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New stability conditions of CPSs with multiple transportation channels under DoS attacks

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Dear editor,

Cyber-physical systems (CPSs) are widely studied owing to their cyclopedic practical value in some fields. In an unreliable network environment, transmission channels may be subject to denial-of-service (DoS) attacks, which greatly affects the performance of systems. In previous studies in this field, most of the studies have focused on a single transmission channel and only a few results have been reported for multiple transmission channels. However, the latter is consider to be more complex than the former because computation exponentially grows as the number of channels linearly increases. In [1], the periodic transmission strategy and the switching controller are introduced and input-to-state stability (ISS) is achieved in CPSs with multiple transmission channels under DoS attacks. To date, the problem on the stability in CPSs with multiple transmission channels under DoS attacks is not completely addressed.

This study proposes an improved transmission mode that has several benefits. First, it makes the conditions of Theorem IV.1 in [1] less conservative. Second, it easily or quickly obtains the data packets under DoS attacks. Third, it improves the performance of the systems. In this study, the transmission cycle changes when DoS attacks occur. The upper bound of the difference value between the current moment and last data receipt time of the ith channel at the current time accurately determines the occurrence of DoS attacks. Concurrently, based on the improved transmission mode, new stability conditions in CPSs with multiple transportation channels under DoS attacks are obtained.

Some preliminaries are provided in Appendix A.

Herein, we consider a class of linear systems of the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_{\varpi}\varpi(t), \\ y_i(t) = C_i x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, and $\varpi(t) \in \mathbb{R}^{n_\varpi}$ are the state vector, control input, and interference input of the system, respectively. $y_i(t) \in \mathbb{R}^{n_u}$ is the output signal of the ith channel, $i \in \mathbb{I} = \{1, 2, 3, \ldots, n_s\}$. A, B, B_ϖ, and C_i are time-invariant matrices with appropriate dimensions. We only consider that the signals $y_i(t)$ and $i \in \mathbb{I}$ are available, and we assume the existence of static feedback matrices K_i, $i \in \mathbb{I} = \{1, 2, 3, \ldots, n_s\}$, such that λ_j, $j \in \mathbb{J} = \{1, 2, 3, \ldots, n_x\}$ have a negative real part, where λ_j, $j \in \mathbb{J} = \{1, 2, 3, \ldots, n_x\}$ are eigenvalues of $A + B\sum_{i\in\mathbb{I}} K_i C_i$. We assume that n_s channels may independently suffer from DoS attacks and $t^i(t)$ is not in the DoS attacks interval, that is, $t^i(t) = \hat{t}^i(t)$. At the same time, we assume that no DoS attacks occur in $[-\delta, 0]$ and that all DoS attacks are valid, that is, each DoS attack changes the transmission cycle.

Assumption 1. The transmission strategy of the ith channel $\nabla_i, \nabla_i^* \in \mathbb{R}_{>0}$ $(\nabla_i^* < \nabla_i)$, $n_i^j \in \mathbb{Z}_{>1}$ satisfies

$$0 \leqslant t - \hat{t}^i(t) \leqslant \begin{cases} \delta^i (t \in \bar{\Theta}^i(0,\infty)), & \delta^i = \nabla^i, \\ \tilde{\delta}^i (t \in \bar{\Xi}^i(0,\infty)), & \tilde{\delta}^i = \nabla^i + n_i^j \nabla_i^* \leqslant \bar{\delta}^i, \end{cases} \tag{2}$$

where $\bar{\Theta}^i(0,\infty)$ and $\bar{\Xi}^i(0,\infty)$ are introduced in Lemma 2, and n_i^j denotes the number of cycles of ∇_i^* experienced by the jth DoS attack on the ith channel. $\bar{\delta}^i$ is known according to the acknowledgement signal and requested signal, described in Appendix A.

Assumption 2 (DoS frequency [2])**.** $\forall a_1, a_2 \in \mathbb{R}_{\geqslant 0}$ with $a_1 \leqslant a_2$, there exist $\gamma_i \in \mathbb{R}_{\geqslant 0}$ and $\tau_D^i \in \mathbb{R}_{\geqslant \nabla_i^*}$, such that

$$n_i(a_1, a_2) \leqslant \gamma_i + \frac{a_2 - a_1}{\tau_D^i}, \tag{3}$$

where $n_i(a_1, a_2)$ denotes the number of DoS attacks on the ith channel in $[a_1, a_2]$.

Assumption 3 (DoS duration [2])**.** $\forall a_1, a_2 \in \mathbb{R}_{\geqslant 0}$ with $a_1 \geqslant a_2$, there exist $q_i \in \mathbb{R}_{\geqslant 0}$ and $L^i \in \mathbb{R}_{\geqslant 1}$, such that

$$|\Xi^i(a_1, a_2)| \leqslant q_i + \frac{a_2 - a_1}{L^i}, \tag{4}$$

where $\Xi^i(a_1, a_2)$ denotes the time intervals suffering from DoS attacks in $[a_1, a_2]$.

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Now, the ISS of the system is discussed without the DoS attacks. The controller is provided as follows:

$$u(t) = \sum_{i\in\mathbb{I}} K_i y_i(\hat{t}^i(t)) = \sum_{i\in\mathbb{I}} K_i C_i x(\hat{t}^i(t)). \tag{5}$$

Based on (5), system (1) can be rewritten as

$$\dot{x}(t) = Ax(t) + B\sum_{i\in\mathbb{I}} K_i C_i x(\hat{t}^i(t)) + B_\varpi \varpi(t). \tag{6}$$

Lemma 1. Given scalars $\nabla^i > 0$ ($\nabla^i = \delta^i$), $a < 0$, if there exist matrices N_i, S_i, $W > 0$, $E_i > 0$, $F_i > 0$, and $i \in \mathbb{I}$ and scalar $b > 0$, such that

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & A_1^{\mathrm{T}}\left(\sum_{i\in\mathbb{I}}\delta^i F_i\right) \\ \star & \Sigma_{22} & 0 \\ \star & \star & -\sum_{i\in\mathbb{I}}\delta^i F_i \end{bmatrix} < 0, \tag{7}$$

where $\Sigma_{11} = \Pi_{11} + \Pi_{12} + \Pi_{12}^{\mathrm{T}}$, $\Pi_{12} = [\sum_{i\in\mathbb{I}} N_i \ S_1 - N_1 \ \cdots \ S_{n_s} - N_{n_s} \ -S_1 \ \cdots \ -S_{n_s} \ 0]$, $\Pi_{12}^{11} = WB[K_1C_1 \ K_2C_2 \ \cdots \ K_{n_s}C_{n_s}]$, $\Sigma_{12} = [\sqrt{\delta^1}N_1 \ \cdots \ \sqrt{\delta^{n_s}}N_{n_s} \ \sqrt{\delta^1}S_1 \ \cdots \ \sqrt{\delta^{n_s}}S_{n_s}]$,

$$\Pi_{11} = \begin{bmatrix} He(WA) - aW + \sum_{i\in\mathbb{I}} E_i & \Pi_{12}^{11} & 0 & WB_\varpi \\ \star & 0 & 0 & 0 \\ \star & \star & \Pi_{33}^{11} & 0 \\ \star & \star & \star & -\mathrm{b}I \end{bmatrix},$$

$$\Pi_{33}^{11} = \begin{bmatrix} -\mathrm{e}^{a\delta^1}E_1 & 0 & 0 \\ \star & & 0 \\ \star & \star & -\mathrm{e}^{a\delta^{n_s}}E_{n_s} \end{bmatrix}, \quad \Sigma_{22} = \begin{bmatrix} \Sigma_{22}^1 & 0 \\ \star & \Sigma_{22}^1 \end{bmatrix},$$

$$\Sigma_{22}^1 = \begin{bmatrix} -\mathrm{e}^{-|a|\delta^1}F_1 & 0 & 0 \\ \star & & 0 \\ \star & \star & -\mathrm{e}^{-|a|\delta^{n_s}}F_{n_s} \end{bmatrix},$$

then the closed-loop system (6) with packet transmission period ∇^i and assumptions is ISS.

To analyze the stability of system (1) with DoS attacks, the following situations are discussed. The modified controller can be written as

$$u(t) = \sum_{i\in\mathbb{I}} \check{K}_i(t) y_i(\hat{t}^i(t)), \tag{8}$$

where $\check{K}_i(t) = \begin{cases} K_i, & t-\hat{t}^i(t) \leqslant \nabla^i, \\ 0, & t-\hat{t}^i(t) > \nabla^i, \end{cases}$ ∇^i is obtained by Lemma 1.

Lemma 2 ([1]). Set $\bar{\Xi}^i(a_1,a_2) = \{t \in [a_1,a_2) \,|\, \check{K}_i(t) = 0\}$ and $\bar{\Theta}^i(a_1,a_2) = \{t \in [a_1,a_2) \,|\, \check{K}_i(t) = K_i\}$. Then, $\bar{\Xi}^i(a_1, a_2)$ and $\bar{\Theta}^i(a_1,a_2)$ satisfy

$$\bar{\Theta}^i(a_1,a_2) = [a_1,a_2)\backslash\bar{\Xi}^i(a_1,a_2), \tag{9}$$

$$|\bar{\Xi}^i(a_1,a_2)| \leqslant |\Xi^i(a_1,a_2)| + (n_i(a_1,a_2)+1)\nabla_i^*. \tag{10}$$

Based on (8), system (1) can be rewritten as

$$\dot{x}(t) = Ax(t) + B\sum_{i\in\mathbb{I}, i\notin\Gamma_\sigma} K_i C_i x(\hat{t}^i(t)) + B_\varpi \varpi(t), \tag{11}$$

where $\Gamma_{\sigma(t)}$ is defined in Appendix A.

For the stability analysis of closed-loop system (11), Theorem 1 is provided.

Theorem 1. Given controller gains K_i; scalars $\tau_D^i > 0$, $L^i > 1$, ∇^i, ∇_i^*, and $\mathrm{b} > 0$; and matrices W, E_i, F_i, N_i, S_i, and $i \in \mathbb{I}$, if there exist real θ_1^i, θ_2^i, and a_p ($p \in \mathbb{P}$), such that

$$\theta_1^i - \theta_2^i \geqslant 0, \tag{12}$$

$$a_p - \left(\sum_{i\in\Gamma_p}\theta_1^i + \sum_{i\in\mathbb{I}\backslash\Gamma_p}\theta_2^i\right) \leqslant 0, \tag{13}$$

$$\sum_{i\in\mathbb{I}}(\bar{\varsigma}_i\theta_1^i + (1-\bar{\varsigma}_i)\theta_2^i) < 0, \tag{14}$$

$$\bar{\Sigma} = \begin{bmatrix} \bar{\Sigma}_{11} & \bar{\Sigma}_{12} & A_1^{\mathrm{T}}\bar{\Sigma}_{13} \\ \star & \bar{\Sigma}_{22} & 0 \\ \star & \star & -\bar{\Sigma}_{13} \end{bmatrix} < 0, \tag{15}$$

where $\bar{\Sigma}_{11} = \bar{\Pi}_{11} + \Pi_{12} + \Pi_{12}^{\mathrm{T}}$, $\Pi_{12} = [\sum_{i\in\mathbb{I}} N_i \ S_1 - N_1 \ \cdots \ S_{n_s} - N_{n_s} \ -S_1 \ \cdots \ -S_{n_s} \ 0]$, $\bar{\Sigma}_{12} = [\sqrt{\delta^1}N_1 \ \cdots \ \sqrt{\delta^{n_s}}N_{n_s} \ \sqrt{\delta^1}S_1 \ \cdots \ \sqrt{\delta^{n_s}}S_{n_s}]$,

$$\bar{\Pi}_{11} = \begin{bmatrix} He(WA) - a_pW + \sum_{i\in\mathbb{I}} E_i & \bar{\Pi}_{12}^{11} & 0 & WB_\varpi \\ \star & 0 & 0 & 0 \\ \star & \star & \bar{\Pi}_{33}^{11} & 0 \\ \star & \star & \star & -\mathrm{b}I \end{bmatrix},$$

$\bar{\Pi}_{12}^{11} = WB[K_1C_1 \ K_2C_2 \ \cdots \ K_{n_s}C_{n_s}]$,

$$\bar{\Pi}_{33}^{11} = \begin{bmatrix} -\mathrm{e}^{a_p\delta^1}E_1 & 0 & 0 \\ \star & & 0 \\ \star & \star & -\mathrm{e}^{a_p\delta^{n_s}}E_{n_s} \end{bmatrix}, \quad \bar{\Sigma}_{22} = \begin{bmatrix} \bar{\Sigma}_{22}^1 & 0 \\ \star & \bar{\Sigma}_{22}^1 \end{bmatrix},$$

$$\bar{\Sigma}_{22}^1 = \begin{bmatrix} -\mathrm{e}^{-|a_p|\delta^1}F_1 & 0 & 0 \\ \star & & 0 \\ \star & \star & -\mathrm{e}^{-|a_p|\delta^{n_s}}F_{n_s} \end{bmatrix},$$

$\bar{\Sigma}_{13} = \sum_{i\in\mathbb{I}}\delta^i F_i$, $\bar{\varsigma}_i = \frac{1}{L^i} + \frac{\nabla_i^*}{\tau_D^i}$, then the closed-loop system (11) with transmission strategy (2) and assumptions is ISS.

Remark 1. N_i, S_i, and b are provided by Lemma 1. When $i \in \Gamma_p$ ($p \in \mathbb{P}$), $K_i = 0$, $\delta^i = \bar{\delta}^i$ in $\bar{\Sigma}_{11}$, $\bar{\Sigma}_{12}$, $\bar{\Sigma}_{22}$, and $\bar{\Sigma}_{13}$.

The proofs of Lemma 1 and Theorem 1 can be found in Appendixes B and C and a numerical example is given in Appendix D.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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