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Low-complexity beamforming design for IRS-aided communication systems

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Dear editor,

Recently, intelligent reflecting surface (IRS) has emerged as a promising technology to achieve high spectrum and energy efficiency for wireless communications. To fully achieve the potential of the IRS, the joint design of active and passive beamforming is of great importance, where the alternating optimization (AO) technique is commonly used to deal with the non-convexity of the joint optimization problem [1–4].

However, the AO-based method involves high computational complexity, which would become prohibitive in a massive IRS setup. Responding to this, in this study, we propose two low-complexity beamforming algorithms for IRS-aided communications, which avoid the complex AO process. The main contributions of our studies are summarized as follows. (1) For the single-user case, we obtain closed-form optimal solutions for both the IRS phase shifts and the BS beamforming vectors, thereby significantly reducing the complexity of beamforming design. (2) For the multi-user case, we propose a simple yet efficient beamforming scheme, which has a low complexity of $\mathcal{O}(M^3)$ and avoids the complex iterative optimization process. (3) Numerical results show that our proposed beamforming scheme for the single-user case has a significant advantage over the AO-based scheme [1]. Also, the proposed beamforming scheme for the multi-user case achieves comparable performance to the SDR-based scheme [1] but with much lower complexity.

System model and problem formulation. We consider an IRS-aided multi-user system, where an IRS with M reflecting elements assists the downlink transmission from the BS to K single-antenna users. The BS and the IRS are equipped with an N-element uniform linear array (ULA) deployed along the y axis and an $M_y \times M_z$ uniform rectangular array (URA) deployed on the y-o-z plane ($M_y \times M_z = M$), respectively. Let $\boldsymbol{G} \in \mathbb{C}^{M\times N}$, $\boldsymbol{h}_{\mathrm{r},k}^{\mathrm{T}} \in \mathbb{C}^{1\times M}$, and $\boldsymbol{h}_{\mathrm{d},k}^{\mathrm{T}} \in \mathbb{C}^{1\times N}$ denote the channels from the BS to the IRS, from the IRS to the k-th user, and from the BS to the k-th user, respectively. We express the equivalent baseband signal received by the k-th user as

$$y_k = \boldsymbol{h}_k^{\mathrm{T}} \sum_{i=1}^{K} \boldsymbol{w}_i s_i + n_k, \tag{1}$$

where $n_k \sim \mathcal{CN}(0,\sigma_0^2)$ denotes the additive white Gaussian noise (AWGN), $\boldsymbol{w}_i$ and s_i, satisfying $\mathbb{E}\{|s_i|^2\} = 1$, are the beamforming vector and symbol for the i-th user, respectively. In addition, we define $\boldsymbol{h}_k^{\mathrm{T}} \triangleq (\boldsymbol{h}_{\mathrm{r},k}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{G} + \boldsymbol{h}_{\mathrm{d},k}^{\mathrm{T}})$ as the effective channel of the k-th user. And $\boldsymbol{\Theta}$ denotes the phase shift matrix of the IRS given by $\boldsymbol{\Theta} = \mathrm{diag}(\boldsymbol{\xi})$, where $\boldsymbol{\xi} = [\mathrm{e}^{\mathrm{j}\vartheta_1}, \ldots, \mathrm{e}^{\mathrm{j}\vartheta_m}, \ldots, \mathrm{e}^{\mathrm{j}\vartheta_M}]^{\mathrm{T}}$ is the phase shift beam with $\vartheta_m \in [0, 2\pi)$ being the phase shift of the m-th reflecting element. Based on (1), we express the achievable rate of the k-th user as

$$R_k = \log_2\left(1 + |\boldsymbol{h}_k^{\mathrm{T}}\boldsymbol{w}_k|^2 \Bigg/ \left(\sum_{i\neq k}^{K} |\boldsymbol{h}_k^{\mathrm{T}}\boldsymbol{w}_i|^2 + \sigma_0^2\right)\right). \tag{2}$$

We aim to minimize the transmit power subject to the quality of service (QoS) constraints, by joint design of the precoding matrix $\boldsymbol{W} \triangleq [\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K]$ and the phase shift beam $\boldsymbol{\xi}$. Let r and ξ_m denote the target rate and the m-th element of the phase shift beam $\boldsymbol{\xi}$, respectively. We formulate the optimization problem as

$$\min_{\boldsymbol{W},\boldsymbol{\xi}} \ \sum_{i=1}^{K} \|\boldsymbol{w}_i\|^2 \tag{3a}$$

$$\text{s.t. } R_k \geqslant r, \ k = 1, \ldots, K, \tag{3b}$$

$$|\xi_m| = 1, \ m = 1, \ldots, M, \tag{3c}$$

where (3b) and (3c) represent the QoS constraint and the unit-modulus constraint imposed by passive phase shifters, respectively.

Single-user case. We first consider the single-user case. To this end, the problem (3) becomes

$$\min_{\boldsymbol{w},\boldsymbol{\xi}} \ \|\boldsymbol{w}\|^2 \tag{4a}$$

$$\text{s.t. } |(\boldsymbol{h}_{\mathrm{r}}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{G} + \boldsymbol{h}_{\mathrm{d}}^{\mathrm{T}})\boldsymbol{w}|^2 \geqslant (2^r - 1)\sigma_0^2, \tag{4b}$$

$$|\xi_m| = 1, \ m = 1, \ldots, M. \tag{4c}$$

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Theorem 1. The optimal solutions for problem (4) are

$$\boldsymbol{\xi} = \mathrm{e}^{-\mathrm{j}\mathrm{angle}(\mathrm{diag}(\boldsymbol{h}_{\mathrm{r}})\boldsymbol{b}(\gamma,\varphi))-\mathrm{j}\mathrm{angle}(\beta \boldsymbol{h}_{\mathrm{d}}^{\mathrm{H}}\boldsymbol{a}(\theta))}, \tag{5}$$

$$\boldsymbol{w} = \sqrt{(2^r-1)\sigma_0^2}\frac{(\boldsymbol{h}_{\mathrm{r}}^{\mathrm{T}}\mathrm{diag}(\boldsymbol{\xi})\boldsymbol{G}+\boldsymbol{h}_{\mathrm{d}}^{\mathrm{T}})^{\mathrm{H}}}{\|\boldsymbol{h}_{\mathrm{r}}^{\mathrm{T}}\mathrm{diag}(\boldsymbol{\xi})\boldsymbol{G}+\boldsymbol{h}_{\mathrm{d}}^{\mathrm{T}}\|^2}, \tag{6}$$

where angle($\boldsymbol{x}$) denotes a vector with each element being the phase of the corresponding element in $\boldsymbol{x}$, $\boldsymbol{b}(\gamma,\varphi)$ and $\boldsymbol{a}(\theta)$ denote the array response vectors of the IRS and the BS, respectively. In addition, γ/φ and θ are the elevation/azimuth angle of arrival (AoA) and angle of departure (AoD) for the BS-IRS link.

Remark 1. For the single-user case, closed-form optimal solutions can be obtained if both the BS-IRS and IRS-user channels are known, but cannot be obtained if only the cascaded channels are available, which indicates the advantage of knowing both the BS-IRS and IRS-user channels.

Multi-user case. We consider the multi-user case. To cancel the inter-user interference, we adopt the zero-forcing (ZF) precoding matrix [5]

$$\boldsymbol{W} = \sqrt{P/\mathrm{tr}(\tilde{\boldsymbol{W}}\tilde{\boldsymbol{W}}^{\mathrm{H}})}\tilde{\boldsymbol{W}}, \tag{7}$$

where $\tilde{\boldsymbol{W}} \triangleq \boldsymbol{H}^*\left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}^*\right)^{-1}$ with $\boldsymbol{H} \triangleq \boldsymbol{H}_{\mathrm{d}} + \boldsymbol{G}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{r}} \in \mathbb{C}^{N\times K}$, $\boldsymbol{H}_{\mathrm{d}} \triangleq [\boldsymbol{h}_{\mathrm{d},1},\ldots,\boldsymbol{h}_{\mathrm{d},K}]$ and $\boldsymbol{H}_{\mathrm{r}} \triangleq [\boldsymbol{h}_{\mathrm{r},1},\ldots,\boldsymbol{h}_{\mathrm{r},K}]$. In addition, P is the total transmit power. As such, problem (3) becomes

$$\min_{P,\boldsymbol{\xi}} \ P \tag{8a}$$

$$\text{s.t. } P \geqslant (2^r-1)\sigma_0^2\mathrm{tr}(\tilde{\boldsymbol{W}}\tilde{\boldsymbol{W}}^{\mathrm{H}}), |\xi_m| = 1, m = 1,\ldots,M. \tag{8b}$$

It can be verified that the optimal P is $P = (2^r-1)\sigma_0^2\mathrm{tr}(\tilde{\boldsymbol{W}}\tilde{\boldsymbol{W}}^{\mathrm{H}})$, based on which, problem (8) can be equivalently transformed into

$$\min_{\boldsymbol{\xi}} \ \mathrm{tr}((\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}^*)^{-1}), \ \text{s.t.} \ |\xi_m| = 1, m = 1,\ldots,M. \tag{9}$$

The above problem is non-convex, and its objective function is too complex. Responding to this, we propose to minimize the lower bound of the objective function, which is given by Lemma 1.

Lemma 1. A lower bound of the objective function $\mathrm{tr}((\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}^*)^{-1})$ is $\frac{K}{\mathrm{tr}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}^*)}$.

To this end, we reformulate problem (9) as

$$\max_{\boldsymbol{\xi}} \ \mathrm{tr}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}^*), \ \ \text{s.t.} \ |\xi_m| = 1, \ m = 1,\ldots,M, \tag{10}$$

which can be further expressed as

$$\max_{\boldsymbol{\xi}} \ \boldsymbol{\xi}^{\mathrm{H}}\boldsymbol{F}_1\boldsymbol{\xi} + 2\mathrm{Re}(\boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{\xi}) + \mathrm{tr}(\boldsymbol{H}_{\mathrm{d}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{d}}) \tag{11a}$$

$$\text{s.t.} \ |\xi_m| = 1, \ m = 1,\ldots,M, \tag{11b}$$

where $\boldsymbol{F}_1 \triangleq \sum_{k=1}^K \mathrm{diag}(\boldsymbol{h}_{\mathrm{r},k}^{\mathrm{H}})\boldsymbol{G}^*\boldsymbol{G}^{\mathrm{T}}\mathrm{diag}(\boldsymbol{h}_{\mathrm{r},k})$ and $\boldsymbol{f}_2^{\mathrm{H}} \triangleq \sum_{k=1}^K \boldsymbol{h}_{\mathrm{d},k}^{\mathrm{H}}\boldsymbol{G}^{\mathrm{T}}\mathrm{diag}(\boldsymbol{h}_{\mathrm{r},k})$. For the case where the line-of-sight (LoS) path of the BS-user link does not exist, the original optimization problem can be approximated as the problem of maximizing the first term. For the case where the LoS path of the BS-user link exists, the original problem can be approximated as the problem of maximizing the second term. Hence, we propose to separately maximize the first term and the second term of the objective function (11a), and then choose a better solution. We first maximize the first term $\boldsymbol{\xi}^{\mathrm{H}}\boldsymbol{F}_1\boldsymbol{\xi}$, and formulate the subproblem as follows:

$$\max_{\boldsymbol{\xi}} \ \boldsymbol{\xi}^{\mathrm{H}}\boldsymbol{F}_1\boldsymbol{\xi}, \ \ \text{s.t.} \ |\xi_m| = 1, \ m = 1,\ldots,M. \tag{12}$$

To solve problem (12), we perform the eigenvalue decomposition of $\boldsymbol{F}_1$ and obtain $\boldsymbol{F}_1 = \boldsymbol{U}_F\mathrm{diag}(\lambda_1,\ldots,\lambda_M)\boldsymbol{U}_F^{\mathrm{H}}$, with its eigenvalues $\lambda_1,\ldots,\lambda_M$ in descending order. Then, we generate a sequence of $\boldsymbol{\zeta}_i, i = 1,\ldots,N_1$ with the form of $\boldsymbol{\zeta}_i \triangleq \mathrm{e}^{-\mathrm{j}\mathrm{angle}(\boldsymbol{U}_F\sqrt{\mathrm{diag}(\lambda_1,\ldots,\lambda_M)}\boldsymbol{r}_i)}$, where N_1 is the number of $\boldsymbol{\zeta}_i$'s, and $\boldsymbol{r}_i \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{I}_M)$ is generated by Gaussian randomization. As such, we obtain

$$\boldsymbol{\xi}^{(1)} = \arg_{\boldsymbol{\zeta}_i, i=1,\ldots,N_1} \max \ \boldsymbol{\zeta}_i^{\mathrm{H}}\boldsymbol{F}_1\boldsymbol{\zeta}_i. \tag{13}$$

With a sufficiently large N_1, we can obtain at least $\frac{\pi}{4}$-approximation of the optimal objective value of the subproblem (12) [6].

Then, we maximize the second term $2\mathrm{Re}\left(\boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{\xi}\right)$, and formulate the following subproblem:

$$\max_{\boldsymbol{\xi}} \ \mathrm{Re}(\boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{\xi}), \ \ \text{s.t.} \ |\xi_m| = 1, \ m = 1,\ldots,M, \tag{14}$$

the optimal solution of which is given by $\boldsymbol{\xi}^{(2)} = \mathrm{e}^{\mathrm{j}\mathrm{angle}(\boldsymbol{f}_2)}$.

Finally, we choose a better one from $\boldsymbol{\xi}^{(i)}, \ i = 1, 2$,

$$\boldsymbol{\xi}^\star = \arg_{\boldsymbol{\xi}^{(i)}, i=1,2} \min \ \mathrm{tr}((\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}^*)^{-1}). \tag{15}$$

By substituting (15) into (7), we obtain the BS precoder $\boldsymbol{W}$.

Remark 2. The phase shift beam is designed either to enhance the cascaded channel or to align the cascaded channel with the direct channel.

Remark 3. The proposed algorithm avoids the complex iterative process and has a low complexity of $\mathcal{O}(M^3)$.

A slightly higher complexity of $\mathcal{O}(M^3 + TM^2)$ is achieved by the majorization-minimization (MM) and complex circle manifold (CCM) algorithms proposed in [4], where the number of iterations, i.e., T, increases with the number of reflecting elements.

The proofs of Theorem 1 and Lemma 1 can be found in Appendixes A and B, respectively. Numerical results and discussions are provided in Appendix C.

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Supporting information Appendixes A–C The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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