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Cyber topology design guaranteed structural controllability for networked systems

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Dear editor,

Structure analysis is of crucial importance for achieving reliable system functionality. In many real-world systems, the exact parameters of system model are not available owing to modeling uncertainties or measurement noise, and then structural controllability is pursued in control configuration design. As first proposed by Lin in [1], a structured system is said to be structurally controllable if there exists one numerical realization such that the associated system is controllable. Since then, structural controllability of linear systems and complex networks has been widely studied from algebraic and graphical perspectives under the assumption that the dimension of each node state is one [2,3]. Recently, structural controllability of networked systems with high-order subsystems has attracted extensive attention [4, 5]. When subsystems are multi-input-multi-output (MIMO), they are coupled via high-dimensional channels [6].

With the development of science and technology, networked systems extend to a broader class of systems — cyber-physical systems (CPSs), in which subsystems are physically distributed and coupled through physical and cyber couplings, such as power systems and traffic systems [7]. When a networked system is constrained by physical conditions or some system structure changes happen, existing control inputs and physical couplings may be unable to ensure structural controllability. Without changing the existing physical structure and control inputs, it is significant to design cyber topology to achieve structural controllability. However, most existing results do not distinguish these two kinds of couplings when analyzing and designing network topology for structural controllability of a networked system.

In this study, we attempt to design cyber topology for the networked systems, which used to be structurally uncontrollable under the restriction of physical conditions. Besides, the study is proposed based on the mild assumption that each input/output connects to at most a single state.

Model and methodology. Now consider a networked system $(A_{\rm sys}, B_{\rm sys})$ consisting of N heterogeneous MIMO subsystems with physical and cyber couplings between subsystems. The i-th subsystem S_i, $i \in [1:N]$ ($[1:N]$ represents the set $\{1, 2, \ldots, N\}$), has the following dynamics:

$$\begin{aligned} \dot{\boldsymbol{x}}_i(t) &= A_{ii}\boldsymbol{x}_i(t) + \sum_{j=1, j\neq i}^{N} A_{ij}\boldsymbol{x}_j(t) + B_i\boldsymbol{v}_i(t), \\ \boldsymbol{y}_i(t) &= C_i\boldsymbol{x}_i(t), \end{aligned} \tag{1}$$

with $\boldsymbol{v}_i(t) = \sum_{j=1}^{N} W_{ij}\boldsymbol{y}_j(t) + \delta_i\boldsymbol{u}_i(t)$. $A_{ij} \in \mathbb{R}^{n_i\times n_j}$, $B_i \in \mathbb{R}^{n_i\times r_i}$, $C_i \in \mathbb{R}^{p_i\times n_i}$; $\boldsymbol{x}_i \in \mathbb{R}^{n_i}$, $\boldsymbol{y}_i \in \mathbb{R}^{p_i}$ are the state and output vector, $\boldsymbol{v}_i, \boldsymbol{u}_i \in \mathbb{R}^{r_i}$ are the input signal and external control input, and matrix-valued weight $W_{ij} \in \mathbb{R}^{r_i\times p_j}$ is the cyber coupling. $\delta_i \in \{0, 1\}$ in which $\delta_i = 1$ means that S_i is directly controlled by $\boldsymbol{u}_i$, and $\delta_i = 0$ means not.

Let $W = [W_{ij}]|_{i=1,j=1}^{N}$ represent cyber couplings of the whole network (W is a block matrix with W_{ij} to be the submatrix in the i-th row, j-th column, $i, j \in [1:N]$).

Unless otherwise specified, the graph theoretic notations used in this study can be referenced in [3, 5].

Lemma 1 ([2])**.** (A, B) is structurally controllable if and only if (i) there exist control inputs on each right-unmatched vertex of a matching set M of $\mathcal{B}(\mathcal{X}, \mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$; (ii) there are directed paths from control inputs to all right-matched vertices with respect to (w.r.t.) the same M.

Firstly, consider the problem formulated as follows.

Problem 1. If $(A_{\rm sys}, B_{\rm sys})$ is structurally uncontrollable without considering cyber couplings, verify whether there is a set of W_{ij}, s.t., $(A_{\rm sys}, B_{\rm sys})$ is structurally controllable.

The following assumption is made to avoid analyzing subsystem dynamics in detail.

Assumption 1. For all subsystems $S_i, i \in [1:N]$, there exists a set of S_j whose elements are different from each other satisfying that S_j can respectively confirm structural controllability of S_i through A_{ij} and B_i.

Similar to the result in [5], the following criterion can be obtained for simplifying networked systems.

Lemma 2. Given S_i with $(A_{ii}, [A_{ij}\ B_i])$ structurally controllable and $(A_{ii}, [A_{ki}^{\rm T}\ C_i^{\rm T}]^{\rm T})$ structurally observable, there

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exists at least one matching set M_i satisfying that, (i) a subset of inputs $\mathcal{V}_i$ and incoming edges $\mathcal{E}_{\mathcal{X}_j,\mathcal{X}_i}$ act on right-unmatched vertices of M_i to ensure structural controllability of S_i, and (ii) a subset of outputs $\mathcal{Y}_i$ and outgoing edges $\mathcal{E}_{\mathcal{X}_i,\mathcal{X}_k}$ come from left-unmatched vertices of M_i to ensure structural observability of S_i at the same time.

To avoid the trivial case, assume subsystems discussed can all be simplified with $q_i = n_i - |M_i| > 0$ ($|\cdot|$ denotes the cardinality of a set). Then S_i can be simplified as follows:

$$\begin{aligned} \dot{\boldsymbol{x}}_i(t) &= A_{ii}\boldsymbol{x}_i(t) + A'_{ij}\boldsymbol{x}_j(t) + b_i\boldsymbol{v}'_i(t), \\ \boldsymbol{y}'_i(t) &= c_i\boldsymbol{x}_i(t), \end{aligned} \tag{2}$$

with $\boldsymbol{v}'_i(t) = \sum_{j=1}^{N} w_{ij}\boldsymbol{y}'_j(t) + \delta'_i\boldsymbol{u}'_i(t)$. $b_i \in \mathbb{R}^{n_i\times r'_i}$, $c_i \in \mathbb{R}^{p'_i\times n_i}$, $w_{ij} \in \mathbb{R}^{r'_i\times p'_j}$; $r'_i \leqslant \min\{r_i, q_i\}$, $p'_i \leqslant \min\{p_i, q_i\}$. Let z_i ($\leqslant r'_i$) denote the number of external inputs in $\boldsymbol{u}'_i$.

For the simplified network $(A'_{\rm sys}, B'_{\rm sys})$, let $w = [w_{ij}]|_{i=1,j=1}^{N}$. Let $L_\alpha = [\alpha_{ij}]|_{i=1,j=1}^{N}$ represent physical topology, where $\alpha_{ij} = 1$ if $A'_{ij} \neq \boldsymbol{0}$, $i \neq j$, otherwise $\alpha_{ij} = 0$. Let $L_\beta = [\beta_{ij}]|_{i=1,j=1}^{N}$ represent cyber topology, where $\beta_{ij} = 1$ if $w_{ij} \neq \boldsymbol{0}$, otherwise $\beta_{ij} = 0$. Let $L = L_\alpha + L_\beta$, $\Delta = {\rm diag}(\delta'_1, \ldots, \delta'_N)$ (Δ is the $N \times N$ diagonal matrix with diagonal elements δ'_i) and $\mathcal{I}_u = \{i \mid \delta'_i \neq 0\}$.

Remark 1. Let $\mathcal{X}_i^l(\mathcal{X}_i^r)$ denote the set of left(right)-unmatched vertices of M_i, and then S_i can be decomposed into q_i disjoint spanning trees from vertices of $\mathcal{X}_i^r$ to vertices of $\mathcal{X}_i^l$. Each tree has single-input-single-output dynamics and is structurally controllable and structurally observable.

To obtain a maximum matching set in Lemma 2 is NP-hard [3]. Here we determine any one M_i for simplifying S_i. Problem 1 can be simplified as follows.

Problem 2. Given $(A_{\rm sys}, B_{\rm sys})$, where each subsystem can be simplified w.r.t. M_i, construct w with $||w||_0$ as few as possible, s.t., $(A_{\rm sys}, B_{\rm sys})$ is structurally controllable.

$||\cdot||_0$ denotes the zero matrix norm. For $\mathcal{X}_i^r, \mathcal{X}_i^l$, let $\mathcal{X}_i^{r1}$ denote the set of vertices driven by $\boldsymbol{u}'_i$, $\mathcal{X}_i^{rp}, \mathcal{X}_i^{lp}$ denote the vertex sets w.r.t. physical couplings and $\mathcal{X}_i^{rc}, \mathcal{X}_i^{lc}$ denote the vertex sets w.r.t. cyber couplings. Let $\mathcal{X}_i^{r0} = \mathcal{X}_i^{rp} + \mathcal{X}_i^{rc}$, $\mathcal{X}^l = \cup_{i=1}^N \mathcal{X}_i^l$, $\mathcal{X}^{r1} = \cup_{i=1}^N \mathcal{X}_i^{r1}$, $\mathcal{X}^{r0} = \cup_{i=1}^N \mathcal{X}_i^{r0}$.

Lemma 3 ([5])**.** $(A'_{\rm sys}, B'_{\rm sys})$ is structurally controllable, if and only if $\mathcal{E}_{\mathcal{X}^l,\mathcal{X}^{r0}}$ has a maximum matching set M^* with $|M^*| = |\mathcal{X}^{r0}|$ and each vertex in $\mathcal{X}^{r0}$ is reachable from $\mathcal{X}^{r1}$.

Considering that constructing M^* directly needs global information, we attempt to give out some design rules of cyber couplings, under which structural controllability of the whole network can be judged by network topology (L, Δ).

There are four cases about constructing cyber couplings: (1) $S_i, i \in \mathcal{I}_u$ with $0 < r'_i - z_i \leqslant p'_i$: $(r'_i - z_i)$ self-loops from $\mathcal{X}_i^{lc}$ to $\mathcal{X}_i^{rc}$ can be constructed to simplify S_i; (2) $S_i, i \in \mathcal{I}_u$ with $r'_i - z_i > p'_i$: p'_i self-loops and $(r'_i - z_i - p'_i)$ cyber couplings from S_j are needed; (3) $S_i, i \notin \mathcal{I}_u$ with $q_j \geqslant q_i$: S_j can ensure structural controllability of S_i by A'_{ij} and w_{ij}; (4) $S_i, i \notin \mathcal{I}_u$ with $q_j < q_i$: $(q_i - q_j)$ self-loops are needed.

Construct a maximum matching set M^*_{iu} with $|M^*_{iu}| = r'_i - z_i$ for $\mathcal{E}_{\mathcal{X}_i^{lc},\mathcal{X}_i^{rc}}$ in Case (1) or $\mathcal{E}_{\mathcal{X}_j^{lc}\cup\mathcal{X}_i^{lc},\mathcal{X}_i^{rc}}$ in Case (2). Construct a maximum matching set M^*_{ij} with $|M^*_{ij}| = r'_i$ for $\mathcal{E}_{\mathcal{X}_j^{lc},\mathcal{X}_i^{rc}}$ in Case (3) or $\mathcal{E}_{\mathcal{X}_j^{lc}\cup\mathcal{X}_i^{lc},\mathcal{X}_i^{rc}}$ in Case (4).

Theorem 1. Given $(A'_{\rm sys}, B'_{\rm sys})$ with cyber couplings constructed under the rules above, it is structurally controllable, if (L, Δ) is structurally controllable.

Proof. If (L, Δ) is structurally controllable, there exists a matching set M_L consisting of edges (j, i) corresponding to non-zero entries in L. Each $i \in \mathcal{I}_u$ corresponds to a right-unmatched vertex of M_L. Let $\mathcal{E}_L = \{\cup(M^*_{ij} \cup \mathcal{E}_{\mathcal{X}_j^{lp},\mathcal{X}_i^{rp}})|(j,i) \in M_L\}$ and $\mathcal{E}_\Delta = \{\cup(M^*_{iu} \cup \mathcal{E}_{\mathcal{X}_j^{lp},\mathcal{X}_i^{rp}})|i \in \mathcal{I}_u\}$. $|\mathcal{E}_L| = \sum_{i\in[1:N]\setminus\mathcal{I}_u} q_i$ and $|\mathcal{E}_\Delta| = \sum_{i\in\mathcal{I}_u}(q_i - z_i)$. $(A'_{\rm sys}, B'_{\rm sys})$ has a matching set $M' = \{\cup_{i=1}^N M_i\} \cup \mathcal{E}_L \cup \mathcal{E}_\Delta$ with $|M'| = n - \sum_{i\in\mathcal{I}_u} z_i$, corresponding to $\sum_{i\in\mathcal{I}_u} z_i$ right-unmatched vertices exactly driven by external inputs. Moreover, all state vertices are input-reachable. Then $(A'_{\rm sys}, B'_{\rm sys})$ is structurally controllable.

Now that some subsystem interconnection links are disregarded when simplifying W_{ij} into w_{ij}, structural controllability of (L, Δ) in Theorem 1 is also sufficient for $(A_{\rm sys}, B_{\rm sys})$. A concrete algorithm is proposed as Algorithm 1.

Algorithm 1 Cyber topology design

Input: Subsystems $(A_{ii}, \sum_{j=1,j\neq i}^N A_{ij}, B_i, C_i)$.
Output: L_β and W.
To each S_i, find out a matching set M_i with simplified form $(A_{ii}, [A'_{ij}\ b_i], c_i)$. $q_i = n_i - |M_i|$ and L_α are obtained.
Let $L_\beta = \boldsymbol{0}$, $w_{ij} = \boldsymbol{0}$, $U = \{i \mid \delta'_i = 1\}$, $V = \{i \mid \delta'_i = 0\}$.
For each $i \in U$ with $z_i < r'_i$, if $r'_i - z_i \leqslant p'_i$, construct $(r'_i - z_i)$ disjoint edges from $\mathcal{X}_i^{lc}$ to $\mathcal{X}_i^{rc}$, and change q_i to $q_i - r'_i + z_i$; else, take $\alpha_{ij} \neq 0$ from L_α, construct $(r'_i - z_i)$ disjoint edges from $\mathcal{X}_i^{lc} \cup \mathcal{X}_j^{lc}$ to $\mathcal{X}_i^{rc}$, and change q_i to $q_i - p'_i$. $\beta_{ii} = 1$, obtain w_{ii} ($\beta_{ij} = 1$ and w_{ij} for the latter).
while $V \neq \emptyset$ **do**
    According to L_α, take j from U and i from V.
    If $q_j \geqslant q_i$, construct r'_i disjoint edges from $\mathcal{X}_j^{lc}$ to $\mathcal{X}_i^{rc}$; else, construct r'_i disjoint edges from $\mathcal{X}_j^{lc} \cup \mathcal{X}_i^{lc}$ to $\mathcal{X}_i^{rc}$, and change q_i to q_j. $\beta_{ij} = 1$, obtain w_{ij} ($\beta_{ii} = 1$ and w_{ii} for the latter).
    Add i to U, and delete i from V and j from U.
end while
Construct each W_{ij} on the basis of w_{ij}.

L_β and W obtained in Algorithm 1 correspond to a cyber topology design for $(A_{\rm sys}, B_{\rm sys})$ to achieve structural controllability. $||W||_0 < \sum_{i=1}^N q_i$, which shows an upper bound of the number of cyber couplings to design. The main complexity of Algorithm 1 lies in determining a matching set M_i of each S_i with complexity $O(m_i\sqrt{n_i})$, where $m_i = |\mathcal{E}_{\mathcal{X}_i,\mathcal{X}_i}|$. Let $m = \sum_{i=1}^N m_i$, $n = \sum_{i=1}^N n_i$. The computational complexity of Algorithm 1 is no more than $O(mn)$.

Conclusion. Through simplifying subsystem dynamics, we proposed design strategies of cyber couplings, under which the whole network can achieve structural controllability if constructing its network topology to be structurally controllable. The presented algorithm can obtain a bounded number of cyber couplings in polynomial time. Further research may focus on minimum cyber topology design.

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