N_i\mathcal{G}_i+\epsilon_i\mathcal{W}_i^{\mathrm{T}}\mathcal{W}_i$, $\mathcal{U}_i=\mathsf{diag}\left\{U_{i1},\ \bar{K}_iU_{i2}\right\}$, $\bar{\Pi}_i=\left[\,\Pi_{i1}^{\mathrm{T}}\ \ \Pi_{i2}^{\mathrm{T}}\,\right]^{\mathrm{T}}$, $P=\mathsf{diag}\left\{P_{\mathrm{h}},\ P_{\mathrm{v}}\right\}$, $H=\left[\,\mathbf{I}_{n_u}\ \ \mathbf{0}_{n_u\times(n_f^{\mathrm{h}}+n_f^{\mathrm{v}}-n_u)}\,\right]$, and

$$\begin{cases} \mathcal{A}_i=\left[\, A_i\ \ B_i \,\right],\ \mathcal{K}_i=\left[\, \bar{K}_iC_i\ \ -R_{i2}^{\mathrm{T}} \,\right], \\ \Pi_{i1}=\left[\, F_i^{\mathrm{T}}PF_i\ \ \mathbf{0}_{(n_x^{\mathrm{h}}+n_x^{\mathrm{v}})\times n_u} \,\right],\ \Pi_{i2}=\left[\, HF_i\ \ \mathbf{I}_{n_u} \,\right], \\ \mathcal{W}_i=\begin{bmatrix} W_{i1} & W_{i3} \\ W_{i1} & \mathbf{0}_{p_2\times n_u} \end{bmatrix},\ \bar{L}_i=\left[\, L_i\ \ \mathbf{0}_{n_z\times n_u} \,\right],\ i\in\mathcal{I}_0, \\ \mathcal{A}_i=\left[\, a_i\ \ A_i\ \ B_i \,\right],\ \mathcal{K}_i=\left[\, \bar{k}_i\ \ \bar{K}_iC_i\ \ -R_{i2}^{\mathrm{T}} \,\right], \\ \Pi_{i1}=\left[\, F_i^{\mathrm{T}}Pf_i\ \ F_i^{\mathrm{T}}PF_i\ \ \mathbf{0}_{(n_x^{\mathrm{h}}+n_x^{\mathrm{v}})\times n_u} \,\right], \\ \Pi_{i2}=\left[\, Hf_i\ \ HF_i\ \ \mathbf{I}_{n_u} \,\right],\ \bar{L}_i=\left[\, \mathbf{0}_{n_z\times 1}\ \ L_i\ \ \mathbf{0}_{n_z\times n_u} \,\right], \\ \mathcal{W}_i=\begin{bmatrix} W_{i2} & W_{i1} & W_{i3} \\ \mathbf{0}_{p_2\times 1} & W_{i1} & \mathbf{0}_{p_2\times n_u} \end{bmatrix},\ i\in\mathcal{I}_1. \end{cases}$$

In addition, feasible solutions of the LMIs in (8) yield the controller gains, $K_i=R_{i2}^{-\mathrm{T}}\bar{K}_i$, $i\in\mathcal{I}$, $k_i=R_{i2}^{-\mathrm{T}}\bar{k}_i$, $i\in\mathcal{I}_1$.

The proof of Theorem 2 is shown in Appendix C.

The $\mathcal{H}_\infty$ performance index γ in Theorem 2 can be minimized by Algorithm 1.

Algorithm 1. $\min_{P_{\mathrm{h}}>0,P_{\mathrm{v}}>0,N_i\succeq 0,R_{i2},\bar{K}_i,\bar{k}_i,\epsilon_i>0}\gamma^2$, subject to LMIs (8).

Conclusion. In this study, a class of 2-D piecewise Lyapunov functions are constructed, together with a descriptor system approach, based on which a novel design method for 2-D PWA SOF controllers is proposed for uncertain Roesser-type continuous-time 2-D PWA systems. It is shown that with the proposed less-conservative control method the closed loop system achieves asymptotic stability with optimal $\mathcal{H}_\infty$ performance $\gamma_{\min}$. Simulation studies (shown in Appendix D) are given to show the effectiveness and less conservativeness of the proposed controller design results.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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