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• **Supplementary File** •

Novel dynamic event-triggered coordination for scaled consensus without continuous communication or controller update

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Appendix A Model transformation

By the event-triggered controller (6) in our LETTER, further, $u_i(t) = K\sum_{j\in N_i} a_{ij}[\alpha_{ij}(x_j(t)+e_j(t)) - (x_i(t)+e_i(t))] + K\tilde{e}_i(t)$. Because $\mathscr{L}L = L\mathscr{L} = L$, we obtain $\dot{\tilde{z}}(t) = (I_N\otimes A - L\otimes BK)\tilde{z}(t) - (L\alpha^{-1}\otimes BK)e(t) + (\mathscr{L}\alpha^{-1}\otimes BK)\tilde{e}(t)$. There exists an orthogonal matrix $T = (\frac{1}{\sqrt{N}}1_N,\ T_2,\ldots,T_N)$ such that $T^TLT = diag\{0,\lambda_2,\ldots,\lambda_N\} \triangleq \Delta$. Denote $\bar{z}(t) = (T^T\otimes I_n)\tilde{z}(t)$, $\bar{z}(t) = [\bar{z}_1^T(t), \bar{z}_{2-N}^T(t)]^T$, then $\bar{z}_1(t) = (\frac{1}{\sqrt{N}}1_N^T\otimes I_n)\tilde{z}(t) \equiv 0$, $\dot{\bar{z}}_{2-N}(t) = (I_N\otimes A - \wedge\otimes BK)\bar{z}_{2-N}(t) - (T_{2-N}^T L\alpha^{-1}\otimes BK)e(t) + (T_{2-N}^T\mathscr{L}\alpha^{-1}\otimes BK)\tilde{e}(t)$, where $T_{2-N} = (T_2,\ldots,T_N)$, $\wedge = diag\{\lambda_2,\ldots,\lambda_N\}$. Note that $\|\bar{z}(t)\|^2 = \|\tilde{z}(t)\|^2 = \|\bar{z}_{2-N}(t)\|^2$.

Appendix B The proof of Theorem 1

Proof. Choose the Lyapunov candidate function as

$$W(t) = V(t) + \sum_{i=1}^{N}\eta_i(t), \tag{B1}$$

where $V(t) = \bar{z}^T(t)(I_N\otimes P)\bar{z}(t) \geqslant 0$, then the time derivative of $V(t)$ along the trajectory of dynamics $\dot{\bar{z}}^T(t)$ can be given by

$$\begin{aligned}\dot{V}(t) &= \dot{\bar{z}}^T(t)(I_N\otimes P)\bar{z}(t) + \bar{z}^T(t)(I_N\otimes P)\dot{\bar{z}}(t)\\ &= \bar{z}^T(t)[I_N\otimes(A^TP+PA) - 2\Delta\otimes PBK]\bar{z}(t)\\ &\quad - 2\bar{z}^T(t)(T^TL\alpha^{-1}\otimes PBK)e(t) + 2\bar{z}^T(t)(T^T\mathscr{L}\alpha^{-1}\otimes PBK)\tilde{e}(t),\end{aligned} \tag{B2}$$

by the relationship between $\bar{z}(t)$ and $\bar{z}_{2-N}(t)$, choose control gain matrix $K = \mu R^{-1}B^TP$, the above equation can be rewritten as

$$\begin{aligned}\dot{V}(t) &= \bar{z}_{2-N}^T(t)[I_{N-1}\otimes(A^TP+PA) - 2\mu\wedge\otimes PBR^{-1}B^TP]\bar{z}_{2-N}(t)\\ &\quad - 2\mu\bar{z}^T(t)(T^TL\alpha^{-1}\otimes PBR^{-1}B^TP)e(t)\\ &\quad + 2\mu\bar{z}^T(t)(T^T\mathscr{L}\alpha^{-1}\otimes PBR^{-1}B^TP)\tilde{e}(t)\\ &\leqslant \bar{z}_{2-N}^T(t)[I_{N-1}\otimes(A^TP+PA-2\mu\lambda_2\otimes PBR^{-1}B^TP)]\bar{z}_{2-N}(t)\\ &\quad - 2\mu\tilde{z}^T(t)(L\alpha^{-1}\otimes PBR^{-1}B^TP)e(t) + 2\mu\tilde{z}^T(t)(\mathscr{L}\alpha^{-1}\otimes PBR^{-1}B^TP)\tilde{e}(t).\end{aligned} \tag{B3}$$

From Lemma 1, since $\mu\lambda_2 \geqslant \frac{1}{2}$,

$$\begin{aligned}\dot{V}(t) &\leqslant -2\mu\tilde{z}^T(t)(L\alpha^{-1}\otimes PBR^{-1}B^TP)e(t) + 2\mu\tilde{z}^T(t)(\mathscr{L}\alpha^{-1}\otimes PBR^{-1}B^TP)\tilde{e}(t)\\ &\quad - \lambda_{min}(Q)\|\bar{z}_{2-N}(t)\|^2\\ &\triangleq -2\mu\tilde{z}^T(t)(L\alpha^{-1}\otimes PBR^{-1}B^TP)e(t) + 2\mu\tilde{z}^T(t)(\mathscr{L}\alpha^{-1}\otimes PBR^{-1}B^TP)\tilde{e}(t) - \delta\|\bar{z}(t)\|^2,\end{aligned} \tag{B4}$$

where $\delta = \lambda_{min}(Q)$.

The combined measurement variable is defined as $q_i(t) = \sum_{j\in N_i} a_{ij}(\alpha_{ij}\hat{x}_j^i(t) - \hat{x}_i(t))$, then one can get its compact form like $q(t) = -(\alpha L\alpha^{-1}\otimes I_n)\hat{x}(t)$, using $\hat{x}(t) = x(t) + e(t)$ yields that

$$\begin{aligned}q(t) &= -(\alpha L\alpha^{-1}\otimes I_n)(x(t)+e(t))\\ &= -(\alpha L\mathscr{L}\alpha^{-1}\otimes I_n)x(t) - (\alpha L\alpha^{-1}\otimes I_n)e(t)\\ &= -(\alpha L\otimes I_n)\tilde{z}(t) - (\alpha L\alpha^{-1}\otimes I_n)e(t).\end{aligned} \tag{B5}$$

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Based on (B5), the following inequalities hold:

$$\begin{aligned}&-2\mu\tilde{z}^T(t)(L\alpha^{-1}\otimes PBR^{-1}B^TP)e(t)\\&=-2\mu\tilde{z}^T(t)(L\alpha\otimes I_n)(\alpha^{-2}\otimes PBR^{-1}B^TP)e(t)\\&\leqslant\frac{\mu}{\rho'}q^T(t)(\alpha^{-4}\otimes(PBR^{-1}B^TP)^2)q(t)+\mu\rho' e^T(t)e(t)\\&\quad+2\mu\lambda_{max}(\alpha^{-1}L\alpha^{-1})\lambda_{max}(PBR^{-1}B^TP)e^T(t)e(t),\end{aligned}\tag{B6}$$

$$\begin{aligned}&2\mu\tilde{z}^T(t)(\mathscr{L}\alpha^{-1}\otimes PBR^{-1}B^TP)\tilde{e}(t)\\&\leqslant\frac{\mu}{\rho}\tilde{z}^T(t)(\mathscr{L}\alpha^{-2}\mathscr{L}\otimes(PBR^{-1}B^TP)^2)\tilde{z}(t)+\mu\rho\tilde{e}^T(t)\tilde{e}(t).\end{aligned}\tag{B7}$$

Consequently, with event-triggered condition (5), substituting (B6) and (B7) into (B4) gives that

$$\begin{aligned}\dot{V}(t)&\leqslant-\delta\|\tilde{z}(t)\|^2+\frac{\mu}{\rho'}\lambda^4_{max}(\alpha^{-1})\lambda^2_{max}(PBR^{-1}B^TP)\|q(t)\|^2\\&\quad+(\mu\rho'+2\mu\lambda_{max}(\alpha^{-1}L\alpha^{-1})\lambda_{max}(PBR^{-1}B^TP))\|e(t)\|^2\\&\quad+\frac{\mu}{\rho}\lambda_{max}(\mathscr{L}\alpha^{-2}\mathscr{L})\lambda^2_{max}(PBR^{-1}B^TP)\|\tilde{z}(t)\|^2+N\gamma_2^2e^{-2\hat{\alpha}_2t}\\&\leqslant-\delta_1\|\tilde{z}(t)\|^2-\varepsilon\|q(t)\|^2+\varepsilon\rho'''\|e(t)\|^2\\&\quad+\frac{\mu}{\rho'}\lambda^4_{max}(\alpha^{-1})\lambda^2_{max}(PBR^{-1}B^TP)\|q(t)\|^2\\&\quad+(\mu\rho'+2\mu\lambda_{max}(\alpha^{-1}L\alpha^{-1})\lambda_{max}(PBR^{-1}B^TP))\|e(t)\|^2\\&\quad+\frac{\mu}{\rho}\lambda_{max}(\mathscr{L}\alpha^{-2}\mathscr{L})\lambda^2_{max}(PBR^{-1}B^TP)\|\tilde{z}(t)\|^2+N\gamma_2^2e^{-2\hat{\alpha}_2t},\end{aligned}\tag{B8}$$

where $\delta_1,\delta_2\in\mathbb{R}^+$ satisfying $\delta=\delta_1+\delta_2$, $\delta_2=\varepsilon[\lambda_{max}(L\alpha^2L)+\frac{\lambda_{max}(L\alpha^2L\alpha^{-2}L\alpha^2L)}{\rho''}]$, $\rho'''=\rho''+\lambda_{max}(\alpha^{-1}L\alpha^2L\alpha^{-1})$, and ρ,ρ',ρ'' are arbitrary positive parameters.

Notice that the following inequality is used in (B8):

$$\begin{aligned}\|q(t)\|^2&=\tilde{z}^T(t)(L\alpha^2L\otimes I_n)\tilde{z}(t)+e^T(t)(\alpha^{-1}L\alpha^2L\alpha^{-1}\otimes I_n)e(t)+2\tilde{z}(t)(L\alpha^2L\alpha^{-1}\otimes I_n)e(t)\\&\leqslant\lambda_{max}(L\alpha^2L)\|\tilde{z}(t)\|^2+\lambda_{max}(\alpha^{-1}L\alpha^2L\alpha^{-1})\|e(t)\|^2\\&\quad+\frac{\lambda_{max}(L\alpha^2L\alpha^{-2}L\alpha^2L)}{\rho''}\|\tilde{z}(t)\|^2+\rho''\|e(t)\|^2\\&\triangleq\frac{\delta_2}{\varepsilon}\|\tilde{z}(t)\|^2+\rho'''\|e(t)\|^2,\end{aligned}\tag{B9}$$

where the regulation parameter $\varepsilon>0$.

Then considering (B1) and the dynamic event-triggered condition (4), one has

$$\begin{aligned}\dot{W}(t)&=\dot{V}(t)+\sum_{i=1}^N\dot{\eta}_i(t)\\&\leqslant-\frac{\delta_1-\frac{\mu}{\rho}\lambda_{max}(\mathscr{L}\alpha^{-2}\mathscr{L})\lambda^2_{max}(PBR^{-1}B^TP)}{\lambda_{max}(P)}V(t)-\sum_{i=1}^N\varphi_i\eta_i(t)\\&\leqslant-rW(t)+N\gamma_2^2e^{-2\hat{\alpha}_2t}+N\gamma_1e^{-\hat{\alpha}_1t}\\&\leqslant-rW(t)+(N\gamma_2^2+N\gamma_1)e^{-\hat{\alpha}_1t}\\&\triangleq-rW(t)+\varrho e^{-\hat{\alpha}_1t},\end{aligned}\tag{B10}$$

where $r=\min\{\frac{\delta_1-\frac{\mu}{\rho}\lambda_{max}(\mathscr{L}\alpha^{-2}\mathscr{L})\lambda^2_{max}(PBR^{-1}B^TP)}{\lambda_{max}(P)},\varphi_1,\varphi_2,\ldots,\varphi_N\}$, $0<\hat{\alpha}_1\leqslant2\hat{\alpha}_2$.

If $\hat{\alpha}_1<r$, it follows that

$$\begin{aligned}W(t)&\leqslant e^{-rt}W(0)+\varrho\int_0^te^{-r(t-s)}e^{-\hat{\alpha}_1s}ds\\&=e^{-rt}W(0)+\frac{\varrho}{r-\alpha_1}(e^{-\hat{\alpha}_1t}-e^{-rt})\\&\leqslant e^{-\hat{\alpha}_1t}(W(0)+\frac{\varrho}{r-\hat{\alpha}_1})\triangleq\tilde{W}e^{-\hat{\alpha}_1t}.\end{aligned}\tag{B11}$$

It can be verified that $W(t)$ will decrease to 0 exponentially. In addition, according to ETM (4), one has $\Gamma_1\|q_i(t)\|^2-\Gamma_2\|e_i(t)\|^2+\gamma_1e^{-\hat{\alpha}_1t}\geqslant-\frac{\eta_i(t)}{\beta_i}$, which shows that $\dot{\eta}_i(t)\geqslant-(\varphi_i+\frac{1}{\beta_i})\eta_i(t)$, then following the Comparison Lemma directly gives that $\eta_i(t)\geqslant\eta_i(0)e^{-(\varphi_i+\frac{1}{\beta_i})t}>0$.

With (B11), it indicates that $\tilde{z}(t)$ and $\eta_i(t)$ can converge to the origin asymptotically which concludes that all agents will reach scaled consensus exponentially with the event-triggered controller (6) under the ETMs (4) and (5). Then the implementation algorithm can be concluded as Algorithm B1.

Remark 1. (4) is a hybrid ETC comprising state- and time-dependent terms. The existence of the exponential term is significant for excluding Zeno behavior and can simultaneously decrease the event triggering number.

Algorithm B1 Implementation Algorithm

Initialization:

(1).Set $t=0, k=0, k_i(0)=0, t_0^i=0, T_0^i=0, e_i(t)=0, \tilde{e}_i(t)=0, \hat{x}_i(0)=x_i(0), q_i(0)=0$ and $\eta_i(0)=\bar{\eta}_i>0$ for each $i\in\mathcal{V}$;

(2).Agent i receives $x_j(0)$ from agent $j, j\in N_i$, agent i sends $x_i(0)$ to agent $j, i\in N_j$.

while $t<t_s$, t_s is the desired lifespan of the controlled system.

for each $i\in\mathcal{V}$

do compute $\hat{x}_i(t)$ and $\hat{x}_j(t), j\in N_i$ at t_k^i, then compute $\eta_i(t)$ with (2) and $e_i(t)$.

if (4) is violated

(1).Update $k=k+1, t_k^i=t, \hat{x}_i(t_k^i)=x_i(t), q_i(t_k^i)=q_i(t)$ and $e_i(t)=0$;

(2).Based on $T_{k_i(t)}^i<t$, compute $q_i(T_{k_i(t)}^i)$, then compute $\tilde{e}_i(t)$.

end if

if (5) is violated

(1).Update $k_i(t)=k_i(t)+1, T_{k_i(t)}^i=t$;

(2).Send $T_{k_i(t)}^i$ to $j, j\in N_i$, get $\hat{x}_j^i(T_{k_i(t)}^i)$, then compute $q_i(T_{k_i(t)}^i)$ from (2);

(3).Update $u_i(t)=\mu R^{-1}B^TPq_i(T_{k_i(t)}^i)$.

end if

end for

end while

Appendix C The proof of Theorem 2

Proof. **(i)** At first, we will prove the Zeno behaviour of event-triggered communication sequence $\{t_k^i\}, k=0,1,\dots,\forall i\in\mathcal{V}$ under the dynamic ETS (4) can not occur.

Recalling $\dot{\bar{z}}_{2-N}(t)$ and $\wedge$, the matrices $A-\mu\lambda_iBR^{-1}B^TP, i\in\{2,3,\dots,N\}$ are Hurwitz if there exists a positive constant μ such that $\mu\lambda_i\geqslant\frac{1}{2}$. Then, for $t\geqslant t_0$, there exists some positive scalars M and $\hat{\alpha}_0$ such that $\|e^{(I_{N-1}\otimes A-\mu\wedge\otimes BR^{-1}B^TP)(t-t_0)}\|\leqslant Me^{-\hat{\alpha}_0(t-t_0)}$.

Given the following event-triggered condition

$$t_{k+1}^i=inf\{t>t_k^i:\gamma_1e^{-\hat{\alpha}_1t}-\Gamma_2\|e_i(t)\|^2\leqslant 0\}, \tag{C1}$$

It is worth to be mentioned that, if ETS (C1) can avoid the Zeno phenomena, then the exclusion of Zeno behaviour for the dynamic ETS (4) can be guaranteed due to $\eta_i(t)>0$ and $\Gamma_1\|q_i(t)\|^2\geqslant 0$ in (4).

Since the new event is triggered at $t_k^i, k=0,1,\dots$ when the event-triggered condition (4) is satisfied, then one obtains

$$\|e_i(t_{k+1}^{i-})\|\geqslant\sqrt{\frac{\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1t_{k+1}^i}. \tag{C2}$$

Moreover, during the inter-event interval $[t_k^i,t_{k+1}^i)$, there has

$$\|e_i(t)\|\leqslant\sqrt{\frac{\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1t}\Rightarrow\|e(t)\|\leqslant\sqrt{\frac{N\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1t}. \tag{C3}$$

Next, by $e_i(t)$, $\tilde{e}_i(t)$ and (6), we will estimate $\|\dot{e_i}(t)\|$, then

$$\dot{e_i}(t)=A\hat{x}_i(t)-Ax_i(t)-Bu_i(t)=Ae_i(t)-BKq_i(T_{k(t)}^i)=Ae_i(t)-BK(q_i(t)+\tilde{e}_i(t)). \tag{C4}$$

From (B5) and (C3), to estimate $\|q(t)\|$, we have

$$\begin{aligned}\|q(t)\|&\leqslant\|\alpha L\otimes I_n\|\|\tilde{z}(t)\|+\|\alpha L\alpha^{-1}\otimes I_n\|\|e(t)\|\\&\leqslant\|\alpha L\otimes I_n\|\|\tilde{z}(t)\|+\sqrt{\frac{N\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1t}\|\alpha L\alpha^{-1}\otimes I_n\|.\end{aligned} \tag{C5}$$

Then based on (C3), (C4), (C5) and event-triggered condition (5), one has

$$\begin{aligned}\|\dot{e_i}(t)\|&\leqslant\|A\|\|e_i(t)\|+\|BK\|(\|q_i(t)\|+\|\tilde{e}_i(t)\|)\\&\leqslant\|A\|\|e_i(t)\|+\|BK\|(\|q(t)\|+\|\tilde{e}(t)\|)\\&\leqslant\sqrt{\frac{\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1t}\|A\|+\|BK\|\|q(t)\|+\sqrt{N}\gamma_2e^{-\hat{\alpha}_2t}\|BK\|.\end{aligned} \tag{C6}$$

In the following, we will estimate $\|\tilde{z}(t)\|$, by the comparison theorem, the solution of $\bar{z}_{2-N}(t)$ is satisfied as

$$\bar{z}_{2-N}(t)=e^{\Xi t}\bar{z}_{2-N}(0)+\int_0^te^{\Xi(t-s)}[-(T_{2-N}^TL\alpha^{-1}\otimes BK)e(s)+(T_{2-N}^T\mathscr{L}\alpha^{-1}\otimes BK)\tilde{e}(s)]ds, \tag{C7}$$

where $\Xi = I_N \otimes A - \wedge \otimes BK$, then in light of $\|e^{(I_{N-1}\otimes A-\mu\wedge\otimes BR^{-1}B^TP)(t-t_0)}\| \leqslant Me^{-\hat{\alpha}_0(t-t_0)}$, the upper bound of $\bar{z}_{2-N}(t)$ is obtained

$$\|\bar{z}_{2-N}(t)\| \leqslant Me^{-\hat{\alpha}_0 t}\|\bar{z}_{2-N}(0)\| + Md_1\int_0^t e^{-\hat{\alpha}_0(t-s)}\|e(s)\|ds + Md_2\int_0^t e^{-\hat{\alpha}_0(t-s)}\|\tilde{e}(s)\|ds, \tag{C8}$$

where $d_1 = \|T_{2-N}^T L\alpha^{-1}\otimes BK\|$, $d_2 = \|T_{2-N}^T \mathscr{L}\alpha^{-1}\otimes BK\|$.

Substituting $\|e(s)\| \leqslant \sqrt{\frac{N\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1 s}$, $\|\tilde{e}(s)\| \leqslant \sqrt{N}\gamma_2 e^{-\hat{\alpha}_2 s}$ into (C8), the further result follows

$$\begin{aligned}
\|\bar{z}_{2-N}(t)\| &\leqslant Me^{-\hat{\alpha}_0 t}\|\bar{z}_{2-N}(0)\| + \sqrt{\frac{N\gamma_1}{\Gamma_2}}\frac{2Md_1}{2\hat{\alpha}_0-\hat{\alpha}_1}(e^{-\frac{1}{2}\hat{\alpha}_1 t}-e^{-\hat{\alpha}_0 t}) + \frac{\sqrt{N}\gamma_2 Md_2}{\hat{\alpha}_0-\hat{\alpha}_2}(e^{-\hat{\alpha}_2 t}-e^{-\hat{\alpha}_0 t})\\
&\leqslant Me^{-\hat{\alpha}_0 t}\|\bar{z}_{2-N}(0)\| + \sqrt{\frac{N\gamma_1}{\Gamma_2}}\frac{2Md_1}{2\hat{\alpha}_0-\hat{\alpha}_1}e^{-\frac{1}{2}\hat{\alpha}_1 t} + \frac{\sqrt{N}\gamma_2 Md_2}{\hat{\alpha}_0-\hat{\alpha}_2}e^{-\hat{\alpha}_2 t}\\
&\leqslant (M\|\bar{z}_{2-N}(0)\| + \sqrt{\frac{N\gamma_1}{\Gamma_2}}\frac{2Md_1}{2\hat{\alpha}_0-\hat{\alpha}_1} + \frac{\sqrt{N}\gamma_2 Md_2}{\hat{\alpha}_0-\hat{\alpha}_2})e^{-\frac{1}{2}\hat{\alpha}_1 t} \triangleq M_1 e^{-\frac{1}{2}\hat{\alpha}_1 t},
\end{aligned}\tag{C9}$$

where $\hat{\alpha}_0 > \hat{\alpha}_2 > \frac{1}{2}\hat{\alpha}_1$.

Combining inequalities (C5), (C6) and (C9) together, noticing the fact that $\|\bar{z}(t)\| = \|\tilde{z}(t)\| = \|\bar{z}_{2-N}(t)\|$, one obtains

$$\begin{aligned}
\|\dot{e}_i(t)\| &\leqslant \|A\|\sqrt{\frac{\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1 t} + M_1\|BK\|\|\alpha L\otimes I_n\|e^{-\frac{1}{2}\hat{\alpha}_1 t} + \|BK\|\|\alpha L\alpha^{-1}\otimes I_n\|\sqrt{\frac{N\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1 t}\\
&\quad + \|BK\|\sqrt{N}\gamma_2 e^{-\frac{1}{2}\hat{\alpha}_1 t} \triangleq M_2 e^{-\frac{1}{2}\hat{\alpha}_1 t}.
\end{aligned}\tag{C10}$$

Notice that $\|e_i(t_k^{i+})\| = 0$, then when $t\in[t_k^i, t_{k+1}^i)$

$$\|e_i(t_{k+1}^{i-})\| \leqslant M_2\int_{t_k^i}^{t_{k+1}^i} e^{-\frac{1}{2}\hat{\alpha}_1 t}dt = -\frac{2M_2}{\hat{\alpha}_1}(e^{-\frac{1}{2}\hat{\alpha}_1 t_{k+1}^i} - e^{-\frac{1}{2}\hat{\alpha}_1 t_k^i}). \tag{C11}$$

Therefore,

$$\sqrt{\frac{\gamma_1}{\Gamma_2}}e^{-\frac{1}{2}\hat{\alpha}_1 t_{k+1}^i} \leqslant \|e_i(t_{k+1}^{i-})\| \leqslant \frac{-2M_2}{\hat{\alpha}_1}(e^{-\frac{1}{2}\hat{\alpha}_1 t_{k+1}^i} - e^{-\frac{1}{2}\hat{\alpha}_1 t_k^i})$$

which implies that $t_{k+1}^i - t_k^i \geqslant \frac{2}{\hat{\alpha}_1}ln(1+\frac{\hat{\alpha}_1}{2M_2}\sqrt{\frac{\gamma_1}{\Gamma_2}})$. Then, for $\forall i\in\mathcal{V}$, the Zeno behaviour of event-triggered communication sequence $\{t_k^i\}, k = 0,1,\ldots$ under the dynamic ETS (4) can be excluded.

(ii) Secondly, the exclusion of Zeno phenomena for controller update triggering sequence $\{T_{k_i(t)}^i\}$ should also be guaranteed. Now we'll prove it.

Based on (3) and $\tilde{e}_i(t)$, we obtain

$$\dot{\tilde{e}}_i(t) = -\dot{q}_i(t) = -\sum_{j\in N_i}a_{ij}(\alpha_{ij}\dot{\hat{x}}_j^i(t) - \dot{\hat{x}}_i(t)) = -A\sum_{j\in N_i}a_{ij}(\alpha_{ij}\hat{x}_j^i(t) - \hat{x}_i(t)) = -Aq_i(t), \tag{C12}$$

with (C5) and (C9), then

$$\begin{aligned}
\|\dot{\tilde{e}}_i(t)\| &\leqslant \|A\|\|q_i(t)\| \leqslant \|A\|\|q(t)\|\\
&\leqslant \|A\|(M_1\|\alpha L\otimes I_n\| + \|\alpha L\alpha^{-1}\otimes I_n\|\sqrt{\frac{N\gamma_1}{\Gamma_2}})e^{-\frac{1}{2}\hat{\alpha}_1 t} \triangleq M_3 e^{-\frac{1}{2}\hat{\alpha}_1 t}.
\end{aligned}\tag{C13}$$

Define $\{\tilde{T}_{\tilde{k}}^i\} = \bigcup_{j\in N_i}\{t_k^j\}\bigcup\{t_k^i\}\bigcup\{T_{k_i(t)}^i\}, k, \tilde{k}, k_i(t) = 0,1,\ldots$. From the definition of event-triggered condition (5), we can see that $\tilde{e}_i(t)$ will be reset at each instant in $\{\tilde{T}_{\tilde{k}}^i\}$ and $\tilde{e}_i(t)$ is not always continuous during $[T_{k_i(t)}^i, T_{k_i(t)+1}^i)$. Strictly speaking, $\tilde{e}_i(t)$ is continuous within any two neighboring instants of $\{\tilde{T}_{\tilde{k}}^i\}$. Denote $\bigcup_{j\in N_i}\{t_k^j\}\bigcup\{t_k^i\} \triangleq \{\tilde{t}_k^i\}$ for simplicity, where the increasing sequence of $\{\tilde{t}_k^i\}$ can be arranged as $0 = \tilde{t}_0^i < \tilde{t}_1^i < \tilde{t}_2^i < \ldots, \forall i$, now we will analyze the exclusion of Zeno phenomena for controller update triggering sequence $\{T_{k_i(t)}^i\}$ from the following cases:

Case 1: When $T_{k_i(t)}^i, T_{k_i(t)+1}^i \in \{\tilde{t}_k^i\}$, then $T_{k_i(t)+1}^i - T_{k_i(t)}^i \geqslant \tilde{t}_{k+1}^i - \tilde{t}_k^i > 0$ since the exclusion for event-triggered condition (4) in **(i)**.

Case 2: When $T_{k_i(t)}^i \in \{\tilde{t}_k^i\}, T_{k_i(t)+1}^i \notin \{\tilde{t}_k^i\}$, then $\exists\, k^*\in\mathbb{N}$, such that $T_{k_i(t)+1}^i \in (\tilde{t}_{k^*}^i, \tilde{t}_{k^*+1}^i)$. Without loss of generality, assume $T_{k_i(t)}^i = \tilde{t}_k^i$. If $\tilde{t}_{k+1}^i \leqslant \tilde{t}_{k^*}^i$ or $\tilde{t}_{k^*}^i < \tilde{t}_{k+1}^i \leqslant T_{k_i(t)+1}^i$, then $T_{k_i(t)+1}^i - T_{k_i(t)}^i \geqslant \tilde{t}_{k+1}^i - \tilde{t}_k^i > 0$. If $\tilde{t}_{k^*}^i < T_{k_i(t)+1}^i < \tilde{t}_{k+1}^i$, then $T_{k_i(t)}^i$ and $T_{k_i(t)+1}^i$ are two neighboring instants in $\{\tilde{T}_{\tilde{k}}^i\}$. That is to say, $\tilde{e}_i(t)$ is continuous during $[T_{k_i(t)}^i, T_{k_i(t)+1}^i)$. Denote the latest controller update instant as $T_{k_i(t)}^i$, then the next controller update time will not occur before $\|\tilde{e}_i(t)\| = \gamma_2 e^{-\hat{\alpha}_2 t}$. Thus, the lower bound of intercommunication time for $\{T_{k_i(t)}^i, T_{k_i(t)+1}^i\}, \forall i$ is given by $\tau_i = t - T_{k_i(t)}^i$. By (B13), then τ_i is the solution of $M_3 e^{-\frac{1}{2}\hat{\alpha}_1 T_{k_i(t)}^i}\tau_i = \gamma_2 e^{-\hat{\alpha}_2 t}$, from the relationship between $\hat{\alpha}_1$ and $\hat{\alpha}_2$, then $\exists\,\gamma_2' > 0$ such that $M_3 e^{-\frac{1}{2}\hat{\alpha}_1 T_{k_i(t)}^i}\tau_i \geqslant \gamma_2' e^{-\frac{1}{2}\hat{\alpha}_1 t}$. It simplifies to $M_3 e^{\frac{1}{2}\hat{\alpha}_1\tau_i}\tau_i \geqslant \gamma_2'$. Easily to see, 0 and negative numbers are not satisfied, so there must exist a strictly positive τ_i such that the above inequality is satisfied. Therefore, the length of the two neighboring instants $T_{k_i(t)}^i$ and $T_{k_i(t)+1}^i$ is strictly positive.

Case 3: For the case when $T_{k_i(t)}^i \notin \{\tilde{t}_k^i\}, T_{k_i(t)+1}^i \in \{\tilde{t}_k^i\}$, similar to the analysis in **Case 2**, it induces that $T_{k_i(t)+1}^i - T_{k_i(t)}^i, \forall i\in\mathcal{V}$

is strictly positive.

Case 4: When $T^i_{k_i(t)} \notin \{\tilde{t}^i_k\}, T^i_{k_i(t)+1} \notin \{\tilde{t}^i_k\}$, then $\exists\ \bar{k}^*, \hat{k}^* \in \mathbb{N}$, such that $T^i_{k_i(t)} \in (\tilde{t}^i_{\bar{k}^*}, \tilde{t}^i_{\bar{k}^*+1})$, $T^i_{k_i(t)+1} \in (\tilde{t}^i_{\hat{k}^*}, \tilde{t}^i_{\hat{k}^*+1})$. If $\tilde{t}^i_{\bar{k}^*+1} < \tilde{t}^i_{\hat{k}^*}$, then $T^i_{k_i(t)+1} - T^i_{k_i(t)} > \tilde{t}^i_{\hat{k}^*} - \tilde{t}^i_{\bar{k}^*+1} > 0$. If $\tilde{t}^i_{\bar{k}^*+1} = \tilde{t}^i_{\hat{k}^*}$, then $T^i_{k_i(t)}$ and $T^i_{k_i(t)+1}$ are two neighboring instants in $\{\tilde{T}^i_{\tilde{k}}\}$. Similar to the procedure in **Case 2**, it obtains that $T^i_{k_i(t)+1} - T^i_{k_i(t)} > 0$. If $\tilde{t}^i_{\bar{k}^*+1} > \tilde{t}^i_{\hat{k}^*}$, then $\tilde{t}^i_{\bar{k}^*} = \tilde{t}^i_{\hat{k}^*} = T^i_{k_i(t)}$ or $\tilde{t}^i_{\bar{k}^*+1} = \tilde{t}^i_{\hat{k}^*+1} = T^i_{k_i(t)+1}$. Thus, $T^i_{k_i(t)+1} - T^i_{k_i(t)} = \tilde{t}^i_{\bar{k}^*+1} - \tilde{t}^i_{\bar{k}^*} > 0$.

Thus, it concludes that for $\forall i \in \mathcal{V}$, the Zeno phenomenon of controller update triggering sequence $\{T^i_{k_i(t)}\}, k_i(t) = 0, 1, \ldots$ under the ETS (5) can be avoided. That is, system performance can be reached without continuous controller update. So far, the exclusion for two event-triggering sequences in this paper have been proved.

Remark 2. Motivated by the original work on dynamic ETS by Girard [1], the exclusion of its static counterpart (C1) indicates the exclusion of Zeno behaviour for (4). The exclusion proof of Zeno behaviour for the ETS (5), we use the analysis method by investigating the relationship between event-triggered communication sequences $\bigcup_{j\in N_i}\{t^j_k\}\bigcup\{t^i_k\}$ and controller update triggering sequence $\{T^i_{k_i(t)}\}$, where four different subordinate cases are considered.

Appendix D Simulation results

To verify the theoretical results, the MASs with 4 agents are considered and the Laplacian matrix is given by $L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$.
The initial states of them are given as $x_{10} = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}$, $x_{20} = \begin{bmatrix} 0.4 \\ 1 \end{bmatrix}$, $x_{30} = \begin{bmatrix} 1.5 \\ -2.5 \end{bmatrix}$, $x_{40} = \begin{bmatrix} -2 \\ -3.6 \end{bmatrix}$. The system matrices are chosen as $A = \begin{bmatrix} -2 & 1 \\ 0.1 & 0.2 \end{bmatrix}$, $B = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix}$. Given $Q = 4I_4$, $R = 0.02$, by solving ARE, one obtains $P = \begin{bmatrix} 0.6389 & -0.6991 \\ -0.6991 & 1.6288 \end{bmatrix}$. The parameters are chosen as $\mu = 0.6$, $\varepsilon = 0.3$, $\gamma_1 = 0.5$, $\gamma_2 = 0.4$, $\hat{\alpha}_1 = 0.02$, $\hat{\alpha}_2 = 0.08$, $\beta_1 = 0.2$, $\beta_2 = 0.3$, $\beta_3 = 0.4$, $\beta_4 = 0.25$, $\rho = 500$, $\rho_1 = 600$, $\rho_2 = 300$, $\varphi_1 = 0.05$, $\varphi_2 = 0.06$, $\varphi_3 = 0.03$, $\varphi_4 = 0.07$.

The simulations of scaled consensus for MASs via dynamic ETS (4) are illustrated in Figures D1-D3. The trajectories of the scaled consensus errors $z_i(t), i = 1, 2, 3, 4$ are depicted in Figure D1 where $z_{i1}(t)$ and $z_{i2}(t)$ are the error components of agent i when scales are defined as $\alpha_1 = 0.6, \alpha_2 = -0.3, \alpha_3 = -1.2, \alpha_4 = 1.1$. The evolutions of controller (6) for each agent i with only piecewise continuous updates required are plotted in Figure D2 which verifies the effectiveness of the employment for two ETSs. The event-triggered communication instants and controller update triggering instants for each agent under the dynamic ETS (4) are presented in Figure D3 where we can see that the Zeno behaviour is excluded for both event-triggered communication sequence $\{t^i_k\}$ and controller update triggering sequence $\{T^i_{k_i(t)}\}$ of each agent.

From the references [1,2] we know that the dynamic ETS (4) will reduce to its standard static counterpart if the internal dynamic variable $\eta_i(t)$ satisfies $\eta_i(t) = 0$, $i \in \mathcal{V}$. That is, $t^i_{k+1} = \inf\{t > t^i_k : \Gamma_1\|q_i(t)\|^2 - \Gamma_2\|e_i(t)\|^2 + \gamma_1 e^{-\hat{\alpha}_1 t} \leqslant 0\}$(4′) Obviously to know, compared with a series of references like [3, 4] and so on, the energy consumption of controller update can be declined with the event-driven scheme (5) since the controller update is continuous in the aforementioned literatures. Thus, for fair comparison, the standard static ETS (4') will be considered here to highlight the effectiveness of the dynamic ETS (4) under event-driven controller update. Then, the triggering instants for communication and controller update under the static ETS (4') are shown in Figure D4. Intuitively, the frequency of communication is significantly reduced. During $[0s, 10s]$, Table D1 and D2 are given for detailedly illustrating where Nt and NT stand for the number of event-triggered communication and controller update, respectively. Also, Mt and MT are the mean value of interevent time for communication sequence $\{t^i_k\}$ and controller update sequence $\{T^i_{k_i(t)}\}$. From Table D1 and D2, we can find that under the dynamic ETS (4), the total of Nt for 4 agents will reduce greatly from 856 to 351 with only the cost of mildly increased NT from 259 to 266. Namely, the case of dynamic ETS (4) will be superior to its static counterpart (4') without requiring for the rapid increasing frequency of controller updates. In addition, both the inter-event time of event-triggered communication sequence $\{t^i_k\}$ and event-triggered controller update sequence $\{T^i_{k_i(t)}\}$ for some agents (agent 2 and 4) can be prolonged with ETSs (4) and (5). It should be noted that it doesn't necessarily minimize communication times by mildly increasing controller update frequency. The interesting phenomena imply that topology structures will impact the event-triggered actions. That is, there may exist some relationship between algebraic graph properties and ETS.

Moreover, in order to show the generality of scaled consensus, illustrations of identical consensus when $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$ and bipartite consensus (a special case of group consensus) when $\alpha_2 = \alpha_3 = -1, \alpha_1 = \alpha_4 = 1$ are given in Figures D5 and D6, respectively.

Agent NO.	Nt	NT	Mt	MT
Agent 1	74	35	0.135	0.286
Agent 2	74	38	0.135	0.263
Agent 3	115	152	0.087	0.066
Agent 4	88	41	0.114	0.244
Total	351	266	-	-

Table D1 Dynamic ETC (4).

Agent NO.	Nt	NT	Mt	MT
Agent 1	166	21	0.060	0.476
Agent 2	223	57	0.045	0.175
Agent 3	243	136	0.041	0.074
Agent 4	224	45	0.045	0.222
Total	856	259	-	-

Table D2 Static ETC (4').

References

1 Girard A. Dynamic triggering mechanisms for event-triggered control. IEEE Trans Autom Control, 2015, 60: 1992-1997
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3 Zhu W, Jiang Z P. Event-based leader-following consensus of multi-agent systems with input time delay. IEEE Trans Autom Control, 2015, 60: 1362-1367
4 Shen H, Chen M S, Wu Z G, et al. Reliable event-triggered asynchronous extended passive control for semi-Markov jump fuzzy systems and its application. IEEE Trans Fuzzy Syst, 2019. doi: 10.1109/TFUZZ.2019.2921264.

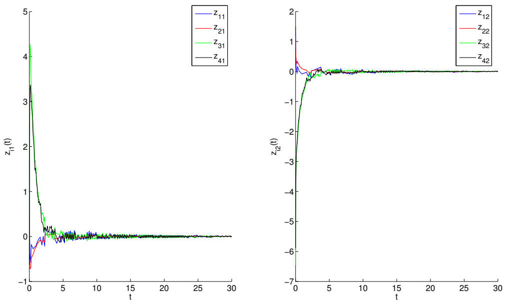

Figure D1 Trajectories of the scaled consensus error components $z_{i1}(t)$, $z_{i2}(t)$ for agent i.

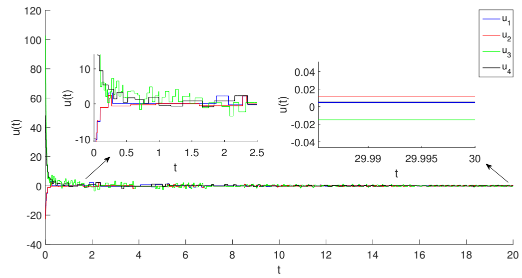

Figure D2 Evolutions of controller (6) for agent i.

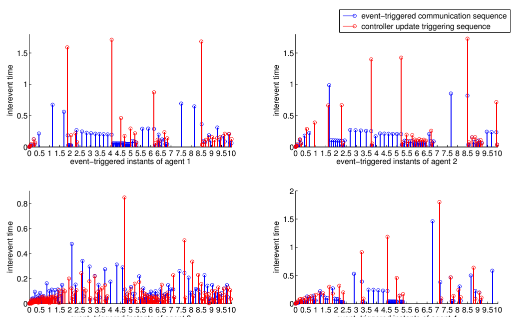

Figure D3 Triggering instants for sequences $\{t_k^i\}$ and $\{T^i_{k_i(t)}\}$ of agent i under dynamic ETS (4).

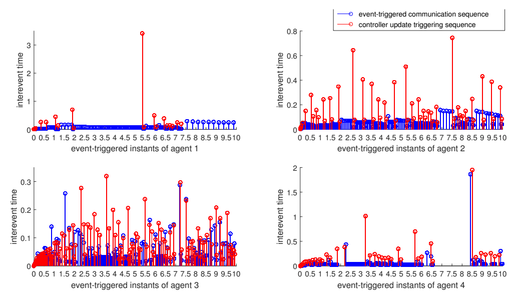

Figure D4 Triggering instants for sequences $\{t_k^i\}$ and $\{T^i_{k_i(t)}\}$ of agent i under static ETS (4').

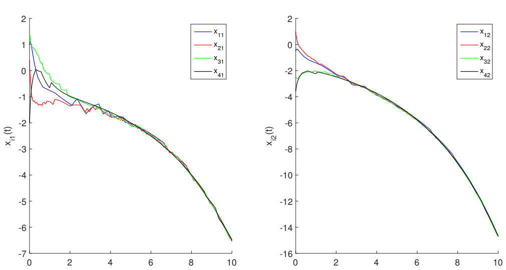

Figure D5 State trajectories of MASs at the case of identical consensus.

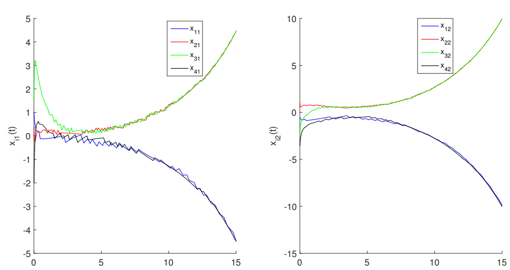

Figure D6 State trajectories of MASs at the case of bipartite consensus.