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# Novel dynamic event-triggered coordination for scaled consensus without continuous communication or controller update

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Dear editor,

As a generalization of "identical consensus", a new notion called "scaled consensus" was first introduced by Sandip [1] that allows each agent to reach a diverse final convergence value based on their individual ratios. To date, scaled consensus problems have attracted great attention for applications in compartmental mass-action systems, closed queuing networks, water distribution systems, task allocation, and webpage-ranking algorithms. Moreover, group/cluster and bipartite consensus [2] are special cases of scaled consensus indicating its generality.

However, most studies have been devoted to continuous sampling and controller update. In practice, the implementation of controller signals relies on an embedded microprocessor with a limited built-in battery. Considering this situation, an event-triggered control mechanism (ETM) was first proposed by Tabuada [3] for conducting real-time control tasks in the background. The core idea of ETM is to predesign a proper event-triggered condition (ETC) related to measurement error (ME) and threshold functions. Using event-triggered schemes (ETSs), fewer sampled states are utilized and considerable energy and resources are saved.

Owing to the advantages of ETSs, a static event-triggered technique was first proposed by Dimarogonas for distributed multiagent systems (MASs). In contrast to such static ETSs [4], a dynamic ETS containing a filtered version of an internal dynamic variable was recently proposed [5] and it can be regarded as a generalization of static ETSs. Because a dynamic ETS can prolong the interevent time compared with its static counterpart, the idea of the dynamic ETS was introduced to MASs [6–8]. In [7, 8], event detection required continuous communication among agents. Following this, an improved dynamic ETM using a model estimation approach was employed in [6]. However, this approach required continuous controller update, which causes potential energy efficiency concerns.

Notably, the requirement of either continuous communication among agents or controller update in [6–8] meant that the process was still energy-intensive. Considering the aforementioned issues, we aimed to design a novel dynamic ETS, which can perform communication and controller update, and studied its scaled consensus coordination via novel double ETSs without the need for either continuous communication or controller update. Using an improved model-based dynamic ETS, continuous communication among individuals is avoided. Moreover, an additional decentralized ETC is designed to execute event-triggered actions for controller update. With the aid of double correlative ETSs, continuous communication among agents and controller update are not required. Furthermore, scaled consensus for MASs is guaranteed without Zeno phenomenon. Finally, a comparison simulation with the proposed dynamic ETS is presented to highlight our proposed control strategy. The contributions can be summarized as follows: (1) In contrast to [6–8], an improved dynamic ETS is developed such that the restriction of continuous communication among agents in the event detector can be removed. To eliminate continuous controller update caused by model-based ETSs, an additional ETS is introduced to reduce the frequency of controller update. Hence, the elimination of both continuous communication and controller update can be guaranteed using the novel double ETSs. (2) As a more general form of collaborative behavior, scaled consensus is proven to be exponentially reached using double ETSs, which is complicated in the case of model transformation. (3) Under the improved dynamic ETS, both the interevent time of communication and controller update sequences for some agents can be prolonged, as verified by comparison simulation. (4) The design of double ETSs incurs difficulties in proving the exclusion of Zeno phenomena, for which existing methods are not effective. A model transformation is given in Appendix A. Rigorous proofs can be found in Appendixes B and C. Appendix D exhibits the simulations.

*Problem formulation.* In this study, we consider a group of MASs in which the dynamics of agent $i$ is formulated as

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \ldots, N, \tag{1}$$

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where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices, $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ denote the agent $i$'s state and control input, respectively. The basic theory of graph topology can be found in related studies on cooperative control and is omitted here.

**Definition 1.** The MASs are said to achieve scaled consensus to $(\alpha_1^{-1}, \ldots, \alpha_N^{-1})$, if $\lim_{t\to\infty}(\frac{x_i}{\alpha_i} - \frac{x_j}{\alpha_j}) = 0, \forall i, j \in \mathcal{E}$, where the scalars $\alpha_1, \ldots, \alpha_N$ are assumed to be nonzero and predefined.

**Definition 2.** The closed-loop system does not exhibit Zeno behavior if $\inf\{t_{k+1}^i - t_k^i > 0 : \forall i \in \mathcal{V}, k = 0, 1, \ldots\}$.

**Assumption 1.** The matrix pair $(A, B)$ is stabilizable.

**Assumption 2.** The graph $\mathcal{G}$ is connected.

**Lemma 1.** Based on Assumption 1, we deduce that for any given matrices $Q = Q^{\mathrm{T}} > 0$ and $R = R^{\mathrm{T}} > 0$, there is a unique solution such that the algebraic Riccati equation (ARE) $A^{\mathrm{T}}P + PA - PBR^{-1}B^{\mathrm{T}}P + Q = 0$ holds.

*Dynamic ETM for communication.* Based on the notions of scaled consensus and $\alpha_{ij} = \frac{\alpha_i}{\alpha_j}$, the distributed event-triggered control $u_i(t)$ for agent $i$ is designed for $t \in [t_k^i, t_{k+1}^i)$:

$$u_i(t) = K \sum_{j\in N_i} a_{ij}(\alpha_{ij}\hat{x}_j^i(t) - \hat{x}_i(t)) \triangleq Kq_i(t). \tag{2}$$

Let $\hat{x}_j^i(t)$ denote the estimate of agent $j$'s state that is produced by agent $i$ and $\hat{x}_i(t)$ denote the estimate of agent $i$'s state produced by itself. The estimators of agent $j$ from the view of agent $i$ are constructed as

$$\begin{cases} \dot{\hat{x}}_j^i(t) = A\hat{x}_j^i(t),\ j \in N_i,\ t \in [t_k^j, t_{k+1}^j), \\ \hat{x}_j^i(t^+) = x_j(t),\ t = t_k^j, \end{cases} \tag{3}$$

and the estimator of agent $i$ produced by itself is expressed as $\dot{\hat{x}}_i(t) = A\hat{x}_i(t)$, $t \in [t_k^i, t_{k+1}^i)$, where $\hat{x}_j^i(t)$ is a right-hand continuous function, $\hat{x}_j^i(t)$ can be reset at event-triggering instant $t_k^j$, and the increasing sequence $\{t_k^i\}$, $i \in \mathcal{V}$, $k = 0, 1, \ldots$ represents the triggering instants for agent $i$ at which the real state $x(t_k^i)$ is permitted to be transmitted. Assume $t_0^i = 0$.

We define the ME as $e_i(t) = \hat{x}_i(t) - x_i(t)$, $t \in [t_k^i, t_{k+1}^i)$. The triggering instants for $i$ are determined using a distributed dynamic ETM with $\eta_i(0) > 0$:

$$\begin{cases} t_{k+1}^i = \inf\{t > t_k^i : \eta_i(t) + \beta_i(\Gamma_1\|q_i(t)\|^2 - \Gamma_2\|e_i(t)\|^2 \\ \qquad + \gamma_1 \mathrm{e}^{-\hat{\alpha}_1 t}) \leqslant 0\}, \\ \dot{\eta}_i(t) = -\varphi_i\eta_i(t) + \Gamma_1\|q_i(t)\|^2 - \Gamma_2\|e_i(t)\|^2 + \gamma_1\mathrm{e}^{-\hat{\alpha}_1 t}, \end{cases} \tag{4}$$

where $\Gamma_1 = \varepsilon - \frac{\mu\lambda_{\max}^4(\alpha^{-1})\lambda_{\max}^2(PBR^{-1}B^{\mathrm{T}}P)}{\rho'} > 0$, $\mu \geqslant \frac{1}{2\lambda_2(L)}$, $\varphi_i, \beta_i, \rho, \rho', \rho'', \gamma_1 > 0$, $\Gamma_2 = \varepsilon\rho''' + \mu\rho' + 2\mu\lambda_{\max}(\alpha^{-1}L\alpha^{-1})\lambda_{\max}(PBR^{-1}B^{\mathrm{T}}P)$, $\rho''' = \lambda_{\max}(\alpha^{-1}L\alpha^2L\alpha^{-1}) + \rho''$, $\delta_1 - \frac{\mu}{\rho}\lambda_{\max}(\mathscr{L}\alpha^{-2}\mathscr{L})$ $\lambda_{\max}^2(PBR^{-1}B^{\mathrm{T}}P) \triangleq \Upsilon > 0$, $r = \min\{\frac{\Upsilon}{\lambda_{\max}(P)}, \varphi_1, \varphi_2, \ldots, \varphi_N\}$, $\delta = \lambda_{\min}(Q)$, $\hat{\alpha}_1 = (0, \min\{r, 2\hat{\alpha}_2, 2\hat{\alpha}_0\})$, $\alpha = \mathrm{diag}\{\alpha_1, \ldots, \alpha_N\}$, $\delta_2 = \varepsilon[\lambda_{\max}(L\alpha^2L) + \frac{\lambda_{\max}(L\alpha^2L\alpha^{-2}L\alpha^2L)}{\rho''}]$, $\delta_1 = \delta - \delta_2$, $\varepsilon > 0$, and $\hat{\alpha}_0 > \hat{\alpha}_2 > 0$.

Let $z_i(t) = x_i(t) - \frac{1}{N}\sum_{j=1}^N \alpha_{ij}x_j(t)$ be the scaled consensus error. With $\tilde{z}_i(t) = \frac{1}{\alpha_i}z_i(t)$, then $\tilde{z}(t) = (\mathscr{L}\alpha^{-1} \otimes I_n)x(t)$, where $\mathscr{L} = I_N - \frac{1}{N}1_N1_N^{\mathrm{T}}$, $\tilde{z}(t) = [\tilde{z}_1^{\mathrm{T}}(t), \ldots, \tilde{z}_N^{\mathrm{T}}(t)]$.

For controller (2), continuous communication between agent $i$ and $j$ is avoided during the new update interval $[t_k^j, t_{k+1}^j)$. However, continuous controller update will still be performed. To tackle this issue, event-based controller update will be dealt with hereon.

*ETM for controller update.* Considering the controller update triggering sequence $\{T_{k_i(t)}^i\}$, $i \in \mathcal{V}$, $k_i(t) = 0, 1, \ldots$, define the ME for controller update as $\tilde{e}_i(t) = q_i(T_{k_i(t)}^i) - q_i(t)$, $t \in [T_{k_i(t)}^i, T_{k_i(t)+1}^i)$. Assume $T_0^i = 0$. For the latest update instant $T_{k_i(t)}^i$, the next updating action at $T_{k_i(t)+1}^i$ is activated when the following condition is satisfied:

$$T_{k_i(t)+1}^i = \inf\{t > T_{k_i(t)}^i : \|\tilde{e}_i(t)\| - \gamma_2\mathrm{e}^{-\hat{\alpha}_2 t} \geqslant 0\}, \tag{5}$$

where $\gamma_2 > 0$, and $\hat{\alpha}_2$ is a positive constant to be determined.

By (5), for $t \in [T_{k_i(t)}^i, T_{k_i(t)+1}^i)$, the piecewise constant event-triggered controller for agent $i$ can be designed as

$$u_i(t) = K \sum_{j\in N_i} a_{ij}\big(\alpha_{ij}\hat{x}_j^i(T_{k_i(t)}^i) - \hat{x}_i(T_{k_i(t)}^i)\big). \tag{6}$$

*Main results for scaled consensus without continuous communication and controller update.*

**Theorem 1.** Consider MAS (1) with the event-triggered controller (6) under Assumptions 1 and 2. Let controller gain $K = \mu R^{-1}B^{\mathrm{T}}P$ with $\mu\lambda_2 \geqslant \frac{1}{2}$. Then, under the dynamic ETM (4) and ETM (5), the scaled consensus for the MAS can be reached asymptotically.

**Theorem 2.** Consider MAS (1) and let controller gain $K$ be designed as in Theorem 1. Then, under the ETMs (4) and (5), no agent will exhibit Zeno behavior.

*Conclusion.* The scaled consensus problem for general linear MASs using dynamic ETS without continuous communication and controller update is presented. Considering power efficiency issues faced by model-based estimation methods, a novel dynamic ETS is proposed that does not require continuous communication in either controller update or ETC monitoring. Moreover, an additional ETM plays the role of reducing the frequency of controller update. In future studies, we will focus on the intriguing problem of how topology structures can impact event-triggered actions.

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**Supporting information** Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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