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Fully actuated system approach to attitude control of flexible spacecraft with nonlinear time-varying inertia

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Abstract A high-order fully actuated (HOFA) system approach is proposed for the attitude control problem of a flexible spacecraft with nonlinear and time-varying inertia. Different from most existing results, the proposed method guarantees a linear time-invariant closed-loop system with arbitrarily assignable eigenvalues. Firstly, the HOFA model for the attitude system is derived from the original dynamic equations of the system by using state transformation and variable elimination. Then, by using the full-actuation characteristics of the obtained HOFA system, the nonlinearity in the system is completely canceled and a linear time-invariant system with arbitrarily assignable eigenvalues is derived. Finally, the control laws are designed for both problems of attitude stabilization and attitude maneuvering, and simulations are carried out based on practical engineering parameters, which demonstrate the effects of the proposed method.

Keywords high-order fully actuated system, flexible spacecraft, time-varying nonlinear systems, attitude stabilization, attitude maneuvering

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1 Introduction

Due to the widespread appearance of flexible devices in modern spacecraft, such as solar panels and large mesh antennas, the control of flexible spacecraft has become one of the core issues in the field of spacecraft control.

1.1 Flexible spacecraft control

Generally, the dynamics of the spacecraft with a flexible device, by omitting the orbital rate ω_0 for the considered spacecraft, can be given by (see [1–3])

$$\begin{cases} I\ddot{\theta} + \dot{\theta}^{\times} I\dot{\theta} + B\ddot{\eta} = T, \\ \ddot{\eta} + 2\xi\Lambda\dot{\eta} + \Lambda^2\eta + B^{\mathrm{T}}\ddot{\theta} = 0, \end{cases} \tag{1}$$

where $\theta = [\,\theta_1\ \theta_2\ \theta_3\,]^{\mathrm{T}}$ and $T = [\,T_1\ T_2\ T_3\,]^{\mathrm{T}}$ denote the attitude angle and control torque vectors, respectively, $\dot{\theta}^{\times}$ is the skew-symmetric matrix of $\dot{\theta}$ which is given by

$$\dot{\theta}^{\times} = \begin{bmatrix} 0 & -\dot{\theta}_3 & \dot{\theta}_2 \\ \dot{\theta}_3 & 0 & -\dot{\theta}_1 \\ -\dot{\theta}_2 & \dot{\theta}_1 & 0 \end{bmatrix}, \tag{2}$$

and $I = \mathrm{diag}\,(I_1, I_2, I_3)$ is the moment of inertia matrix; $\eta = [\,\eta_1\ \eta_2\ \cdots\ \eta_N\,]^{\mathrm{T}}$ is the flexible mode; $\xi = \mathrm{diag}\,(\xi_i, i = 1, 2, \ldots, N)$ and $\Lambda = \mathrm{diag}\,(\Lambda_i, i = 1, 2, \ldots, N)$ are the modal damping and modal frequency

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coefficients with N being the number of flexible modes; $B \in \mathbb{R}^{3\times N}$ is the coupling matrix between the flexible structures and the rigid body.

The theoretical results on the control of the three-axis coupling model (1) are very fruitful, and many control methods have been developed, such as fault-tolerant control [4–6], variable structure control [7–9], and backstepping control [10–12]. However, in most practical projects we have $N = 3$ and $B = \mathrm{diag}\,(b_1, b_2, b_3)$. Thus, under the following control input transformation:

$$T = \dot{\theta}^{\times} I \dot{\theta} + u, \tag{3}$$

the system (1) is easily decoupled into the following three linear time-invariant subsystems:

$$\begin{cases} I_i \ddot{\theta}_i + B_i \ddot{\eta}_i = u_i, \\ \ddot{\eta}_i + 2\xi_i \Lambda_i \dot{\eta}_i + \Lambda_i^2 \eta_i + B_i \ddot{\theta}_i = 0, \ i = 1, 2, 3. \end{cases} \tag{4}$$

Therefore, a three-axis decoupling control strategy can be applied to fulfil the attitude control in practical engineering.

Quite a few approaches have been proposed fully or partially based on the above idea. In [1], on the basis of canceling the nonlinearity and assigning the linear terms, a robust term is added to the control law to suppress the disturbances. Based on the nonlinear inversion theory, control laws are developed in [13, 14] to realize three-axis decoupling control for the case of state feedback and output feedback, respectively. The feedback linearization method is used to completely [3, 15] or partially [16] cancel the nonlinearity of the flexible spacecraft system to realize attitude stabilization or tracking. The adaptive sliding mode fault tolerant attitude tracking control law developed in [2] is also based on the idea of three-axis decoupling and cancellation of nonlinearity. Some references even directly focus on the control of the above models (4), including attitude stabilization [17] and attitude maneuvering [18].

Along this research direction, taking into account the widespread liquid sloshing and fuel consumption phenomena in practical engineering, this paper further considers the situation where the moment of inertia in (4) depends on the attitude angle and angular velocity in a nonlinear and time-varying manner. In such a case, the system (4) is certainly no longer a constant linear one, and this time-varying nonlinearity brings additional difficulties to the design of the control system. In recent years, there have been quite a few results regarding this issue, and various control methods have been developed, such as proportional-derivative control [19, 20], sliding mode control [21, 22], backstepping control [23], and dynamic output-feedback control [24]. However, the closed-loop system obtained in all these results is still a nonlinear one rather than a linear time-invariant one with an arbitrarily assignable eigenstructure, and many of the results can only be obtained in a local sense [19, 20, 22, 23].

1.2 HOFA system approach

Very recently, a novel method, termed as high-order fully actuated (HOFA) system approach, is proposed via a series of studies, aiming at establishing a unified architecture for the control of general nonlinear systems. Particularly, it is discussed in [25,26] that the newly proposed HOFA model, although subject to a so-called full-actuation condition, is actually a general model for dynamic control systems in parallel with the state-space models, rather than representing a small class of nonlinear systems. On one hand, such HOFA models can be derived by directly modeling controllable physical systems using variable elimination techniques [25, 27]. On the other hand, many types of nonlinear systems in the state-space form can also be converted into HOFA systems, these include, but not limited to, strict-feedback systems [28, 29] and generalized strict-feedback systems [26], nonlinear systems in a kind of controllable canonical form [30], feedback linearizable systems [28, 29], and more general nonlinear systems under certain conditions [25].

The full-actuation characteristic of the HOFA system allows us to directly eliminate the nonlinearity in the system, thereby to obtain a linear time-invariant closed-loop system with arbitrarily assignable eigenstructures. This advantage provides great convenience for the control of the system, and consequently the HOFA systems approach has been demonstrated to be very effective in dealing with various control problems, such as robust control, adaptive control, optimal control, tracking control, and disturbance attenuation and decoupling (see the other studies in the series).

In this paper, the control of the system (4) is considered for the case of time-varying nonlinear moment of inertia based on the HOFA system approach. The HOFA model for the system is firstly derived from (4) directly through state transformation and variable elimination, and then the time-varying nonlinearity is canceled by using the full-actuation characteristics of the obtained HOFA system. Finally, parametric control laws are designed for the attitude stabilization and attitude maneuver, respectively. Different from the methods in [19–24], the proposed method guarantees that the closed-loop system is a linear time-invariant one with arbitrarily assignable eigenvalues, which is the most important advantage of the proposed approach.

The rest of this paper is organized as follows. In Section 2, the problem to be solved is formulated, and in Section 3, some preliminary results are presented. In Section 4, the process of deriving the HOFA model is given in detail. Based on the derived HOFA model the control laws are designed in Section 5. Simulation results are presented in Section 6 based on a set of practical engineering parameters, followed by a brief conclusion.

In the following, for a vector $x \in \mathbb{R}^n$ and a set of matrices $A_i \in \mathbb{R}^{r\times r}$, $i = 0, 1, \ldots, n-1$, the following symbols introduced in [25, 26] are used:

$$x^{(0\sim k)} = \begin{bmatrix} x \\ \dot{x} \\ \vdots \\ x^{(k)} \end{bmatrix}, \quad A^{0\sim n-1} = \begin{bmatrix} A_0 & A_1 & \cdots & A_{n-1} \end{bmatrix}, \tag{5}$$

$$\Phi(A^{0\sim n-1}) = \begin{bmatrix} 0 & I_r & & \\ & & \ddots & \\ & & & I_r \\ -A_0 & -A_1 & \cdots & -A_{n-1} \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_r \end{bmatrix}. \tag{6}$$

2 Problem formulation

In order to cope with more complicated applications in which liquid sloshing and fuel consumption have to be taken into account, let us consider the following time-varying nonlinear model of a flexible spacecraft:

$$\begin{cases} I(\theta, \dot{\theta}, t)\ddot{\theta} + b\ddot{q} = u, \\ \ddot{q} + 2\xi\Lambda\dot{q} + \Lambda^2 q + b\ddot{\theta} = 0, \end{cases} \tag{7}$$

where θ is the attitude angle, q is the flexible mode, u is the control torque generated by the momentum wheel, b is the modal coupling coefficient, ξ is the damping coefficient, Λ is the modal frequency, $I(\theta, \dot{\theta}, t)$ is the moment of inertia associated with θ, $\dot{\theta}$ and t, in view of the liquid sloshing and fuel consumption phenomena. All the variables mentioned above are scalars.

In this paper, we impose the following assumptions which are almost always met in practical engineering.

Assumption 1. Λ, ξ and b are nonzero constant scalars.

Assumption 2. For any $\theta, \dot{\theta} \in \mathbb{R}$ and $t \in [0, +\infty)$,

$$\Delta\left(\theta, \dot{\theta}, t\right) \triangleq I\left(\theta, \dot{\theta}, t\right) - b^2 \neq 0. \tag{8}$$

Remark 1. The above two assumptions are in fact not strict at all and fully comply with engineering practice. Assumption 1 is a common one, which can be found in most reported studies about flexible spacecraft control (see [1–18]). Assumption 2 is essentially the controllability condition of the system (7), which has been proven to be a necessary and sufficient one for the system (7) to be controllable in the case that $I(\theta, \dot{\theta}, t) = I$ is constant (see [17]). On the one hand, Assumption 2 can almost always be met in practical engineering. On the other hand, for a specific mission, it can also be further relaxed, that is, it only needs to hold for any $\theta, \dot{\theta} \in \mathbb{R}$ and $t \in [0, T)$, with T being the duration of the mission.

This paper considers both the attitude stabilization and attitude maneuver problems, which are specifically stated in turn as follows.

Problem 1. Let the system (7) satisfy Assumptions 1 and 2, and $\Gamma = \{\lambda_i, i = 1, 2, 3, 4\}$ be an arbitrarily chosen set of distinct self-conjugate complex numbers. Find a feedback control law $u = u(\theta, \dot{\theta}, q, \dot{q}, t)$ such that the closed-loop system is a linear time-invariant one with Γ being its set of eigenvalues.

Problem 2. On the basis of the above Problem 1, the feedback controller is additionally required to make the attitude angle θ asymptotically track a given constant reference signal θ_c.

Remark 2. Besides the attitude angle and angular velocity, the flexible mode and its derivative are also assumed to be measurable in this paper. This is because the main difficulty lies in how to establish an HOFA approach for the proposed attitude control problems, instead of estimating or observing the flexible mode. In the situation where the flexible modes cannot be measured in practical engineering, a state observer can be called into use to estimate the unmeasured flexible modes, as demonstrated in [17, 18].

3 Preliminaries

Consider the system

$$\begin{cases} \dot{x}^{(0\sim n-1)} = Ax^{(0\sim n-1)} + Bu + Dd, \\ y = Cx^{(0\sim n-1)}, \end{cases} \tag{9}$$

where $x^{(0\sim n-1)} \in \mathbb{R}^{nr}, u \in \mathbb{R}^r, y \in \mathbb{R}^m, d \in \mathbb{R}^p$ are the state, the control input, the regulated output and the constant disturbance, respectively, and A, B, C, D are constant matrices of appropriate dimensions.

First, let us introduce the following basic result about the output regulation.

Lemma 1 ([31])**.** Let system (9) satisfy

$$\operatorname{rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = nr + m. \tag{10}$$

If K_{PD} and K_{I} are matrices making

$$\begin{bmatrix} A + BK_{\mathrm{PD}} & BK_{\mathrm{I}} \\ C & 0 \end{bmatrix} \tag{11}$$

Hurwitz, then the controller

$$u = K_{\mathrm{PD}} x^{(0\sim n-1)} + K_{\mathrm{I}} \int_0^t \left[y\left(\sigma\right) - y_c\right] \mathrm{d}\sigma \tag{12}$$

guarantees

$$\lim_{t\to\infty} y(t) = y_c \text{ and } \lim_{t\to\infty} x^{(1\sim n)} = 0. \tag{13}$$

Next, an eigenstructure assignment result is presented. According to [31], if the matrix pair (A, B) is controllable, then there exist a pair of right coprime polynomial matrices $N(s) \in \mathbb{R}^{nr\times r}[s]$ and $D(s) \in \mathbb{R}^{r\times r}[s]$ such that the following right coprime factorization (RCF) holds:

$$\left(sI - A\right)^{-1} B = N(s)D^{-1}(s). \tag{14}$$

If we let $D(s) = [d_{ij}(s)]$ and

$$\omega = \max\left\{\deg\left(d_{ij}(s)\right),\ i, j = 1, 2, \ldots, r\right\}, \tag{15}$$

then $N(s)$ and $D(s)$ can be rewritten into the following form:

$$\begin{cases} N(s) = \sum\limits_{i=0}^{\omega} N_i s^i,\ N_i \in \mathbb{R}^{nr\times r}, \\ D(s) = \sum\limits_{i=0}^{\omega} D_i s^i,\ D_i \in \mathbb{R}^{r\times r}. \end{cases} \tag{16}$$

With the above preparation, the following result can be given.

Lemma 2 ([31])**.** Let (A, B) be controllable, and $N(s)$ and $D(s)$ be a pair of right coprime polynomial matrices in the form of (16) satisfying the RCF (14). Then, for an arbitrarily given $F \in \mathbb{R}^{nr\times nr}$, all the matrices $V \in \mathbb{R}^{nr\times nr}$ and $K \in \mathbb{R}^{r\times nr}$ satisfying $\det V \neq 0$ and

$$A + BK = VFV^{-1} \tag{17}$$

are given by

$$\begin{cases} K = WV^{-1}, \\ V = N_0 Z + N_1 ZF + \cdots + N_\omega Z F^\omega, \\ W = D_0 Z + D_1 ZF + \cdots + D_\omega Z F^\omega, \end{cases} \tag{18}$$

where $Z \in \mathbb{R}^{r\times nr}$ is any parameter matrix satisfying

$$\det\left(N_0 Z + N_1 ZF + \cdots + N_\omega Z F^\omega\right) \neq 0. \tag{19}$$

Finally, the RCF of two specific matrix pairs is discussed.

Lemma 3 ([31])**.** The pair of right coprime polynomial matrices $N(s)$ and $D(s)$ satisfying the RCF

$$\left(sI - \Phi\left(0_{0\sim n-1}\right)\right)^{-1} B_c = N(s)D^{-1}(s) \tag{20}$$

are given by

$$N(s) = \begin{bmatrix} I_r & I_r s & \cdots & I_r s^{n-1} \end{bmatrix}^{\mathrm{T}}, \quad D(s) = I_r s^n. \tag{21}$$

Let

$$\tilde{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \tag{22}$$

and then the RCF of $(\tilde{A}, \tilde{B})$ can be derived by using the RCF of (A, B), which is summarized as follows.

Lemma 4 ([31])**.** Let $N(s)$ and $D(s)$ be a pair of right coprime polynomial matrices given by (16) satisfying the RCF (14). Then the matrix pair $(\tilde{A}, \tilde{B})$ given by (22) is controllable if and only if

$$\operatorname{rank}\left(CN_0\right) = m. \tag{23}$$

In such a case, the pair of right coprime polynomial matrices $\tilde{N}(s)$ and $\tilde{D}(s)$ satisfying the RCF

$$\left(sI - \tilde{A}\right)^{-1} \tilde{B} = \tilde{N}(s)\tilde{D}^{-1}(s) \tag{24}$$

are given by

$$\begin{cases} \tilde{N}(s) = \left[\begin{bmatrix} N(s) \\ C\bar{N}(s) \end{bmatrix} T_2 \quad \begin{bmatrix} sN(s) \\ CN(s) \end{bmatrix} T_1 P\right], \\ \tilde{D}(s) = \left[D(s)T_2 \quad sD(s)T_1 P\right], \end{cases} \tag{25}$$

where

$$\bar{N}(s) = \frac{1}{s}\left(N(s) - N_0\right) = \sum_{i=1}^{\omega} N_i s^{i-1},$$

and $P \in \mathbb{R}^{m\times m}$ and $[T_1 \quad T_2]$ are nonsingular matrices satisfying

$$PCN_0\left[T_1 \quad T_2\right] = \left[I_m \quad 0\right], \quad T_1 \in \mathbb{R}^{r\times m}. \tag{26}$$

4 Direct HOFA system modeling

The modeling of the HOFA model of the system (7) is divided into the following steps.

4.1 Step 1: applying a transformation

Define the following variable transformation:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{1}{4\xi^2} \begin{bmatrix} -2\xi b\Lambda & 2\xi\Lambda\left(4\xi^2-1\right) & 4\xi^2 b & 4\xi^2 \\ b\Lambda^2 & \Lambda^2 & 0 & 0 \\ 0 & 0 & b\Lambda^2 & \Lambda^2 \\ 0 & -\Lambda^4 & 0 & -2\xi\Lambda^3 \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \dot\theta \\ \dot q \end{bmatrix}, \tag{27}$$

and then we have the following result.

Proposition 1. Under the transformation (27), the system (7) is equivalent to the following one:

$$\begin{cases} \dot x_1 = -x_2 - \frac{\Lambda}{2\xi} x_1, \\ \dddot x_2 = f_1(\dot x_1, \dot x_2, \ddot x_2, \theta, \dot\theta, t) + \frac{b\Lambda^3}{2\xi\Delta\left(\theta,\dot\theta,t\right)} u, \end{cases} \tag{28}$$

where

$$f_1(\cdot) = -\frac{\Lambda^3}{8\xi^3}\dot x_1 - \frac{\Lambda^2}{4\xi^2}\dot x_2 - \frac{\Lambda[b^2 + I(\theta,\dot\theta,t)\left(4\xi^2-1\right)]}{2\xi\Delta(\theta,\dot\theta,t)}\ddot x_2. \tag{29}$$

Proof. It follows from (27) that

$$\begin{cases} \theta = -\frac{1}{2b\Lambda^3\xi}\left(x_1\Lambda^2 - 8x_2\Lambda\xi^3 + 2x_2\Lambda\xi - 4x_3\xi^2\right), \\ q = \frac{1}{2\Lambda^3\xi}\left(x_1\Lambda^2 + 2x_2\Lambda\xi - 4x_3\xi^2\right), \\ \dot\theta = \frac{1}{4b\Lambda^3\xi^2}\left(x_1\Lambda^3 + 2x_2\Lambda^2\xi + 16x_3\Lambda\xi^4 - 4x_3\Lambda\xi^2 + 8x_4\xi^3\right), \\ \dot q = -\frac{1}{4\Lambda^3\xi^2}\left(x_1\Lambda^3 + 2x_2\Lambda^2\xi - 4x_3\Lambda\xi^2 + 8x_4\xi^3\right). \end{cases} \tag{30}$$

Taking the derivative on both sides of the above equation, we have

$$\begin{cases} \dot\theta = -\frac{1}{2b\Lambda^3\xi}\left(\dot x_1\Lambda^2 - 8\dot x_2\Lambda\xi^3 + 2\dot x_2\Lambda\xi - 4\dot x_3\xi^2\right), \\ \dot q = \frac{1}{2\Lambda^3\xi}\left(\dot x_1\Lambda^2 + 2\dot x_2\Lambda\xi - 4\dot x_3\xi^2\right), \\ \ddot\theta = \frac{1}{4b\Lambda^3\xi^2}\left(\dot x_1\Lambda^3 + 2\dot x_2\Lambda^2\xi + 16\dot x_3\Lambda\xi^4 - 4\dot x_3\Lambda\xi^2 + 8\dot x_4\xi^3\right), \\ \ddot q = -\frac{1}{4\Lambda^3\xi^2}\left(\dot x_1\Lambda^3 + 2\dot x_2\Lambda^2\xi - 4\dot x_3\Lambda\xi^2 + 8\dot x_4\xi^3\right). \end{cases} \tag{31}$$

Substituting the second equation of (30) and the last three equations in (31) into (7), we have

$$\begin{cases} \dot x_1 = -\frac{1}{2}\frac{\Lambda}{\xi}x_1 - x_2, \\ w_1\dot x_1 + w_2\dot x_2 + w_3\dot x_3 + w_4\dot x_4 = 4b\Lambda^3\xi^2 u, \end{cases} \tag{32}$$

where

$$\begin{aligned} w_1 &= \Delta\left(\theta,\dot\theta,t\right)\Lambda^3, \\ w_2 &= 2\Lambda^2\xi\Delta\left(\theta,\dot\theta,t\right), \\ w_3 &= 4\Lambda\xi^2\left[b^2 + I\left(\theta,\dot\theta,t\right)\left(4\xi^2-1\right)\right], \\ w_4 &= 8\xi^3\Delta\left(\theta,\dot\theta,t\right). \end{aligned}$$

Using (27) and (7), we can easily verify that

$$\dot x_2 = \frac{1}{4}b\frac{\Lambda^2}{\xi^2}\dot\theta + \frac{1}{4}\frac{\Lambda^2}{\xi^2}\dot q = x_3 \tag{33}$$

and

$$\dot x_3 = \frac{1}{4}\frac{\Lambda^2}{\xi^2}\left(b\ddot\theta + \ddot q\right) = -\frac{1}{4}\frac{\Lambda^2}{\xi^2}\left(2\xi\Lambda\dot q + \Lambda^2 q\right) = x_4. \tag{34}$$

With the above two relations, Eq. (32) can be rewritten into (28) finally.

4.2 Step 2: applying variable elimination

It follows from the first equation of (28) that

$$\begin{cases} \dot{x}_2 = -\ddot{x}_1 - \frac{1}{2}\frac{\Lambda}{\xi}\dot{x}_1, \\ \ddot{x}_2 = -\dddot{x}_1 - \frac{1}{2}\frac{\Lambda}{\xi}\ddot{x}_1, \\ \dddot{x}_2 = -x_1^{(4)} - \frac{1}{2}\frac{\Lambda}{\xi}\dddot{x}_1. \end{cases} \tag{35}$$

Substituting the above relations into the second equation of (28) yields

$$x_1^{(4)} = f\left(\ddot{x}_1, \dddot{x}_1, \theta, \dot{\theta}, t\right) + B\left(\theta, \dot{\theta}, t\right) u, \tag{36}$$

where

$$f\left(\ddot{x}_1, \dddot{x}_1, \theta, \dot{\theta}, t\right) = -\frac{2\Lambda\xi I(\theta, \dot{\theta}, t)}{\Delta(\theta, \dot{\theta}, t)}\dddot{x}_1 - \frac{\Lambda^2 I(\theta, \dot{\theta}, t)}{\Delta(\theta, \dot{\theta}, t)}\ddot{x}_1 \tag{37}$$

and

$$B\left(\theta, \dot{\theta}, t\right) = -\frac{b\Lambda^3}{2\xi\Delta(\theta, \dot{\theta}, t)} \neq 0. \tag{38}$$

With the above procedure, we derive the following result.

Proposition 2. The system (7) is equivalent to the HOFA system (36)–(38).

4.3 Step 3: deriving the interrelations

Before using the standard fully actuated system approach in [25] to design the controller for the HOFA system (36)–(38), to make the control law available, we need to express the variables $x_1, \dot{x}_1, \ddot{x}_1$ and $\dddot{x}_1$ as functions with $\theta, q, \dot{\theta}$ and $\dot{q}$ as the independent variables.

First, it follows from the first equation of (27) that

$$x_1 = -\frac{b\Lambda}{2\xi}\theta + \frac{\Lambda}{2\xi}\left(4\xi^2 - 1\right) q + b\dot{\theta} + \dot{q}. \tag{39}$$

Taking the derivative on both sides of the above equation and using the second equation of (7), we have

$$\begin{aligned} \dot{x}_1 &= -\frac{b\Lambda}{2\xi}\dot{\theta} + \frac{\Lambda}{2\xi}\left(4\xi^2 - 1\right)\dot{q} + \left(b\ddot{\theta} + \ddot{q}\right) \\ &= -\frac{b\Lambda}{2\xi}\dot{\theta} + \frac{\Lambda}{2\xi}\left(4\xi^2 - 1\right)\dot{q} - 2\xi\Lambda\dot{q} - \Lambda^2 q \\ &= -\frac{b\Lambda}{2\xi}\dot{\theta} - \frac{\Lambda}{2\xi}\dot{q} - \Lambda^2 q. \end{aligned} \tag{40}$$

Taking the derivative on both sides of the above equation and using the second equation of (7) again, we obtain

$$\begin{aligned} \ddot{x}_1 &= -\frac{\Lambda}{2\xi}b\ddot{\theta} - \frac{\Lambda}{2\xi}\ddot{q} - \Lambda^2\dot{q}, \\ &= -\frac{\Lambda}{2\xi}\left(b\ddot{\theta} + \ddot{q}\right) - \Lambda^2\dot{q}, \\ &= \frac{\Lambda}{2\xi}\left(2\xi\Lambda\dot{q} + \Lambda^2 q\right) - \Lambda^2\dot{q}, \\ &= \frac{\Lambda^3}{2\xi}q, \end{aligned} \tag{41}$$

from which we have

$$\dddot{x}_1 = \frac{\Lambda^3}{2\xi}\dot{q}. \tag{42}$$

Combining (39)–(42) clearly gives the following result.

Proposition 3. The variables $x_1, \dot{x}_1, \ddot{x}_1$ and $\dddot{x}_1$ in the HOFA system (36) are linked to the original system variables $\theta, q, \dot{\theta}$ and $\dot{q}$ by the following variable transformation:

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ \ddot{x}_1 \\ \dddot{x}_1 \end{bmatrix} = \frac{1}{2\xi} \begin{bmatrix} -b\Lambda & \Lambda\left(4\xi^2-1\right) & 2\xi b & 2\xi \\ 0 & -2\xi\Lambda^2 & -b\Lambda & -\Lambda \\ 0 & \Lambda^3 & 0 & 0 \\ 0 & 0 & 0 & \Lambda^3 \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \dot{\theta} \\ \dot{q} \end{bmatrix}. \tag{43}$$

5 Control system design

Let us first consider the solution to Problem 1.

5.1 Attitude stabilization

According to the Proposition 2.2 in [25], choose for the system (36)–(38) the following control law:

$$u = -B^{-1}\left(\theta, \dot{\theta}, t\right)\left(f\left(\ddot{x}_1, \dddot{x}_1, \theta, \dot{\theta}, t\right) + \sum_{i=0}^{3} a_i x_1^{(i)}\right), \tag{44}$$

and then we obtain the following closed-loop system:

$$x_1^{(4)} + \sum_{i=0}^{3} a_i x_1^{(i)} = 0. \tag{45}$$

To design the controller gains $a_i, i = 0, 1, 2, 3$, the following lemma presented in [32] is needed.

Lemma 5. For a given matrix $F \in \mathbb{R}^{nr\times nr}$, all the matrices $A_{0\sim n-1}$ and $V \in \mathbb{R}^{nr\times nr}$ satisfying $\det V \neq 0$ and

$$\Phi\left(A_{0\sim n-1}\right) = VFV^{-1} \tag{46}$$

are given by

$$\begin{cases} A_{0\sim n-1} = -ZF^n V^{-1}(Z, F), \\ V = V(Z, F) = \begin{bmatrix} Z \\ ZF \\ \vdots \\ ZF^{n-1} \end{bmatrix}, \end{cases} \tag{47}$$

where $Z \in \mathbb{R}^{r\times nr}$ is a parameter matrix satisfying

$$\det V(Z, F) \neq 0. \tag{48}$$

Using the above Lemma 5, we can prove the following result.

Theorem 1. Let system (36)–(38) satisfy Assumptions 1 and 2, and α, β, s_1 and s_2 be a set of negative scalars and $s_1 \neq s_2$. Then, the control law which guarantees that the closed-loop poles are $\alpha \pm \beta\,\mathrm{i}, s_1$ and s_2 are given by (44), where the variables $x_1, \dot{x}_1, \ddot{x}_1$ and $\dddot{x}_1$ is derived by (43), and the controller gains $a_i, i = 0, 1, 2, 3$, are taken as

$$a_{0\sim 3} = -\Psi V^{-1}, \tag{49}$$

where

$$V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ v_{1-} & v_{1+} & s_1 & s_2 \\ v_{2-} & v_{2+} & s_1^2 & s_2^2 \\ v_{3-} & v_{3+} & s_1^3 & s_2^3 \end{bmatrix}, \quad \Psi = \begin{bmatrix} \varphi_- & \varphi_+ & s_1^4 & s_2^4 \end{bmatrix}, \tag{50}$$

with

$$\begin{cases} v_{1\pm} = \alpha \pm \beta, \\ v_{2\pm} = \alpha^2 - \beta^2 \pm 2\alpha\beta, \\ v_{3\pm} = \alpha^3 - 3\alpha\beta^2 \pm \left(3\alpha^2\beta - \beta^3\right) \end{cases} \tag{51}$$

and

$$\varphi_{\pm} = \alpha^4 - 6\alpha^2\beta^2 + \beta^4 \pm 4\alpha\beta\left(\alpha^2 - \beta^2\right). \tag{52}$$

Proof. Choose

$$Z = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} \alpha & \beta & 0 & 0 \\ -\beta & \alpha & 0 & 0 \\ 0 & 0 & s_1 & 0 \\ 0 & 0 & 0 & s_2 \end{bmatrix}, \tag{53}$$

and then the matrix V can be obtained by (50), and it is easy to deduce that

$$\begin{aligned} \det V &= -2\beta\left(s_1 - s_2\right)\left(s_1^2 - 2s_1\alpha + \alpha^2 + \beta^2\right)\left(s_2^2 - 2s_2\alpha + \alpha^2 + \beta^2\right) \\ &= -2\beta\left(s_1 - s_2\right)\left[\left(s_1 - \alpha\right)^2 + \beta^2\right]\left[\left(s_2 - \alpha\right)^2 + \beta^2\right], \end{aligned} \tag{54}$$

from which we know that V is nonsingular when $s_1 \neq s_2$ and $\alpha, \beta, s_1, s_2 < 0$. Thus the result can be immediately obtained by using Lemma 5.

5.2 Attitude maneuver

This subsection further considers the solution to Problem 2. The generalized proportional-integral-derivative (PID) control method proposed in [31] is applied.

5.2.1 *Generalized PID control law*

Following the generalized PID approach proposed in [31], take for the system (36)–(38) the following control law:

$$\begin{cases} \dot{\eta}(t) = \theta(t) - \theta_c, \\ u = -B^{-1}(\cdot)\left(f(\cdot) - K_{\mathrm{PD}}x_1^{(0\sim3)} - K_{\mathrm{I}}\eta\right), \end{cases} \tag{55}$$

where K_{PD} and K_{I} are two parameters to be designed, θ_c is a given constant reference signal, and $x_1, \dot{x}_1, \ddot{x}_1$ and $\dddot{x}_1$ are derived using the relations in (43).

With the above control law (55), the closed-loop system is a linear one and appears as

$$\begin{cases} \dot{\eta}(t) = \theta(t) - \theta_c, \\ x_1^{(4)} = K_{\mathrm{PD}}x_1^{(0\sim3)} + K_{\mathrm{I}}\eta. \end{cases} \tag{56}$$

Define

$$C = -\frac{2\xi}{b\Lambda^3}\begin{bmatrix} \Lambda^2 & 2\xi\Lambda & 1 & 0 \end{bmatrix} \tag{57}$$

and

$$\tilde{A} = \begin{bmatrix} \Phi\left(0_{0\sim3}\right) & 0 \\ C & 0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0_{3\times1} \\ 1 \\ 0 \end{bmatrix}. \tag{58}$$

The following result is immediately obtained by using Lemma 1.

Lemma 6. Let the system (36) satisfy Assumptions 1 and 2, and θ_c be a given constant reference angle. If the parameter matrices K_{PD} and K_{I} are chosen to make

$$A_c = \tilde{A} + \tilde{B}\begin{bmatrix} K_{\mathrm{PD}} & K_{\mathrm{I}} \end{bmatrix} \tag{59}$$

Hurwitz, then the closed-loop eigenvalues are given by those of matrix A_c, and the following tracking relation is guaranteed:

$$\lim_{t\to\infty}\theta(t) = \theta_c. \tag{60}$$

Proof. It follows from (43) that $\theta = Cx_1^{(0\sim3)}$. Then, the conclusion is directly obtained by applying Lemma 1 to the HOFA system (36)–(38) with the above output equation.

5.2.2 *Parametric design of* $K_{\rm PD}$ *and* $K_{\rm I}$

In this part we further discuss the design of $K_{\rm PD}$ and $K_{\rm I}$, for which a parametric approach is given based on Lemmas 2–4.

Theorem 2. Let system (36) satisfy Assumptions 1 and 2. α, β, s_1, s_2 and s_3 be a set of negative scalars and $s_i \neq s_j, i \neq j$, and

$$\operatorname{rank}\begin{bmatrix} \Phi\left(0_{0\sim3}\right) & B_c \\ C & 0 \end{bmatrix} = 5 \quad \text{with } B_c = \begin{bmatrix} 0_{3\times1} \\ 1 \end{bmatrix}. \tag{61}$$

Then, all the matrices $K_{\rm PD}$ and $K_{\rm I}$ making $\alpha \pm \beta\,\mathrm{i}, s_1, s_2$ and s_3 be the eigenvalues of A_c are given by

$$\begin{cases} \begin{bmatrix} K_{\rm PD} & K_{\rm I} \end{bmatrix} = \Upsilon \tilde{V}^{-1}, \\ \tilde{V} = \begin{bmatrix} \alpha-\beta & \alpha+\beta & s_1 & s_2 & s_3 \\ \tilde{\alpha}_1-\tilde{\beta}_1 & \tilde{\alpha}_1+\tilde{\beta}_1 & s_1^2 & s_2^2 & s_3^2 \\ \tilde{\alpha}_2-\tilde{\beta}_2 & \tilde{\alpha}_2+\tilde{\beta}_2 & s_1^3 & s_2^3 & s_3^3 \\ \tilde{\alpha}_3-\tilde{\beta}_3 & \tilde{\alpha}_3+\tilde{\beta}_3 & s_1^4 & s_2^4 & s_3^4 \\ \tilde{\alpha}_4-\tilde{\beta}_4 & \tilde{\alpha}_4+\tilde{\beta}_4 & \nu(1) & \nu(2) & \nu(3) \end{bmatrix}, \\ \Upsilon = \begin{bmatrix} \tilde{\alpha}_5-\tilde{\beta}_5 & \tilde{\alpha}_5+\tilde{\beta}_5 & s_1^5 & s_2^5 & s_3^5 \end{bmatrix}, \end{cases} \tag{62}$$

where

$$\begin{cases} \tilde{\alpha}_1 = \alpha^2-\beta^2, \\ \tilde{\alpha}_2 = \alpha\left(\alpha^2-3\beta^2\right), \\ \tilde{\alpha}_3 = \alpha^4-6\alpha^2\beta^2+\beta^4, \\ \tilde{\alpha}_4 = -\frac{2\xi}{b\Lambda^3}\left[\left(\alpha+2\xi\Lambda\right)\alpha-\beta^2+\Lambda^2\right], \\ \tilde{\alpha}_5 = \alpha^5-10\alpha^3\beta^2+5\alpha\beta^4, \end{cases} \tag{63}$$

$$\begin{cases} \tilde{\beta}_1 = 2\alpha\beta, \\ \tilde{\beta}_2 = \beta\left(3\alpha^2-\beta^2\right), \\ \tilde{\beta}_3 = 4\alpha\beta\left(\alpha^2-\beta^2\right), \\ \tilde{\beta}_4 = -\frac{4\xi}{b\Lambda^3}\left(\alpha+\xi\Lambda\right)\beta, \\ \tilde{\beta}_5 = \beta\left(5\alpha^4-10\alpha^2\beta^2+\beta^4\right), \end{cases} \tag{64}$$

and

$$\nu\left(i\right) = -\frac{2\xi}{b\Lambda^3}\left(s_i^2+2\xi\Lambda s_i+\Lambda^2\right), \quad i=1,2,3. \tag{65}$$

Proof. It follows from Lemma 3 that the pair of right coprime polynomial matrices $N(s)$ and $D(s)$ satisfying the following RCF

$$\left[sI_4-\Phi\left(0_{0\sim3}\right)\right]^{-1}B_c = N(s)D^{-1}(s) \tag{66}$$

are given by

$$N(s) = \begin{bmatrix} 1 & s & s^2 & s^3 \end{bmatrix}^{\rm T}, \quad D(s) = s^4. \tag{67}$$

Then, using the Lemma 4, the pair of right coprime polynomial matrices $\tilde{N}(s)$ and $\tilde{D}(s)$ satisfying the RCF

$$\left(sI-\tilde{A}\right)^{-1}\tilde{B} = \tilde{N}(s)\tilde{D}^{-1}(s) \tag{68}$$

are given by

$$\tilde{N}(s) = \begin{bmatrix} sN(s) \\ CN(s) \end{bmatrix} = \sum_{i=0}^{4}\tilde{N}_i s^i, \quad \tilde{D}(s) = sD(s) = \sum_{i=0}^{5}\tilde{D}_i s^i, \tag{69}$$

when Eq. (61) holds.

Let

$$\tilde{F} = \begin{bmatrix} \alpha & \beta & 0 & 0 & 0 \\ -\beta & \alpha & 0 & 0 & 0 \\ 0 & 0 & s_1 & 0 & 0 \\ 0 & 0 & 0 & s_2 & 0 \\ 0 & 0 & 0 & 0 & s_3 \end{bmatrix}, \quad \tilde{Z} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}. \tag{70}$$

Then, according to the second conclusion in Lemma 2, all the gain matrix $K = [\, K_{\mathrm{PD}} \;\; K_{\mathrm{I}} \,]$ satisfying

$$A_c = \tilde{A} + \tilde{B} \begin{bmatrix} K_{\mathrm{PD}} & K_{\mathrm{I}} \end{bmatrix} = \tilde{V}\tilde{F}\tilde{V}^{-1} \tag{71}$$

is given by

$$\begin{cases} \begin{bmatrix} K_{\mathrm{PD}} & K_{\mathrm{I}} \end{bmatrix} = \tilde{W}\tilde{V}^{-1}, \\ \tilde{V} = \sum\limits_{i=0}^{4} \tilde{N}_i \tilde{Z} \tilde{F}^i, \\ \tilde{W} = \sum\limits_{i=0}^{5} \tilde{D}_i \tilde{Z} \tilde{F}^i, \end{cases} \tag{72}$$

and V is certainly nonsingular in view of

$$\det V = \frac{4\xi}{b\Lambda}\beta\,(s_3 - s_4)\,(s_3 - s_5)\,(s_4 - s_5)\left[(s_3-\alpha)^2 + \beta^2\right]\left[(s_4-\alpha)^2 + \beta^2\right]\left[(s_5-\alpha)^2 + \beta^2\right]. \tag{73}$$

Finally, the expression (62) can be obtained by substituting (69) and (70) into (72).

6 Simulation results

In this section, numerical simulations are carried out based on the parameters of a practical spacecraft with a large antenna, as shown below (see [17, 18]):

$$\begin{aligned} b &= -108.88\,\sqrt{\mathrm{kg}}\cdot\mathrm{m}, \quad \xi = 0.005, \quad \Lambda = 2\pi \times 0.151, \\ I &= 527.78\sin[3.424\theta(t) + 1.072\dot{\theta}(t)] - 0.3921t + 20667.25\ \mathrm{kg}\cdot\mathrm{m}^2. \end{aligned}$$

6.1 Attitude stabilization

Select the set of desired closed-loop poles as

$$\Gamma = \{(-1.2186 \pm 1.4625) \times 10^{-2}\,\mathrm{i}, -0.24485, -6.7860 \times 10^{-3}\} \tag{74}$$

and thus we have

$$\alpha = -1.2186 \times 10^{-2}, \quad \beta = -1.4625 \times 10^{-2}, \tag{75}$$

$$s_1 = -0.24485, \quad s_2 = -6.7860 \times 10^{-3}. \tag{76}$$

Then, according to Theorem 1, the gains $a_i, i = 0, 1, 2, 3$, in (44) can be computed as

$$\begin{aligned} a_0 &= 6.0214 \times 10^{-7}, \quad a_1 = 1.3169 \times 10^{-4}, \\ a_2 &= 8.1569 \times 10^{-3}, \quad a_3 = 0.27601. \end{aligned} \tag{77}$$

When the initial values are chosen as

$$\theta(0) = -45^\circ, \quad \dot{\theta}(0) = 0.5\,(^\circ/\mathrm{s}), \quad \dot{q}(0) = q(0) = 0, \tag{78}$$

the simulation results are obtained and shown in Figure 1.

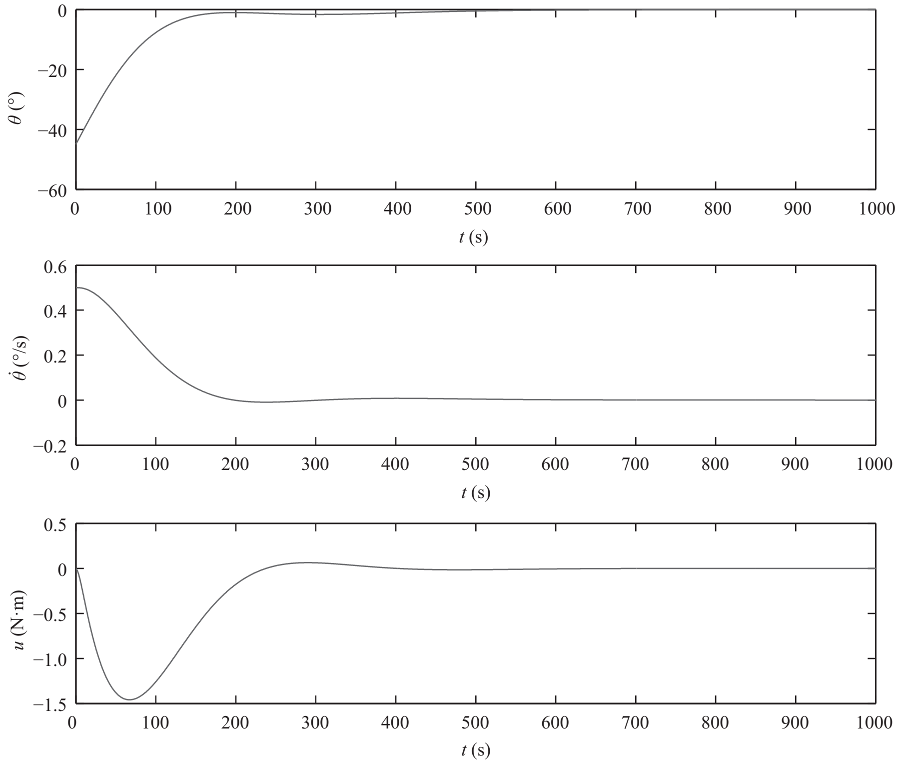

Figure 1 Simulation results for the attitude stabilization using the HOFA system approach.

For comparison, we also design the control law using the method in [17] for the nominal system with $I = 20667.25\ \mathrm{kg}\cdot\mathrm{m}^2$. In order to make an equal comparison, the desired closed-loop poles are taken the same as in (74), and the corresponding control law is designed as

$$u = k_1\theta + k_2 q + k_3\dot{\theta} + k_4\dot{q}, \tag{79}$$

where

$$\begin{aligned} &k_1 = -5.8947\times 10^{-3}, \quad k_2 = 1.7020\times 10^{2}, \\ &k_3 = -1.2891, \quad k_4 = -20.526. \end{aligned} \tag{80}$$

Choose the initial value as

$$\theta\,(0) = 2^\circ, \quad \dot{\theta}\,(0) = 0.5^\circ/\mathrm{s}, \quad \dot{q}\,(0) = q\,(0) = 0, \tag{81}$$

and then the simulation result is obtained and shown in Figure 2.

On one hand, it can be seen from Figure 1 that the attitude angle and the angular velocity converge to zero smoothly without overshoot at about 800 s. What is more, although the control torque has taken the burden to cancel the time-varying nonlinearity in the system, its amplitude is still within $1.5\ \mathrm{N}\cdot\mathrm{m}$, which is within an acceptable range in practical engineering, from which the effect of the proposed HOFA system approach is well verified.

On the other hand, as shown in Figure 2, although the initial values of the states are chosen very small, the states of the system diverge after about 250 s when the control method in [17] is applied, and the amplitude of the control torque quickly exceed $250\ \mathrm{N}\cdot\mathrm{m}$, revealing the failure of the traditional method.

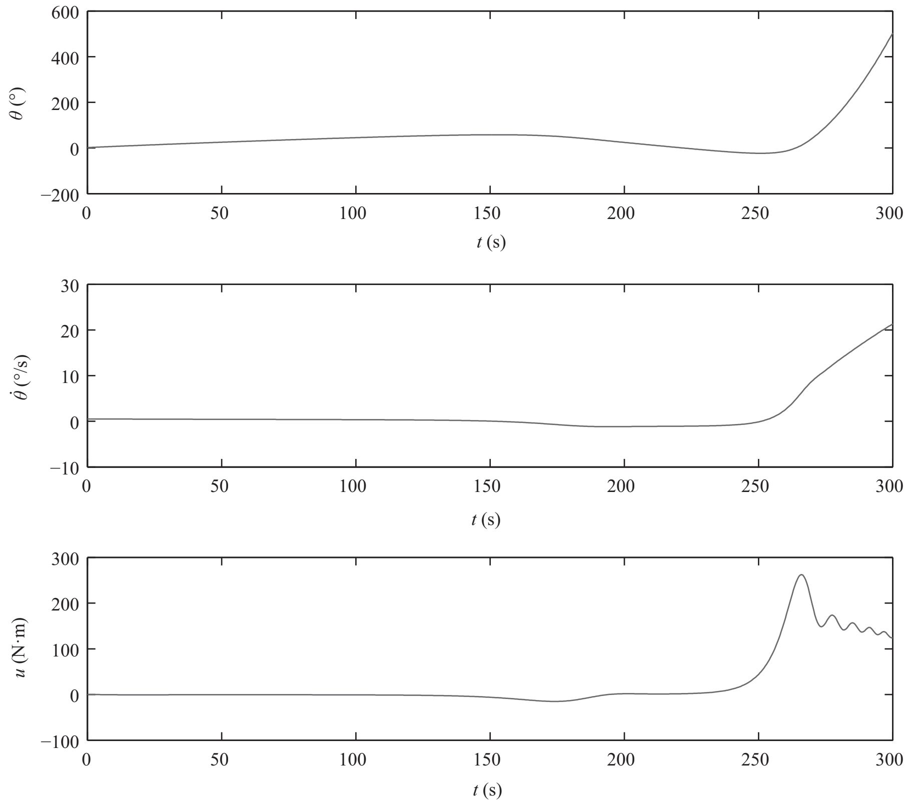

Figure 2 Simulation results for the attitude stabilization using the method in [17].

6.2 Attitude maneuver

The first four desired closed-loop poles are also taken as (74), and the fifth one is chosen as $s_3 = -0.10082$. Then, according to Theorem 2, the gains $K_{\rm PD}$ and $K_{\rm I}$ in the control law (55) can be computed as

$$K_{\rm PD} = \begin{bmatrix} -1.3878 \times 10^{-5} \\ -9.5399 \times 10^{-4} \\ -3.5984 \times 10^{-2} \\ -0.37683 \end{bmatrix}^{\rm T} \quad \text{and} \quad K_{\rm I} = -6.2710 \times 10^{-4}. \tag{82}$$

With the reference angle $\theta_c = 45^\circ$ and the initial values still chosen as in (78) and adding $\eta(0) = 0$, the corresponding simulation result is carried out and shown in Figure 3.

It can be seen from Figure 3 that the attitude angle asymptotically converges the preset reference angle at about 800 s without overshoot, and the amplitude of the control torque is within 1.5 N · m, which is still at an acceptable level in practical engineering. This verifies that the proposed method is also well-suited for the attitude maneuvering task of practical spacecraft.

7 Conclusion

Due to liquid sloshing and fuel consumption, the moment of inertia of a flexible spacecraft in practical engineering usually depends on the attitude angle, the angular velocity and the time t in a nonlinear and time-varying manner. In such situations, the attitude control of a flexible spacecraft turns out to be a problem of time-varying nonlinear control and hence becomes much more challenging. With the help of a recently proposed HOFA system approach for nonlinear control system design, the problem is satisfactorily solved. As a result, a linear time-invariant closed-loop system whose eigenvalues can be arbitrarily assigned is derived, which is the most important advantage compared with existing results (see [19–24]).

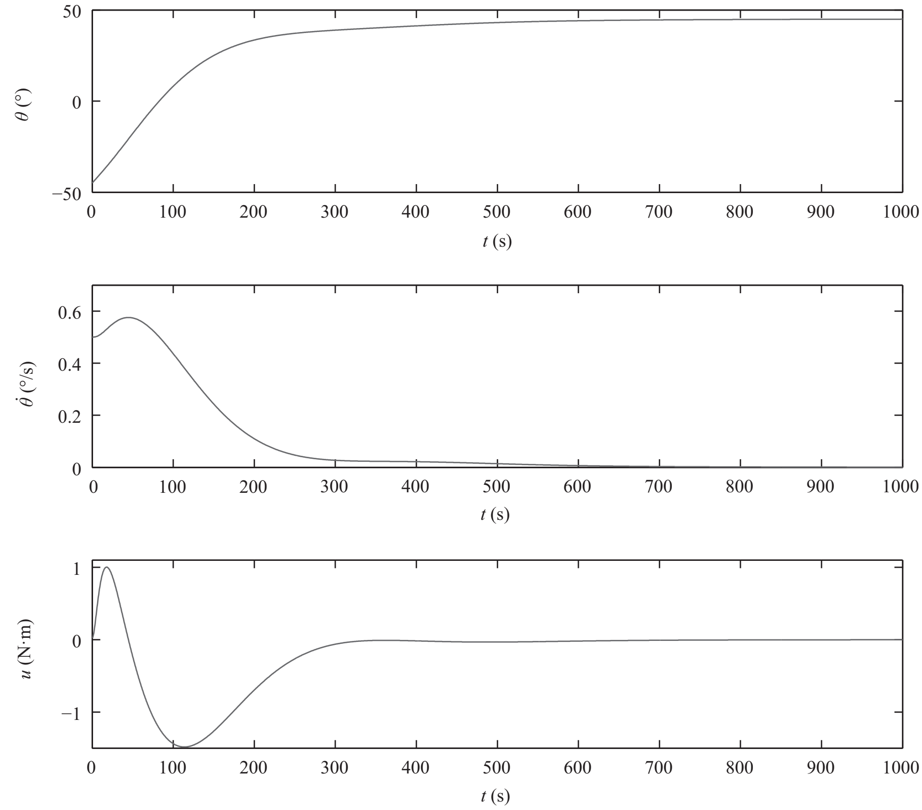

Figure 3 Simulation results for the attitude maneuver.

Again, this article only considers the case of full state feedback. For the case where the flexible modes are not measurable, an observer-based state feedback control law can be designed instead, and treatment will be presented in future work.

It should be noted that although the proposed method is based on an accurately known open-loop system, it can also be extended to the case of uncertain systems. At the theoretical level, the cases where the considered system is subject to uncertainties [33], external disturbance [32], and unknown parameters [34] are all investigated in depth. All these results can be readily used based on the HOFA model and the basic control law obtained in our current paper. Specific application-level issues may be further discussed in other papers.

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