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Plug-and-Play algorithm for under-sampling Fourier single-pixel imaging

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Dear editor,

Single-pixel imaging (SPI) is an indirect image measurement technique that can capture images using a detector without spatial resolution [1]. The principle of SPI is using a sequence of structured light patterns to sample the object image and recording the reflected or refracted light intensity using a single-pixel detector. The image can be reconstructed by correlating the structured patterns and the detected light intensity results. Compared to capturing images using a detector array, SPI offers multiple unique advantages such as wide spectrum range, low cost, and high signal-to-noise ratio.

Despite these advantages, SPI requires numerous measurements to reconstruct a high-quality image [1]. Therefore, the balance between the imaging quality and speed is important in the SPI technique. The reconstruction of higher-quality target images with fewer structured patterns has become a research hotspot. Recently, in SPI, deterministic orthogonal basis patterns have been used. Compared with random patterns, the deterministic orthogonal basis patterns form a completely orthogonal set, with the benefits of imaging quality improvement and measurement reduction. Fourier single-pixel imaging (FSI) is one of the representative SPI techniques using deterministic orthogonal basis patterns. FSI uses grayscale Fourier basis patterns for illumination and an inverse Fourier transform to reconstruct the object image. However, with an increase in the number of image pixels, the FSI speed is considerably limited by the modulator's frequency. FSI requires considerable time to obtain full sampling of object spectrum, rendering it unfeasible for real-time imaging [2]. To solve this limitation, previous studies [3, 4] only made use of the low-frequency region of the Fourier spectrum to reduce the number of samples as the spectrum energy is mostly concentrated in its low-frequency component. However, doing so reduces the imaging resolution and causes a ringing effect in the reconstructed image. To remove the ringing effect and improve the image quality, the deep learning method can be used for under-sampling FSI. The image recovered from the low-frequency Fourier spectrum is processed via a deep neural network to achieve denoising-deringing and detail retention [5]. However, this method requires a long-time network training and must retrain a new network when the image size or other parameters change. Moreover, it attempts to fit a brute-force mapping between the reconstructed and the desirable images, thus ignoring the FSI model. Recently, researchers have proposed a new sparse Fourier single-pixel imaging based on the compressed sensing (CS) algorithm to reduce the number of samples [6]. Compared to the method of only acquiring the low-frequency region of the Fourier space, it can improve the FSI reconstruction quality at the same sampling ratio. However, it uses the total variation (TV) regularization as a hand-crafted prior, which results in watercolor-like artifacts. Therefore, a method for high-quality image reconstruction with a low sampling ratio is desired in FSI.

In this study, we present a new high-quality under-sampling FSI method, namely PnP-FSI. By combining the benefits of Plug-and-Play (PnP) framework and generalized alternating projection algorithm (GAP), the proposed PnP-FSI method can well reconstruct the object images and overcome the ringing effect in the case of under-sampling. Extensive simulation and real experimental results demonstrate that the PnP-FSI method outperforms the existing FSI algorithms and is quite suitable for practical SPI applications.

Fourier single-pixel imaging. According to the Fourier transform theorem, FSI uses grayscale Fourier basis patterns for illumination and receives light reflected from the object image using a single-pixel detector. Mathematically, the received reflection light intensity I_φ can be expressed as follows:

$$I_\varphi(u,v) = \sum_x \sum_y P_\varphi(x,y;u,v)O(x,y), \tag{1}$$

where $O(x,y)$ is the object image, x and y are the coordinates in the spatial domain, u and v are the coordinates in the Fourier domain, and φ is the phase parameter. P_φ is the

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grayscale Fourier basis pattern, which can be represented as follows:

$$P_\varphi(x,y;u,v) = a + b\cos\left(2\pi x\frac{u}{M} + 2\pi y\frac{v}{N} + \varphi\right), \quad (2)$$

where a is the average intensity of patterns and b is the amplitude of the modulation pattern. M and N are the sizes of the object image. In this study, we adopt the four-step phase-shifting illumination. Therefore, the values of φ are 0, $\pi/2$, π, and $3\pi/2$.

The Fourier spectrum of the image $O(x,y)$ is given as follows:

$$\begin{aligned} F(u,v) &= \mathscr{F}\{O(x,y)\} \\ &= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} O(x,y)\exp\left(-\mathrm{j}2\pi\left(\frac{ux}{M}+\frac{vy}{M}\right)\right). \end{aligned} \quad (3)$$

Using trigonometric transforms, the relationship between the FSI measurements I_φ and the Fourier spectrum $F(u,v)$ can be obtained and written as follows:

$$F(u,v) = \frac{1}{2b}[(I_0 - I_\pi) + \mathrm{j}(I_{\pi/2} - I_{3\pi/2})]. \quad (4)$$

From (4), the Fourier spectrum of the target object can be obtained using the four-step phase-shifting approach. After the Fourier spectrum acquisition, the object image can be reconstructed via inverse Fourier transformation.

In theory, the reconstruction of an image with $M\times N$ pixels requires $4\times M\times N$ measurements. To reduce the number of measurements, the most commonly used method only samples the low-frequency spectrum. Three sampling order schemes exist for the low-frequency acquisition method, namely square, diamond, and circle sampling orders. It has been demonstrated that the circular sampling order can realize higher-quality images in a low-frequency situation [3]. However it usually reduces the imaging resolution and exhibits the ringing effect. To acquire a larger range of frequency spectrum in fewer measurements rather than the low frequencies only, we adopt a variable density random sampling scheme to select the spectrum [6]. This strategy is based on the circular sampling order and the object spectrum characteristics, i.e., high-frequency sparseness and low-frequency concentration. The probability density function of the variable density random sampling can be expressed as follows:

$$\rho(d) = \begin{cases} 1, & d \leqslant D, \\ 1/(1-d)^p, & \text{otherwise}, \end{cases} \quad (5)$$

where d is the distance from the sampling point to the spatial center and p is the polynomial coefficient. Reportedly, a high-quality image cannot be well reconstructed from this under-sampling Fourier spectrum using inverse Fourier transformation [6]. Therefore, we propose the PnP-FSI method for high-quality FSI reconstruction in under-sampling situation.

Plug-and-Play based FSI algorithm. To recover high-quality image from the variable density random sampling of the Fourier spectrum, FSI reconstruction can be modeled as an optimization problem. Based on the FSI principle, we define this optimization problem as follows:

$$\arg\min_{O,Z} \frac{1}{2}\|O-Z\|_2^2 + \lambda g(Z), \quad \text{subject to} \quad FO = I_m, \quad (6)$$

where O is the object image and Z is the auxiliary parameter for variable splitting, I_m is the under-sampling measurements acquired by the single-pixel detector, F is the under-sampling Fourier transform, $g(Z)$ is a regularization term containing certain prior information of the object image, and λ is the regularization parameter.

According to GAP algorithm [7], this optimization problem can be divided into two sub-problems in which O and Z are alternately updated. Letting k denote the iteration number, we derive a closed-form expression of O update as follows:

$$O^{k+1} = Z^k + \mathscr{F}^{-1}\{I_m - \mathscr{F}\{Z^k\}\}, \quad (7)$$

where $\mathscr{F}^{-1}\{\cdot\}$ is the inverse Fourier transform and $\mathscr{F}\{\cdot\}$ is the Fourier transform.

Inspired by the PnP framework [8], the sub-problem Z is regarded as a denoising problem and replaced by an off-the-shelf image denoising algorithm. Hence, the Z update is expressed as follows:

$$Z^{k+1} = \mathcal{D}_\sigma(O^{k+1}), \quad (8)$$

where D_σ is the denoiser that can use the state-of-the-art image denoising algorithms, such as pre-trained CNN denoising networks [9]. $\sigma = \sqrt{\lambda}$ is the denoising strength of the denoiser. The proposed PnP-FSI algorithm is summarized in Appendix A.

Experimental results. The performance of the proposed PnP-FSI method is confirmed by extensive simulations and real experiments are shown in Appendix B.

Conclusion. This study proposes a novel method for under-sampling FSI. By utilizing the advantages of the Plug-and-Play framework and the advanced denoising algorithm, the reconstructed image quality of FSI is considerably improved. The proposed algorithm is confirmed by simulations and experiments, demonstrating its great potential in the practical SPI application, particularly in large-size image reconstruction.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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