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• **Supplementary File** •

Low-Complexity Beamforming Design for IRS-Aided Communication Systems

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Appendix A Proof of Theorem 1

Recall the joint optimization problem in the single-user case

$$\min_{\mathbf{w},\boldsymbol{\xi}} \ \|\mathbf{w}\|^2 \tag{A1a}$$
$$\text{s.t.} \ \left|\left(\mathbf{h}_{\rm r}^T\boldsymbol{\Theta}\mathbf{G}+\mathbf{h}_{\rm d}^T\right)\mathbf{w}\right|^2 \geqslant \left(2^r-1\right)\sigma_0^2, \tag{A1b}$$
$$|\xi_m|=1, m=1,...,M. \tag{A1c}$$

Let P denote the transmit power of the BS. For any given $\boldsymbol{\xi}$, the optimal beamforming vector should satisfy [1]

$$\mathbf{w}=\sqrt{P}\frac{\left(\mathbf{h}_{\rm r}^T\boldsymbol{\Theta}\mathbf{G}+\mathbf{h}_{\rm d}^T\right)^H}{\left\|\mathbf{h}_{\rm r}^T\boldsymbol{\Theta}\mathbf{G}+\mathbf{h}_{\rm d}^T\right\|}, \tag{A2}$$

which is obtained by observing the QoS constraint (A1b).

Substituting the optimal beamforming vector (A2) into the problem (A1), we have

$$\min_{P,\boldsymbol{\xi}} \ P \tag{A3a}$$
$$\text{s.t.} \ P\left\|\mathbf{h}_{\rm r}^T\boldsymbol{\Theta}\mathbf{G}+\mathbf{h}_{\rm d}^T\right\|^2 \geqslant \left(2^r-1\right)\sigma_0^2, \tag{A3b}$$
$$|\xi_m|=1, m=1,...,M. \tag{A3c}$$

Note that the optimal transmit power should satisfy

$$P=\frac{\left(2^r-1\right)\sigma_0^2}{\left\|\mathbf{h}_{\rm r}^T\boldsymbol{\Theta}\mathbf{G}+\mathbf{h}_{\rm d}^T\right\|^2}. \tag{A4}$$

Then, the problem (A3) is equivalently transformed into

$$\max_{\boldsymbol{\xi}} \ \left\|\mathbf{G}^T\boldsymbol{\Theta}\mathbf{h}_{\rm r}+\mathbf{h}_{\rm d}\right\|^2 \tag{A5a}$$
$$\text{s.t.} \ |\xi_m|=1, m=1,...,M. \tag{A5b}$$

Since the IRS-BS channel is dominated by the line-of-sight (LoS) component, we have the following approximation [2]

$$\mathbf{G}^T \approx \beta\mathbf{a}\left(\theta\right)\mathbf{b}^T\left(\gamma,\varphi\right), \tag{A6}$$

where β is the complex channel gain, $\mathbf{b}\left(\gamma,\varphi\right)$ and $\mathbf{a}\left(\theta\right)$ denote the array response vectors of the IRS and the BS respectively. In addition, γ/φ and θ are the elevation/azimuth angle of arrival (AoD) and angle of departure (AoA) for the BS-IRS link.

As such, we rewrite the objective function (A5a) as

$$\begin{aligned}&\left\|\mathbf{G}^T\boldsymbol{\Theta}\mathbf{h}_{\rm r}+\mathbf{h}_{\rm d}\right\|^2\\ &=\left\|\mathbf{h}_{\rm r}\right\|^2+N|\beta|^2\left|f\left(\boldsymbol{\xi}\right)\right|^2+2{\rm Re}\left(\beta\mathbf{h}_{\rm d}^H\mathbf{a}\left(\theta\right)f\left(\boldsymbol{\xi}\right)\right),\end{aligned} \tag{A7}$$

where

$$f\left(\boldsymbol{\xi}\right)=\mathbf{b}^T\left(\gamma,\varphi\right){\rm diag}\left(\mathbf{h}_{\rm r}\right)\boldsymbol{\xi}. \tag{A8}$$

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According to (A7), we convert the optimization problem (A5) into

$$\max_{\boldsymbol{\xi}} \ N|\beta|^2 \left|f\left(\boldsymbol{\xi}\right)\right|^2 + 2\mathrm{Re}\left(\beta \mathbf{h}_{\mathrm{d}}^H \mathbf{a}\left(\theta\right) f\left(\boldsymbol{\xi}\right)\right) \tag{A9a}$$

$$\text{s.t. } |\xi_m| = 1, m = 1, ..., M. \tag{A9b}$$

The above problem is still non-convex. However, noticing the special structure of the objective function, we can obtain the optimal solution. Specifically, the objective function (A9a) satisfies the following inequality

$$\mathrm{Re}\left(\beta \mathbf{h}_{\mathrm{d}}^H \mathbf{a}\left(\theta\right) f\left(\boldsymbol{\xi}\right)\right) \leqslant \left|\beta \mathbf{h}_{\mathrm{d}}^H \mathbf{a}\left(\theta\right)\right| \left|f(\boldsymbol{\xi})\right|, \tag{A10}$$

where the equality can be achieved with angle $\left(\beta \mathbf{h}_{\mathrm{d}}^H \mathbf{a}\left(\theta\right)\right) = -\mathrm{angle}\left(f(\boldsymbol{\xi})\right)$.

Next, we demonstrate that a solution always exists, which satisfies the constraint (A9b) and (A10) with equality. Based on (A10), the problem (A9) can be equivalently transformed into

$$\max_{\boldsymbol{\xi}} \ f\left(\boldsymbol{\xi}\right) = \mathbf{b}^T\left(\gamma, \varphi\right) \mathrm{diag}\left(\mathbf{h}_{\mathrm{r}}\right) \boldsymbol{\xi} \tag{A11a}$$

$$\text{s.t. } |\xi_m| = 1, m = 1, ..., M, \tag{A11b}$$

$$\mathrm{angle}\left(\beta \mathbf{h}_{\mathrm{d}}^H \mathbf{a}\left(\theta\right)\right) = -\mathrm{angle}\left(f(\boldsymbol{\xi})\right), \tag{A11c}$$

where angle($\mathbf{x}$) denotes a vector with each element being the phase of the corresponding element in $\mathbf{x}$.

If we only consider the first constraint $|\xi_m| = 1, m = 1, ..., M$, the optimal solution of the above problem should have the following form

$$\boldsymbol{\xi} = e^{-j\mathrm{angle}(\mathrm{diag}(\mathbf{h}_{\mathrm{r}})\mathbf{b}(\gamma,\varphi)) + j\phi}, \tag{A12}$$

where ϕ can be any value.

Then, by letting $\phi = -\mathrm{angle}\left(\beta \mathbf{h}_{\mathrm{d}}^H \mathbf{a}\left(\theta\right)\right)$, the second constraint angle $\left(\beta \mathbf{h}_{\mathrm{d}}^H \mathbf{a}\left(\theta\right)\right) = -\mathrm{angle}\left(f(\boldsymbol{\xi})\right)$ is satisfied.

To this end, we obtain the optimal solution of the optimization problem (A11) as

$$\boldsymbol{\xi} = e^{-j\mathrm{angle}(\mathrm{diag}(\mathbf{h}_{\mathrm{r}})\mathbf{b}(\gamma,\varphi)) - j\mathrm{angle}\left(\beta \mathbf{h}_{\mathrm{d}}^H \mathbf{a}(\theta)\right)}. \tag{A13}$$

Combining (A2), (A4) and (A13) yields the optimal BS beamforming vector $\mathbf{w}$.

Appendix B Proof of Lemma 1

Performing eigendecomposition of $\mathbf{H}^T\mathbf{H}^*$, we have

$$\mathbf{H}^T\mathbf{H}^* = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H, \tag{B1}$$

where $\boldsymbol{\Lambda} \triangleq \mathrm{diag}\left(\lambda_1, \cdots, \lambda_K\right)$.

Based on the property of trace, the following equality holds

$$\mathrm{tr}\left(\left(\mathbf{H}^T\mathbf{H}^*\right)^{-1}\right) = \sum_{k=1}^{K} \frac{1}{\lambda_k}. \tag{B2}$$

Note that $f(x) = \frac{1}{x}$ is a convex function. As such, by applying the Jensen's inequality, we have

$$\frac{1}{K}\sum_{k=1}^{K} \frac{1}{\lambda_k} \geqslant \frac{1}{\sum\limits_{k=1}^{K} \lambda_k}. \tag{B3}$$

Due to $\mathrm{tr}\left(\mathbf{H}^T\mathbf{H}^*\right) = \sum\limits_{k=1}^{K} \lambda_k$, the above inequality becomes

$$\mathrm{tr}\left(\left(\mathbf{H}^T\mathbf{H}^*\right)^{-1}\right) \geqslant \frac{K}{\mathrm{tr}\left(\mathbf{H}^T\mathbf{H}^*\right)}. \tag{B4}$$

Hence, $\frac{K}{\mathrm{tr}\left(\mathbf{H}^T\mathbf{H}^*\right)}$ is a lower bound of $\mathrm{tr}\left(\left(\mathbf{H}^T\mathbf{H}^*\right)^{-1}\right)$.

Appendix C Numerical Results and Discussions

In this section, we provide numerical results to verify the effectiveness of our proposed beamforming schemes and to investigate the performance of the IRS-assisted communication system. The BS-IRS and IRS-user channels are modelled as Rician fading channels, while the BS-user channels are modelled as Rayleigh fading channels. We adopt the following distance-dependent path loss model [1]

$$\beta(d) = C_0\left(\frac{d}{d_0}\right)^{-\alpha_0}, \tag{C1}$$

where d is the distance, α_0 is the path loss exponent, $C_0 = 30$dB is the path loss at the reference distance of $d_0 = 1$ m. The BS-user path loss exponent is set to be 4, while the BS-IRS and IRS-user path loss exponents are set to be 2.2. Unless otherwise specified, the following system parameters are used: $N = 8$, $M = 256$, Rician K-factor for both BS-IRS and IRS-user channels $\kappa = 2$, noise power $\sigma_0^2 = -80$ dBm, the BS-IRS and IRS-user distances are set to be $d_{\mathrm{B2I}} = 50$ m and $d_{\mathrm{I2U}} = 6$ m, respectively.

Appendix C.1 Single-User Case

In this section, we provide numerical results to verify the good performance of the proposed beamforming scheme in the single-user case. The simulation setup is illustrated in Fig. C1, where the IRS, BS and user are located at $(0, 0, 5\text{ m})$, $(49.43\text{ m}, -85.62\text{ m}, 20\text{ m})$ and $(3.32\text{ m}, 0, 0)$, respectively.

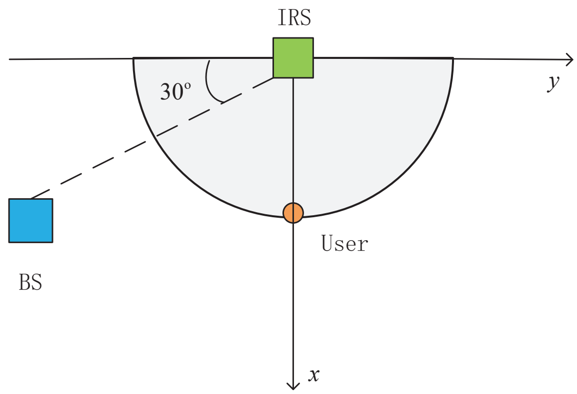

Figure C1 Simulation setup in the single-user case (top view).

Fig.C2 shows the required transmit power under different target rates by adopting the proposed beamforming scheme. For comparison, the AO-based scheme [1] and the random phase shift scheme are presented as two benchmarks. As can be readily seen, the proposed beamforming scheme performs much better than both the AO-based scheme and the random scheme. For example, with a given target rate of 5 bits/s/Hz, our proposed beamforming scheme achieves a transmit power saving of about 10 dB, compared with the AO-based scheme.

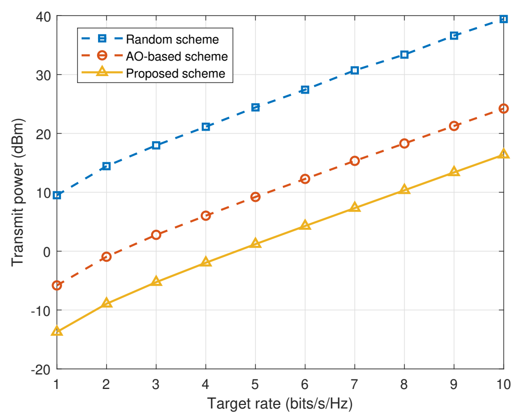

Figure C2 Performance of the proposed beamforming scheme in the single-user case.

Fig.C3 shows the performance of the proposed beamforming scheme under different Rician K-factors. Although the proposed beamforming scheme is designed according to LoS components, it achieves much better performance than both the AO-based scheme and the random scheme. Moreover, our proposed scheme improves as the Rician K-factor becomes larger, while the two benchmarks degrades with the increase of the Rician K-factor. This is because the proposed beamforming algorithm is designed by approximating the BS-IRS channel as a LoS channel. As the Rician K-factor increases, this approximation becomes more accurate and the proposed algorithm gradually achieves the global optimum.

Fig.C4 investigates the impact of the number of reflecting elements. For all the three beamforming schemes, the transmit power decreases with the number of reflecting elements, which indicates the benefit of IRS in power saving. Moreover, as the number of reflecting elements increases, the advantage of our proposed scheme becomes more significant.

Appendix C.2 Multi-User Case

In this section, we present numerical results to verify the effectiveness of the proposed beamforming scheme in the multi-user case, as well as to investigate the performance of the IRS-aided multi-user communication system. The simulation setup is shown in Fig. C5, where the IRS, BS and $K = 3$ users are located at $(0, 0, 5\text{ m})$, $(49.43\text{ m}, -85.62\text{ m}, 20\text{ m})$, $(2.35\text{ m}, -2.35\text{ m}, 0)$, $(3.32\text{ m}, 0, 0)$ and $(2.35\text{ m}, 2.35\text{ m}, 0)$, respectively.

Fig. C6 compares the proposed beamforming scheme with two benchmarks, i.e., the SDR-based scheme [1] and the random phase shift scheme. As can be seen, the proposed beamforming scheme achieves comparable performance to the SDR scheme, but with a much lower complexity. Moreover, as the target rate increases, the performance of the proposed scheme asymptotically approaches that of the SDR scheme.

Fig. C7 investigates the impact of the Rician K-factor on the system performance. For the multi-user case, more power is consumed as the Rician K-factor becomes larger. This is because with a larger Rician K-factor, inter-user interference would

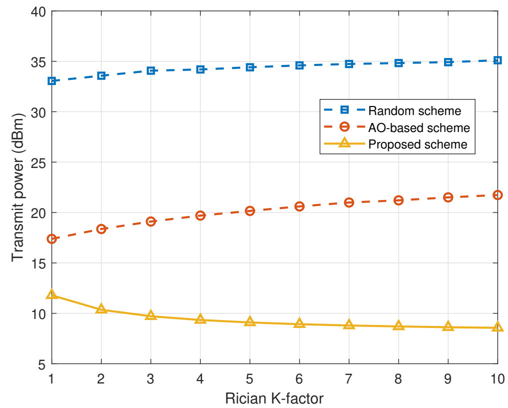

Figure C3 The impact of the Rician K-factor in the single-user case.

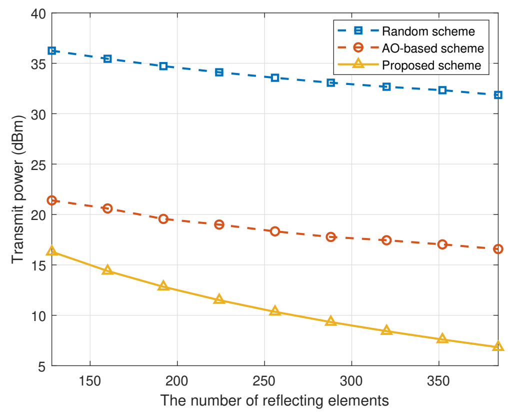

Figure C4 The impact of the number of reflecting elements in the single-user case.

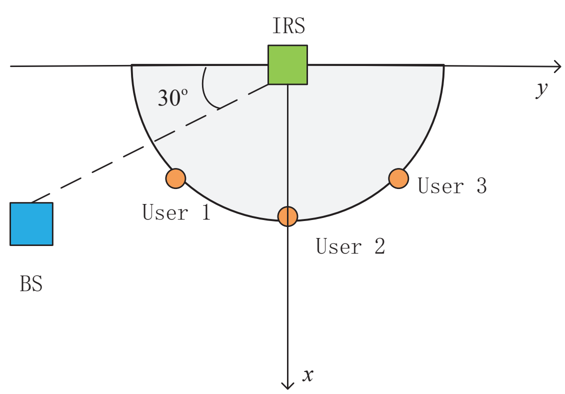

Figure C5 Simulation setup in the multi-user case (top view).

become more severe, and more transmit power is needed to eliminate the interference. For a fixed target rate, as the Rician K-factor increases from 1 to 10, more power than 10 dBm is consumed.

Fig. C8 investigates the impact of the number of reflecting elements on the system performance. We can observe that less power is required by adding more reflecting elements. For a given target rate of 7 bits/s/Hz, a transmit power saving of 10 dB is achieved by increasing the number of reflecting elements from 64 to 256.

References

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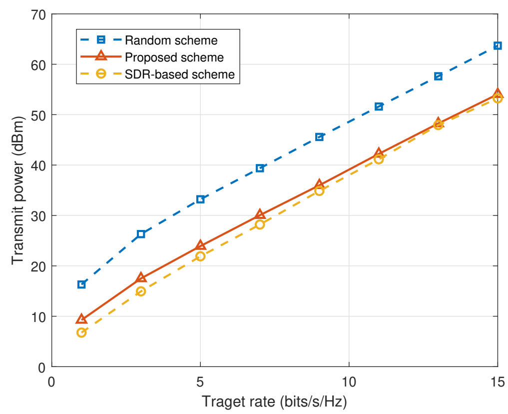

Figure C6 Performance of the proposed beamforming scheme in the multi-user case.

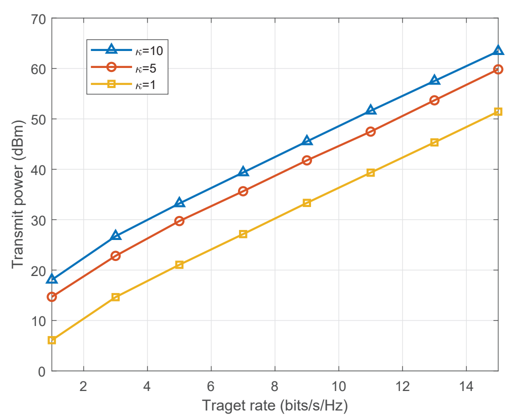

Figure C7 The impact of Rician K-factor in the multi-user case.

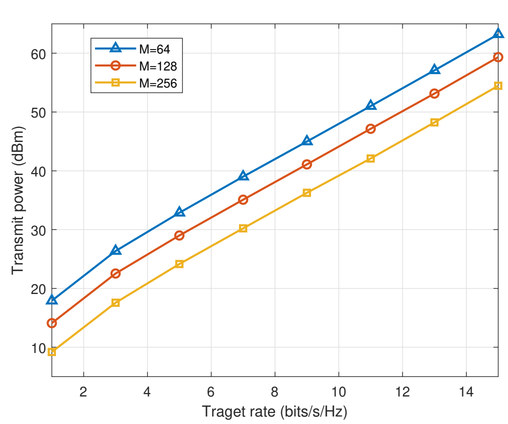

Figure C8 The impact of the number of reflecting elements in the multi-user case.