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• **Supplementary File** •

# New Stability Conditions of CPSs with Multiple Transportation Channels and DoS Attacks

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## Appendix A Preliminaries

**Notations**. $W^T$ is the transpose of the given matrix $W$, $He(W) = W + W^T$, $W > 0$ ($W < 0$) denotes positive definite symmetric (negative definite symmetric) matrix. $\lambda_{min}(W)$ and $\lambda_{max}(W)$ denote the minimum and maximum eigenvalues of matrix $W$, respectively. $\mathbb{R}$ denotes real number, $\mathbb{R}^i$ is a vector space made up of $i$ dimensional real vectors. $\mathbb{R}^{i\times j}$ denotes $i \times j$ real matrixes. $t^i(t)$ ($t^i(t) \leqslant t$) and $\hat{t}^i(t)$ ($\hat{t}^i(t) \leqslant t$) are the packet transmission time and packet reception time of the $i^{th}$ channel closest to $t$, respectively. Given an interval $\pi$, $|\pi|$ denotes the length of the interval. Given a vector $a$, $\|a\|$ denotes its spectral norm. Given a time interval $[0, t)$ and function $f(t)$, $\|f_t\|_\infty = ess\ sup_{s\in[0,t)}\|f(s)\|$ denotes the $L_\infty$ norm of $f(\cdot)$ on $[0, t)$. Given a continuous function $x(t)$ and scalars $\delta^i$, $\delta = max\{\delta^i\}$, $i \in \mathbb{I} = \{1, 2, 3 \cdots n_s\}$, $\|x_t\|_\delta \triangleq sup_{-\delta\leqslant d\leqslant 0}\{\|\dot{x}(t + d)\|, \|x(t + d)\|\}$. For $a_1$, $a_2 \in \mathbb{R}_{\geqslant 0}$ with $a_2 \geqslant a_1$, $[a_1, a_2]$ denotes the time interval that contains the left and right endpoints. similarly, $[a_1, a_2)$, $(a_1, a_2]$ and $(a_1, a_2)$ denote the time intervals that contain the left endpoint, right endpoint and none of them, respectively. Given a decimal number $c$, $\lceil c\rceil$ represents the smallest integer greater than or equal to itself.

The transmission mode adopted in [1] is constant periodic sampling. The transmission cycle changes when DoS attacks occur in this paper. The Table A1 shows some comparisons of changes caused by the two transmission modes. We need to set the same variables in the purpose of better comparison. From Table A1, we can get the following statements. Firstly, it makes the condition 3 of Theorem IV.1 less conservative in [1]. Because the parameters that satisfy $\tilde{\mu} < 0$ don't necessarily satisfy $\bar{\mu} < 0$, and the parameters that satisfy $\bar{\mu} < 0$ must satisfy $\tilde{\mu} < 0$. Secondly, the data packets are earlier to obtain with DoS attacks. This is mainly due to taking a sampling period of $\nabla_i^*$ ($< \nabla^i$) after the DoS attacks occur. Thirdly, according to Table A1, the system converges faster and is more resistant to interference.

**Table A1** The comparison of results for two transmission modes

| | | | |
|---|---|---|---|
| [1] | $\bar{\mu}_i = \frac{1}{\mu_i} + \frac{\Delta_*^i}{\tau_{D_i}}$ | $\bar{\eta}_i = \varsigma_i + (\eta_i + 1)\Delta_*^i$ | $\bar{\eta} = \Sigma_{i\in\mathbb{I}}(\theta_1^i - \theta_2^i)\bar{\eta}_i$ |
| | $\bar{\mu} = \Sigma_{i\in\mathbb{I}}(\bar{\mu}_i\theta_1^i + (1 - \bar{\mu}_i)\theta_2^i)$ | | |
| | the condition 3 $\bar{\mu} < 0$ | $\|x(t)\| \leqslant e^{\frac{\bar{\mu}}{2}t}\sqrt{\frac{\hat{a}}{\check{a}}e^{\bar{\eta}}}\|x(0)\| + \sqrt{\frac{\beta}{\check{a}}e^{\bar{\eta}}}\|d_t\|_\infty$ | |
| this paper | $\tilde{\mu}_i = \frac{1}{\mu_i} + \frac{\nabla_i^*}{\tau_{D_i}}$ | $\tilde{\eta}_i = \varsigma_i + (\eta_i + 1)\nabla_i^*$ | $\tilde{\eta} = \Sigma_{i\in\mathbb{I}}(\theta_1^i - \theta_2^i)\tilde{\eta}_i$ |
| | $\tilde{\mu} = \Sigma_{i\in\mathbb{I}}(\tilde{\mu}_i\theta_1^i + (1 - \tilde{\mu}_i)\theta_2^i)$ | | |
| | the condition 3 $\tilde{\mu} < 0$ | $\|x(t)\| \leqslant e^{\frac{\tilde{\mu}}{2}t}\sqrt{\frac{\hat{a}}{\check{a}}e^{\tilde{\eta}}}\|x(0)\| + \sqrt{\frac{\beta}{\check{a}}e^{\tilde{\eta}}}\|d_t\|_\infty$ | |
| results | $\tilde{\mu}_i < \bar{\mu}_i$ | $\tilde{\eta}_i < \bar{\eta}_i$ | $\tilde{\eta} < \bar{\eta}$ |
| | $\tilde{\mu} < \bar{\mu}$ | | |
| | the condition 3 $\tilde{\mu} < \bar{\mu} < 0$ | $e^{\frac{\tilde{\mu}}{2}} < e^{\frac{\bar{\mu}}{2}}$ and $e^{\tilde{\eta}} < e^{\bar{\eta}}$ | |

As shown in Figure A1, once the controller $i$ receives a packet or a signal, it will send an acknowledgement (ACK) signal to logic. In the case of no DoS attacks, the receiver (controller $i$) will send an ACK signal to the sender (sensor $i$), so as to ensure that the sender knows that the receiver has received packets that were sent previously. Assuming that signals are an instant transmission without network-induced delay in this case. Besides, we also assume that after a time interval of at most $\nabla_i^*$, the receiver cannot receive the signal from the sender and still cannot make a corresponding response signal to the sender, that is, we judge that the $i^{th}$ channel of the system suffers from DoS attacks at this time. The requested (REQ) signal is used to detect whether DoS attacks end. See [2] in detail.

**Definition 1** ( [3]). $\forall\ t \in \mathbb{R}_{\geqslant 0}$, if there exist a $\mathcal{KL}$-function $h_1$ and a $\mathcal{K}_\infty$-function $h_2$ such that, for each $\varpi(t) \in \mathcal{L}_\infty(\mathbb{R}_{\geqslant 0})$ and $x(0) \in \mathbb{R}^n$

$$\|x(t)\| \leqslant h_1(\|x(0)\|, t) + h_2(\|\varpi(t)\|) \tag{A1}$$

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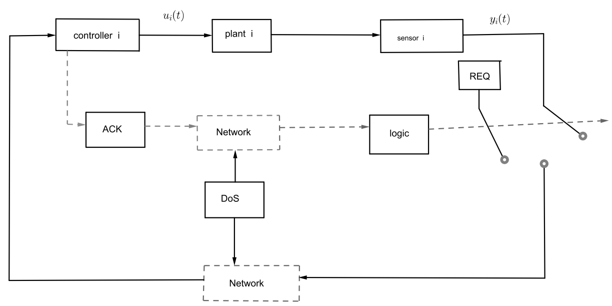


**Figure A1** The setup of ACK and REQ signals

then system (1) is identified as input-to-state stable (ISS). If (A1) holds when $\varpi(t)=0$, then systems (1) is globally asymptotically stable.

Next we cite the definitions in [1]. Define

$$\begin{aligned} &\Gamma_{\sigma(t)} = \{i \in \mathbb{I} \mid n^i(t) = 1\} \\ &\sigma(t) = 1 + \textstyle\sum_{i\in\mathbb{I}} 2^{i-1} n^i(t) \end{aligned}$$

where $n^i(t) = \begin{cases} 1, t \in \bar{\Xi}^i(0,\infty) \\ 0, t \notin \bar{\Xi}^i(0,\infty) \end{cases}$, $\sigma \in \mathbb{P} = \{1, \cdots, 2^{n_s}\}$.

$$\Re_p(a_1,a_2) = (\cap_{i\in\Gamma_p} \bar{\Xi}^i(a_1,a_2)) \cap (\cap_{i\in\mathbb{I}\setminus\Gamma_p} \bar{\Theta}^i(a_1,a_2))$$

where $p \in \mathbb{P}$.

$$\begin{aligned} &\bar{\Xi}^i(a_1,a_2) = \cup_{p\in\mathbb{P}, i\in\Gamma_p} \Re_p(a_1,a_2) \\ &\bar{\Theta}^i(a_1,a_2) = \cup_{p\in\mathbb{P}, i\notin\Gamma_p} \Re_p(a_1,a_2) \end{aligned} \tag{A2}$$

## Appendix B The Proof of Lemma 1

Consider the Lyapunov function as follows:

$$\begin{aligned} V_1(t) = &x^T(t)Wx(t) + \textstyle\sum_{i\in\mathbb{I}} \int_{t-\delta^i}^t x^T(s)e^{\mathrm{a}(t-s)}E_i x(s)ds \\ &+ \textstyle\sum_{i\in\mathbb{I}} \int_{-\delta^i}^0 \int_{t+\theta}^t \dot{x}^T(s)e^{\mathrm{a}(t-s)}F_i\dot{x}(s)dsd\theta \end{aligned} \tag{B1}$$

where $W \in \mathbb{R}^{n_x\times n_x}$, $E_i \in \mathbb{R}^{n_x\times n_x}$, $F_i \in \mathbb{R}^{n_x\times n_x}$ and $W>0$, $E_i>0$, $F_i>0$, $i\in\mathbb{I}$. The derivative of (B1) can be obtained:

$$\begin{aligned} \dot{V}_1(t) \leqslant\, &2x^T(t)W\dot{x}(t) + \mathrm{a}\sum_{i\in\mathbb{I}} \int_{t-\delta^i}^t x^T(s)e^{\mathrm{a}(t-s)}E_i x(s)ds \\ &+ \sum_{i\in\mathbb{I}} x^T(t)E_i x(t) - \sum_{i\in\mathbb{I}} x^T(t-\delta^i)e^{\mathrm{a}\delta^i}E_i x(t-\delta^i) \\ &+ \mathrm{a}\sum_{i\in\mathbb{I}} \int_{-\delta^i}^0 \int_{t+\theta}^t \dot{x}^T(s)e^{\mathrm{a}(t-s)}F_i\dot{x}(s)dsd\theta + \sum_{i\in\mathbb{I}} \delta^i \dot{x}^T(t)F_i\dot{x}(t) \\ &- \sum_{i\in\mathbb{I}} \int_{t-\delta^i}^t \dot{x}^T(s)e^{-|\mathrm{a}|\delta^i}F_i\dot{x}(s)ds \end{aligned} \tag{B2}$$

setting $\xi^T = \begin{bmatrix} x^T(t) & x^T(\hat{t}^1(t)) & \cdots & x^T(\hat{t}^{n_s}(t)) & x^T(t-\delta^1) & \cdots & x^T(t-\delta^{n_s}) & \varpi^T(t) \end{bmatrix}$,

$$\delta^i\dot{x}^T(t)F_i\dot{x}(t) = \delta^i\xi^T A_1^T F_i A_1\xi, i\in\mathbb{I} \tag{B3}$$

where $A_1 = \begin{bmatrix} A & BK_1C_1 & \cdots & BK_{n_s}C_{n_s} & 0 & \cdots & 0 & B_\varpi \end{bmatrix}$.

$$-2\xi^T N_i \int_{\hat{t}^i(t)}^t \dot{x}(s)ds \leqslant \delta^i\xi^T N_i e^{|\mathrm{a}|\delta^i}F_i^{-1}N_i^T\xi + \int_{\hat{t}^i(t)}^t \dot{x}(s)e^{-|\mathrm{a}|\delta^i}F_i\dot{x}(s)ds, i\in\mathbb{I} \tag{B4}$$

$$-2\xi^T S_i \int_{t-\delta^i}^{\hat{t}^i(t)} \dot{x}(s)ds \leqslant \delta^i\xi^T S_i e^{|\mathrm{a}|\delta^i}F_i^{-1}S_i^T\xi + \int_{t-\delta^i}^{\hat{t}^i(t)} \dot{x}(s)e^{-|\mathrm{a}|\delta^i}F_i\dot{x}(s)ds, i\in\mathbb{I} \tag{B5}$$

Substituting (B3), (B4), (B5) into (B2) yields

$$\dot{V}_1(t) \leqslant \mathrm{a}V_1(t) + \xi^T[\Sigma_{11} + \sum_{i\in\mathbb{I}} \delta^i N_i e^{|\mathrm{a}|\delta^i}F_i^{-1}N_i^T$$

$$+\sum_{i\in\mathbb{I}}\delta^i S_i e^{|\mathrm{a}|\delta^i}F_i^{-1}S_i^T+\sum_{i\in\mathbb{I}}\delta^i A_1^T F_i A_1]\xi$$
$$+\mathrm{b}\|\varpi(t)\|^2$$

Applying schur complement lemma to inequality (7), we get

$$\Sigma_{11}+\textstyle\sum_{i\in\mathbb{I}}\delta^i N_i e^{|\mathrm{a}|\delta^i}F_i^{-1}N_i^T+\sum_{i\in\mathbb{I}}\delta^i S_i e^{|\mathrm{a}|\delta^i}F_i^{-1}S_i^T+\sum_{i\in\mathbb{I}}\delta^i A_1^T F_i A_1<0$$

thus, one gets

$$\dot V_1(t)\leqslant \mathrm{a}V_1(t)+\mathrm{b}\|\varpi(t)\|^2 \tag{B6}$$

From (B6), we get

$$V_1(t)\leqslant e^{\mathrm{a}t}V_1(0)+\int_0^t e^{\mathrm{a}(t-\tau)}\mathrm{b}\|\varpi(\tau)\|^2d\tau \tag{B7}$$

According to (B1), we have

$$m_1\|x(t)\|^2\leqslant V_1(t),V_1(0)\leqslant m_5\|x_0\|_\delta^2 \tag{B8}$$

Substituting (B8) into (B7) yields

$$\|x(t)\|\leqslant\sqrt{\frac{m_5}{m_1}e^{\mathrm{a}t}}\|x_0\|_\delta+\sqrt{-\frac{\mathrm{b}}{m_1\mathrm{a}}}\|\varpi_t\|_\infty$$

where $m_1=\lambda_{min}(W)$, $m_2=\lambda_{max}(W)$, $m_3=\sum_{i\in\mathbb{I}}\delta^i\lambda_{max}(E_i)$, $m_4=\sum_{i\in\mathbb{I}}(\frac{(\delta^i)^2}{2})\lambda_{max}(F_i)$, $m_5=m_2+m_3+m_4$. Then, according to the Definition 1, the closed-loop system (6) with packets transmission period $\nabla^i$ and assumptions is ISS.

## Appendix C The Proof of Theorem 1

Consider Lyapunov function as follows:

$$\begin{aligned}V_2(t)=x^T(t)Wx(t)&+\textstyle\sum_{i\in\mathbb{I},i\in\Gamma_p}\int_{t-\bar\delta^i}^t x^T(s)e^{\mathrm{a}_p(t-s)}E_ix(s)ds\\&+\textstyle\sum_{i\in\mathbb{I},i\notin\Gamma_p}\int_{t-\delta^i}^t x^T(s)e^{\mathrm{a}_p(t-s)}E_ix(s)ds\\&+\textstyle\sum_{i\in\mathbb{I},i\in\Gamma_p}\int_{-\bar\delta^i}^0\int_{t+\theta}^t\dot x^T(s)e^{\mathrm{a}_p(t-s)}F_i\dot x(s)dsd\theta\\&+\textstyle\sum_{i\in\mathbb{I},i\notin\Gamma_p}\int_{-\delta^i}^0\int_{t+\theta}^t\dot x^T(s)e^{\mathrm{a}_p(t-s)}F_i\dot x(s)dsd\theta\end{aligned} \tag{C1}$$

where $W\in\mathbb{R}^{n_x\times n_x}$, $E_i\in\mathbb{R}^{n_x\times n_x}$, $F_i\in\mathbb{R}^{n_x\times n_x}$, $i\in\mathbb{I}$ are given by Lemma 1. $\mathrm{a}_p$ $(p\in\mathbb{P})$ is a positive or negative scalar. Similar to the proof of Lemma 1, we get

$$\dot V_2(t)\leqslant \mathrm{a}_pV_2(t)+\mathrm{b}\|\varpi(t)\|^2,\sigma=p \tag{C2}$$

From (C2), we have

$$V_2(t)\leqslant e^{\mathrm{a}_{\sigma_\kappa}(t-\zeta_\kappa)}V_2(\zeta_\kappa)+\int_{\zeta_\kappa}^t e^{\mathrm{a}_{\sigma_\kappa}(t-z)}\mathrm{b}\|\varpi(z)\|^2dz,t\in[\zeta_\kappa,\zeta_{\kappa+1}) \tag{C3}$$

where $\zeta_\kappa$ $(\kappa\in\mathbb{N})$ are assumed to be the time instants at which $\sigma$ changes, $\sigma_\kappa=\sigma(\zeta_\kappa)$. By iterating on (C3) yields

$$\begin{aligned}V_2(t)\leqslant{}& e^{S_\kappa(t,0)}V_2(0)+e^{S_\kappa(t,1)}\int_0^{\zeta_0}e^{\mathrm{a}_1}(\zeta_0-z)\mathrm{b}\|\varpi(z)\|^2dz\\&+e^{S_\kappa(t,2)}\int_{\zeta_0}^{\zeta_1}e^{\mathrm{a}_{\sigma_0}}(\zeta_1-z)\mathrm{b}\|\varpi(z)\|^2dz\\&+\cdots+e^{S_\kappa(t,\kappa+1)}\int_{\zeta_{\kappa-1}}^{\zeta_\kappa}e^{\mathrm{a}_{\sigma_{\kappa-1}}}(\zeta_\kappa-z)\mathrm{b}\|\varpi(z)\|^2dz\\&+\int_{\zeta_\kappa}^t e^{\mathrm{a}_{\sigma_\kappa}}(t-z)\mathrm{b}\|\varpi(z)\|^2dz\\&\leqslant e^{S_V(0,t)}V_2(0)+\int_0^t e^{S_V(z,t)}\mathrm{b}\|\varpi(z)\|^2dz\end{aligned} \tag{C4}$$

where $S_\kappa(t,r)=\mathrm{a}_{\sigma_\kappa}(t-\zeta_\kappa)+\sum_{j=r}^{\kappa}\mathrm{a}_{\sigma_{j-1}}(\zeta_j-\zeta_{j-1})$, $\sigma(0)=1=\sigma_{-1}$, $\zeta_{-1}=0$, $S_V(z,t)$=$\sum_{p\in\mathbb{P}}\mathrm{a}_p|\Re_p(z,t)|$. From (13), we get

$$\begin{aligned}S_V(z,t)&\leqslant\sum_{p\in\mathbb{P}}\left(\sum_{i\in\Gamma_p}\theta_1^i+\sum_{i\in\mathbb{I}\setminus\Gamma_p}\theta_2^i\right)|\Re_p(z,t)|\\&=\sum_{i\in\mathbb{I}}\left(\theta_1^i\sum_{p\in\mathbb{P},i\in\Gamma_p}|\Re_p(z,t)|+\theta_2^i\sum_{p\in\mathbb{P},i\notin\Gamma_p}|\Re_p(z,t)|\right)\end{aligned} \tag{C5}$$

For $\forall p_1\neq p_2$, $\Re_{p_1}(z,t)\cap\Re_{p_2}(z,t)=\emptyset$, from (A2) and (9), we have

$$\sum_{p\in\mathbb{P},i\in\Gamma_p}|\Re_p(z,t)|=|\bar\Xi^i(z,t)| \tag{C6}$$

$$\sum_{p\in\mathbb{P},i\notin\Gamma_p}|\Re_p(z,t)|=|\bar{\Theta}^i(z,t)|=t-z-|\bar{\Xi}^i(z,t)| \tag{C7}$$

According to (C5), (C6) and (C7), we have

$$S_V(z,t)\leqslant\sum_{i\in\mathbb{I}}((\theta_1^i-\theta_2^i)|\bar{\Xi}^i(z,t)|+\theta_2^i(t-z)) \tag{C8}$$

From (3), (4) and (10), we have

$$\begin{aligned}|\bar{\Xi}^i(z,t)|&\leqslant q_i+\frac{t-z}{L^i}+\left(\gamma_i+\frac{t-z}{\tau_D^i}+1\right)\nabla_i^*\\&=\bar{\gamma}_i+\bar{\varsigma}_i(t-z)\end{aligned} \tag{C9}$$

where $\bar{\gamma}_i=q_i+(\gamma_i+1)\nabla_i^*$. (12), (C8) and (C9) imply

$$S_V(z,t)\leqslant\bar{\gamma}+\bar{\varsigma}(t-z) \tag{C10}$$

where $\bar{\gamma}=\sum_{i\in\mathbb{I}}(\theta_1^i-\theta_2^i)\bar{\gamma}_i$, $\bar{\varsigma}=\sum_{i\in\mathbb{I}}(\bar{\varsigma}_i\theta_1^i+(1-\bar{\varsigma}_i)\theta_2^i)$. (14) implies $\bar{\varsigma}<0$. From (C4) and (C10), we have

$$\begin{aligned}V_2(t)&\leqslant e^{\bar{\gamma}+\bar{\varsigma}t}V_2(0)+\int_0^t e^{\bar{\gamma}+\bar{\varsigma}(t-z)}\mathrm{b}\|\varpi(z)\|^2dz\\&\leqslant e^{\bar{\varsigma}t}e^{\bar{\gamma}}V_2(0)+(-\frac{1}{\bar{\varsigma}})e^{\bar{\gamma}}\mathrm{b}\|\varpi_t\|_\infty^2\end{aligned} \tag{C11}$$

According to (C1), we have

$$m_1\|x(t)\|^2\leqslant V_2(t),V_2(0)\leqslant m_5\|x_0\|_\delta^2 \tag{C12}$$

(C11) and (C12) imply

$$\|x(t)\|\leqslant e^{\frac{\bar{\varsigma}}{2}t}\sqrt{\frac{m_5e^{\bar{\gamma}}}{m_1}}\|x_0\|_\delta+\sqrt{-\frac{\mathrm{b}e^{\bar{\gamma}}}{\bar{\varsigma}m_1}}\|\varpi_t\|_\infty$$

Then, according to the Definition 1, the closed-loop system (11) with transmission strategy (2) and assumptions is ISS.

**Remark 1.** In (15), all the matrices and scalars are known except for the variable $\mathrm{a}_p$. Solving for the value of $\mathrm{a}_p$ is equivalent to solving for a nonlinear matrix inequality. The nonlinear matrix inequality needs to be enlarged to get a linear matrix inequality, then the value of $\mathrm{a}_p$ can be obtained. But the value of $\mathrm{a}_p$ should not be too large when $\mathrm{a}_p$ is positive. The main reasons are as follows. Firstly, in the simulation, (12)-(14) are unsolvable when the value of $\mathrm{a}_p$ is too large. Secondly, if the positive value of $\mathrm{a}_p$ is too large, stability is hard to achieve for systems.

## Appendix D Numerical example

In this section, consider the system (1) under DoS attacks as follows:

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t)+B_\varpi\varpi(t)\\y_i(t)&=C_ix(t)\end{aligned} \tag{D1}$$

where $A=\begin{bmatrix}0&1\\0&-5\end{bmatrix}$, $B=\begin{bmatrix}0\\1.2\end{bmatrix}$, $B_\varpi=\begin{bmatrix}0.01\\0\end{bmatrix}$, $C_1=\begin{bmatrix}1&0\end{bmatrix}$, $C_2=\begin{bmatrix}0&-1\end{bmatrix}$, $i\in\mathbb{I}$, $\varpi(t)=0.1e^{-0.01t}sin(5t)$, The control gains are set as $K_1=-1, K_2=-1$.

(1) Given $\mathrm{a}=-0.56$, $\nabla^1=0.15, \nabla^2=0.15$, the solution of (7) can be obtained: $W=\begin{bmatrix}5.8098&2.3831\\2.3831&3.9100\end{bmatrix}$, $E_1=\begin{bmatrix}0.8794&1.1151\\1.1151&4.1361\end{bmatrix}$,

$$N_1=\begin{bmatrix}-2.7485&0.0585\\0.1974&0.6668\\1.4898&0.2473\\-0.2123&1.0764\\-1.1852&0.0583\\-1.1549&-0.5013\\1.0246&0.0873\\-0.3839&0.3815\\-0.8265&0.0436\\-0.0064&-0.3249\\-0.0243&-0.0107\end{bmatrix}, N_2=\begin{bmatrix}-1.6800&0.7222\\0.7853&1.2772\\-0.2991&0.4225\\0.1508&-0.0761\\0.9560&0.2860\\-0.7383&1.2067\\-0.5239&0.2661\\0.2000&-0.1782\\0.9177&0.1160\\-0.5180&0.4076\\-0.0220&-0.0094\end{bmatrix}, S_1=\begin{bmatrix}-0.2622&-0.0331\\0.5115&-0.0782\\-0.6162&-0.0703\\-0.0724&-0.7655\\-0.4530&0.0030\\-0.0473&-0.1568\\1.2383&-0.1656\\-0.6297&-0.0010\\-0.4010&0.0156\\-0.0160&-0.1488\\-0.0112&-0.0045\end{bmatrix}, S_2=\begin{bmatrix}-0.6729&-0.3984\\0.2821&-0.2843\\-0.2862&0.0726\\0.0823&-0.1088\\-0.7081&-0.2199\\-0.3687&-1.0190\\-0.2707&0.1035\\0.0872&-0.0851\\1.0583&-0.3897\\-0.8854&-0.1536\\-0.0106&-0.0045\end{bmatrix},$$

$E_2=\begin{bmatrix}1.2490&1.6228\\1.6228&4.4274\end{bmatrix}$, $F_1=\begin{bmatrix}5.5711&0.4716\\0.4716&2.0683\end{bmatrix}$, $F_2=\begin{bmatrix}4.8528&0.3419\\0.3419&2.2734\end{bmatrix}$, $\mathrm{b}=4.2204$.

(2) $\mathrm{a}_p$, $p\in\{1,2,3,4\}$ can be obtained to make (15) hold as shown in the Table D1. $L^1=5$, $L^2=4$, $\tau_D^1=0.2$, $\tau_D^2=0.2$ are given, and $\bar{\varsigma}_1=0.25$, $\bar{\varsigma}_2=0.3$ are obtained from the definition of $\bar{\varsigma}_i$, then $\theta_1^1=0.6063$, $\theta_2^1=-0.2118$, $\theta_1^2=0.2256$ and $\theta_2^2=-0.1071$ are obtained to make (12)-(14) hold.

Next, we get the following figures from the example simulation. In Figure D1, The first two subgraphs show the DoS attacks ranges of channel 1 and channel 2 for (D1) respectively, the third subgraph shows the control signal of system (D1). In Figure D2, both state 1 and state 2 tend to 0 as time increases, which indicates that the ISS of system (D1) is obtained.

**Table D1** The solutions of $\mathrm{a}_p$, $p \in \{1,2,3,4\}$

| cases | $\nabla^i$ $(i \in \{1,2\})$ | $\nabla_i^*$ $(i \in \{1,2\})$ | $\delta^1$ | $\delta^2$ | $K_1$ | $K_2$ | a |
|---|---|---|---|---|---|---|---|
| $\Gamma_1 = \{\emptyset\}$ | 0.15 | 0.01 | 0.15 | 0.15 | -1 | -1 | -0.3321 |
| $\Gamma_2 = \{1\}$ | 0.15 | 0.01 | 0.19 | 0.15 | 0 | -1 | 0.4889 |
| $\Gamma_3 = \{2\}$ | 0.15 | 0.01 | 0.15 | 0.18 | -1 | 0 | -0.0058 |
| $\Gamma_4 = \{1,2\}$ | 0.15 | 0.01 | 0.19 | 0.19 | 0 | 0 | 0.5801 |

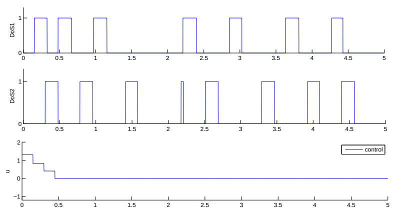


**Figure D1** DoS attacks and controllers for (D1)

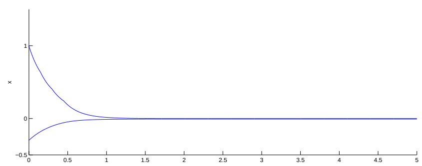


**Figure D2** State responses for (D1)

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