SCIENCE CHINA
Information Sciences

• LETTER •

October 2022, Vol. 65 209201:1–209201:2
https://doi.org/10.1007/s11432-020-2948-y

Leader-follower formation control of underactuated surface vessels

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Received 23 January 2020/Revised 6 April 2020/Accepted 29 May 2020/Published online 13 May 2021

Citation He X D, Geng Z Y. Leader-follower formation control of underactuated surface vessels. Sci China Inf Sci, 2022, 65(10): 209201, https://doi.org/10.1007/s11432-020-2948-y

Dear editor,

Leader-follower formation control of underactuated surface vessels has attracted much attention in past decades. The challenge arises from the second-order nonholonomic constraint on acceleration. To overcome the difficulty, plenty of nonlinear techniques are applied, such as robust adaptive control [1], dynamic surface control [2], and input-output linearization [3]. In addition, many papers study the event-triggered technique to enhance the management of communication resources between surface vessels, e.g., [4, 5].

Generally, the surface vessel's states contain both of position and orientation. Most existing results consider these states in a linear space. However, the actual configuration space is a nonlinear manifold [6]. An important class of manifolds which arises naturally in rigid body kinematics is called matrix Lie groups. Regarding surface vessels, the system can be established on the special Euclidean group in two dimensions, i.e., SE(2). Moreover, in many studies, another fact is that the desired formation pattern is defined in the earth-fixed frame. However, the formation cannot rotate in the scenario of curved reference paths but only has the movement of translation. Thus, it will be more practical if the desired pattern is given in the body-fixed frame.

Therefore, we investigate the leader-follower formation control of underactuated surface vessels under the framework of SE(2). The desired formation pattern is not predefined in the earth-fixed frame but in the leader's body-fixed frame. By designing a virtual leader, the formation problem is transformed into the trajectory tracking problem. Then, with relative systems on Lie groups, we convert the trajectory tracking to the stabilization of a relative system. Utilizing a stabilization controller on SE(2), we derive the formation controller eventually. Regarding the advantages, the surface vessel is described on a differentiable manifold without any local coordinates, so that globally effective results can be derived under such a framework. Another advantage is that we consider the desired formation pattern given in the leader's body-fixed frame, which allows the formation to have rotation as well as translation.

Preliminaries. Let $g = (R, p) \in \mathrm{SE}(2)$ denote the configuration of the surface vessel, where R is the rotation matrix through attitude angle θ, and $p = [x\ y]^{\mathrm{T}}$ is the position vector. Let $\hat{\xi} \in \mathfrak{se}(2)$ denote the velocity, where $\mathfrak{se}(2)$ is the Lie algebra associated with SE(2). Then the kinematic equation of the surface vessel is $\dot{g} = g\hat{\xi}$, which is a global description in the sense that it does not rely on local coordinates. Let $\mathbb{I}$ represent the inertia tensor; then, the dynamic equation of the surface vessel is given by $\dot{\hat{\xi}} = \mathbb{I}^{\sharp}(\mathrm{ad}^{*}_{\hat{\xi}}\mathbb{I}^{\flat}\hat{\xi} + \hat{b}(\hat{\xi}) + \sum_{i=1}^{2}\hat{f}_i u_i)$, where $\hat{f}_i$ is the control vector field, and u_i is the scalar control input. We assume there are only yaw moment and surge force on the surface vessel. For the simplicity, we define a new control input $\hat{\tau} = \sum_{i=1}^{2}\hat{f}_i u_i$, which can be expressed in $\mathbb{R}^3$ as $\tau = [\tau^r\ \tau^x\ 0]^{\mathrm{T}}$. Therefore, we will design the yaw moment τ^r and surge force τ^x. Note that the control input $\hat{\tau}$ lies in Lie algebra $\mathfrak{se}(2)$, while the configuration g is on Lie group SE(2). In order to construct state feedback, g should be mapped into $\mathfrak{se}(2)$. Thus, we introduce the logarithm map $\log_{\mathrm{SE}(2)} : \mathrm{SE}(2) \to \mathfrak{se}(2)$.

Definition 1. For $g = (R, p) \in \mathrm{SE}(2)$ with $\mathrm{trace}(g) \neq -1$, the logarithm map $\log_{\mathrm{SE}(2)}$ is defined as $\hat{X} = \log_{\mathrm{SE}(2)}(g) = [\hat{\theta}\ A^{-1}(\theta)p;\ 0_{1\times2}\ 0]$, where $\hat{\theta} = [0\ \ -\theta;\ \theta\ 0]$, $A^{-1}(\theta) = [\alpha(\theta)\ \theta/2;\ \ -\theta/2\ \alpha(\theta)]$, $\alpha(\theta) = \frac{\theta}{2}\cot\frac{\theta}{2}$. Due to the fact that $\mathbb{R}^3$ and $\mathfrak{se}(2)$ are isomorphic, $\hat{X}$ can also be mapped into $\mathbb{R}^3$ and it is denoted by $X = [\theta\ q^x\ q^y]^{\mathrm{T}}$, where $q = [q^x\ q^y]^{\mathrm{T}} = A^{-1}(\theta)p$.

Problem formulation. Consider the formation of two surface vessels, i.e., one leader and one follower. We use subscript "0" and "1" to represent leader and follower respectively, so that their configurations are denoted by g_0 and g_1. Due to the fact that the desired formation is provided in the leader's body-fixed frame, we firstly define the relative configuration g_{01}. Referring to errors on Lie groups, the relative configuration of the follower with respect to the leader can be defined as $g_{01} = g_0^{-1}g_1$. Then, g_{01} can be uniquely decided by θ_{01} and $r_{01} = [r_{01}^x\ r_{01}^y]^{\mathrm{T}}$, where $\theta_{01} = \theta_1 - \theta_0$, $r_{01}^x = (x_1 - x_0)\cos\theta_0 + (y_1 - y_0)\sin\theta_0$, $r_{01}^y = -(x_1 - x_0)\sin\theta_0 + (y_1 - y_0)\cos\theta_0$. Let $\bar{\theta}_{01}$ denote the desired relative attitude angle and $\bar{r}_{01} = [\bar{r}_{01}^x\ \bar{r}_{01}^y]^{\mathrm{T}}$ denote the desired relative position in the leader's body-fixed

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frame. Then, we can define a constant configuration $\bar{g}_{01}$, which serves as the desired formation pattern. Thus, the leader and the follower realize the formation $\bar{g}_{01}$ if and only if $g_{01} = \bar{g}_{01}$.

Formation control strategy. In order to achieve the desired formation, we define an auxiliary configuration $g_a = g_0\bar{g}_{01}$, where g_0 is the leader's configuration and $\bar{g}_{01}$ is the desired formation pattern. From the definition, we can see that g_a is attached to the leader's body-fixed frame, and will move with the leader while keeping a relative configuration $\bar{g}_{01}$. In this sense, g_a is a virtual leader for the follower.

Lemma 1. The leader-follower formation $\bar{g}_{01}$ is achieved if the follower tracks the trajectory of the virtual leader.

Lemma 1 transforms the leader-follower formation to the trajectory tracking for the virtual leader. Before we solve the tracking problem, the virtual leader's kinematics can be derived as $\dot{g}_a = g_a\hat{\xi}_a$, where the virtual leader's velocity is $\hat{\xi}_a = \mathrm{Ad}_{\bar{g}_{01}^{-1}}\hat{\xi}_0$. Next, computing the time derivative of $\hat{\xi}_a$, we can obtain the dynamics of the virtual leader as $\dot{\hat{\xi}}_a = \hat{c}_a(\hat{\xi}_0, \bar{g}_{01}) + \hat{b}_a(\hat{\xi}_0, \bar{g}_{01}) + \hat{\tau}_a(\hat{\tau}_0, \bar{g}_{01})$. Now, according to Lemma 1, we should make the follower track the trajectory of the virtual leader. To this end, we define a relative configuration $g_{a1} = g_a^{-1}g_1$ and a relative velocity $\hat{\xi}_{a1} = \hat{\xi}_1 - \mathrm{Ad}_{g_{a1}^{-1}}\hat{\xi}_a$. In this way, we obtain a relative system of the follower with respect to the virtual leader, and its kinematics and dynamics are $\dot{g}_{a1} = g_{a1}\hat{\xi}_{a1}$ and $\dot{\hat{\xi}}_{a1} = \hat{c}_{a1}(\hat{\xi}_1, \hat{\xi}_0, \bar{g}_{01}) + \hat{b}_{a1}(\hat{\xi}_1, \hat{\xi}_0, \bar{g}_{01}) + \hat{\tau}_{a1}(\hat{\tau}_1, \hat{\tau}_0, \bar{g}_{01})$, where the control input is

$$\hat{\tau}_{a1}(\hat{\tau}_1, \hat{\tau}_0, \bar{g}_{01}) = \mathbb{I}_1^{\sharp}\hat{\tau}_1 - \mathrm{Ad}_{g_{a1}^{-1}}\hat{\tau}_a(\hat{\tau}_0, \bar{g}_{01}). \tag{1}$$

Lemma 2 ([7], Lemma 3)**.** The follower can track the trajectory of the virtual leader if the relative system is stabilized to the identity, that is, $\lim_{t\to\infty} g_{a1} = I_3$ and $\lim_{t\to\infty}(\hat{\xi}_1 - \mathrm{Ad}_{g_{a1}^{-1}}\hat{\xi}_a) = 0_{3\times 3}$.

From Lemma 2, as long as we stabilize the relative system, the trajectory tracking can be guaranteed. Then, we can achieve the formation of the follower with respect to the leader. Once the stabilization control law $\hat{\tau}_{a1}$ is designed, the follower's controller can be deduced from (1) as

$$\hat{\tau}_1 = \mathbb{I}_1^{\flat}\hat{\tau}_{a1} + \mathbb{I}_1^{\flat}\mathrm{Ad}_{g_{a1}^{-1}}\hat{\tau}_a. \tag{2}$$

For the convenience of deriving $\hat{\tau}_{a1}$, we let θ_a, x_a, y_a denote the attitude angle and position of the virtual leader, let τ_a^r and τ_a^x denote the angular and linear acceleration (control inputs) of the virtual leader, and the control inputs of the follower are denoted by τ_1^r and τ_1^x. In addition, the exponential coordinates of g_{a1} can be denoted by $X_{a1} = [\theta_{a1}\ \ q_{a1}^x\ \ q_{a1}^y]^{\mathrm{T}}$. The following lemma shows how to design $\hat{\tau}_{a1}$ to stabilize the relative system.

Lemma 3 ([7], Theorem 1)**.** If the control law $\hat{\tau}_{a1}$ for the relative system is designed as

$$\hat{\tau}_{a1} = \begin{bmatrix} 0 & -\tau_{a1}^r & \tau_{a1}^x \\ \tau_{a1}^r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3}$$

where $\tau_{a1}^r = -k_1\theta_{a1} - k_2\beta_{a1} - k_3(\theta - \tilde{\theta})$, $\tau_{a1}^x = -k_1 q_{a1}^x$, $\tilde{\theta}_1 = \theta_0 + \arctan\frac{(\tau_0^r/J_0)r_{01}^x}{(\tau_0^x/m_0^x) - (\tau_0^r/J_0)r_{01}^y}$, $\beta_{a1} = -\arctan(q_{a1}^y/q_{a1}^x)$, and k_1, k_2, k_3 are positive scalars, then $\hat{\tau}_{a1}$ stabilizes the relative system to the identity globally and asymptotically.

Substituting (3) into (2) and mapping it to $\mathbb{R}^3$, we obtain the formation controller for the follower, i.e.,

$$\tau_1^r = -J_1(k_1\theta_{a1} + k_2\beta_{a1} + k_3(\theta_1 - \tilde{\theta}_1) - \tau_a^r), \tag{4}$$

$$\begin{aligned}\tau_1^x = &-m_1^x(k_1 q_{a1}^x - (\tau_a^x - \tau_a^r)r_{a1}^y)\cos\theta_{a1} \\ &- m_1^x\tau_a^r r_{a1}^x \sin\theta_{a1},\end{aligned} \tag{5}$$

where $r_{a1}^x = (x_1 - x_a)\cos\theta_a + (y_1 - y_a)\sin\theta_a$ and $r_{a1}^y = -(x_1 - x_a)\sin\theta_a + (y_1 - y_a)\cos\theta_a$. The result is summarized below.

Theorem 1. Consider two underactuated vessels evolving on SE(2), where one is the leader and the other is the follower. Let $\bar{g}_{01}$ denote the desired rigid formation. Then, the yaw moment and surge force in (4) and (5) can globally and asymptotically make the follower achieve formation $\bar{g}_{01}$.

Proof. Substituting (4) and (5) into (1), we obtain the relative control law in (3). According to Lemma 3, the relative system with (3) can be stabilized to the identity. Then, due to Lemma 2, the follower tracks the virtual leader. Finally, from Lemma 1, the lead-follower formation is achieved.

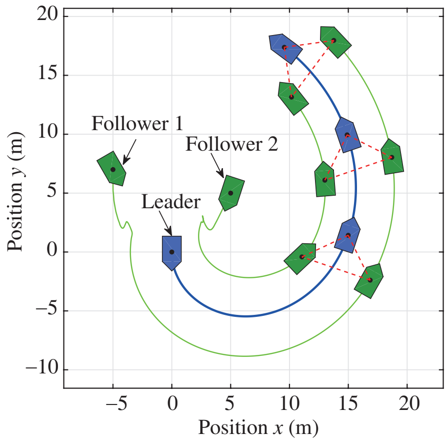

Figure 1 (Color online) Simulation result of the leader-follower formation.

Simulation. We make a simulation of three surface vessels. The trajectories of surface vessels are provided in Figure 1, which implies the formation is successfully realized.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant No. 61773024).

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