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Observer-based l_2-l_∞ control for singularly perturbed semi-Markov jump systems with an improved weighted TOD protocol

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Dear editor,

In practical scenarios, many engineering systems possess switching features. When the factors incurring the switch phenomenon occur with a certain probability, semi-Markov chains can be adopted to describe them [1, 2]. Besides, dynamics in real systems generally present multiple time scales which can induce serious numerical problems. These problems, however, can be tackled effectively by introducing a singularly perturbed model [3]. Therefore, it is fascinating to analyze the l_2-l_∞ performance issue for the singularly perturbed semi-Markov jump systems (SPSMJSs). Meanwhile, limited communication resources pose a major problem in digital communication scheduling. A widely recognized way for saving limited resources is to adopt specific communication protocols, such as event-triggering, round-robin and try-once-discard protocols (TODP) [4, 5]. Inspired by their mechanisms, one will naturally wonder whether we can develop a novel transmission protocol that can transmit a specific part of the required data at a certain time point. When the current piece of data is similar to the previous one, the transmission is suspended. To explore the possibilities of an effective protocol design, this study is devoted to conducting an improvement on the conventional weighted TODP (WTODP).

This study investigates the observer-based l_2-l_∞ controller design issue for SPSMJSs with measurement quantization under the improved WTODP. The main contributions can be synthesized as below: (1) A new analytical approach based on [1] is proposed, under which utilizing the iteration of $x(k)$ between k_m and k_{m+1} based on state space expression is avoided. (2) Inspired by [5], an improved WTODP, which can further increase the utilization ratio of the limited bandwidth, is proposed. Different from the traditional WTODP, the transmission between the sensor and observer is suspended when the error between the current and previous piece of data of the nodes is zero. Then, two novel constraints revealing this communication feature are constructed by virtue of the Kronecker sign function. (3) An observer-based controller is designed with the aid of several contract matrices.

Notations. See Appendix A.

Problem formulation. Consider a discrete-time SPSMJS:

$$x(k+1) = A_{\sigma(k)}E_\epsilon x(k) + B_{\sigma(k)}u(k) + D_{\sigma(k)}\omega(k)\,, \quad (1)$$

where $x(k) \triangleq [\, x_s^{\mathrm{T}}(k)\;\, x_f^{\mathrm{T}}(k)\,]^{\mathrm{T}} \in \mathbb{R}^{n_x}$ with $x_s(k) \in \mathbb{R}^{n_1}$ and $x_f(k) \in \mathbb{R}^{n_2}$ being the slow and fast states, respectively; $\omega(k)$ is the exogenous disturbance; $E_\epsilon \triangleq \mathrm{diag}\{I_{n_1}, \epsilon I_{n_2}\}$ with $\epsilon > 0$ being the singularly perturbation parameter. $y(k) = C_{\sigma(k)}E_\epsilon x(k) \in \mathbb{R}^{Nn}$ and $z(k) = G_{\sigma(k)}E_\epsilon x(k)$ denote the measurement and practical outputs, respectively. Taking values in the set $\bar{M} \triangleq \{1, 2, \ldots, M\}$, $\{\sigma(k)\}_{k\in\mathbb{Z}_{>0}}$ represents a semi-Markov chain.

Based on [6], the quantized outputs are written as

$$\tilde{y}(k) \triangleq \mathcal{G}(y(k)) = [g_1(y_{11}(k)), g_1(y_{12}(k)), \ldots, g_N(y_{Nn}(k))]^{\mathrm{T}}. \quad (2)$$

Quantization error is $\tilde{y}(k) - y(k) \triangleq \Delta(k)y(k)$ with $\Delta(k) \triangleq \mathrm{diag}\{\delta_1(k), \ldots, \delta_{Nn}(k)\}$. $y(k) \triangleq [y_1^{\mathrm{T}}(k), \ldots, y_N^{\mathrm{T}}(k)]^{\mathrm{T}}$.

For $\forall i \in \bar{N} \triangleq \{1, 2, \ldots, N\}$, let $\Phi(i) \triangleq \mathrm{diag}\{\vartheta(i-1), \vartheta(i-2), \ldots, \vartheta(i-N)\} \otimes I_n$, where $\vartheta(i)$ is Kronecker sign function. Then, the node obtaining transmission permission at time k under the improved WTODP is determined by

$$\xi(k) \triangleq \begin{cases} \arg\max\limits_{1\leqslant i\leqslant N} \|y(k) - \bar{y}(k-1)\|^2_{\hat{Q}\Phi(i)}\,, & \varepsilon_y(k) > 0, \\ 0, & \varepsilon_y(k) = 0, \end{cases} \quad (3)$$

where $\bar{y}(k-1)$ represents the previously transmitted data; $\varepsilon_y(k) \triangleq \|y(k) - \bar{y}(k-1)\|^2_{\hat{Q}}$; $\hat{Q} \triangleq \mathrm{diag}\{Q_1, Q_2, \ldots, Q_N\}$ denotes weight matrix; $\bar{y}(k) \triangleq [\bar{y}_1^{\mathrm{T}}(k), \ldots, \bar{y}_N^{\mathrm{T}}(k)]^{\mathrm{T}}$. Then, the

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data received by the observer are given as follows:

$$\bar{y}(k) = \Phi(\xi(k))\tilde{y}(k) + (I_{Nn} - \Phi(\xi(k)))\bar{y}(k-1). \quad (4)$$

For $\sigma(k) = a \in \bar{M}$, the observer-based controller can be designed as

$$\begin{cases} \hat{x}(k+1) = A_a E_\epsilon \hat{x}(k) + B_a u(k) + H_{a\xi(k)}(\bar{y}(k) - \hat{y}(k)), \\ \hat{y}(k) = C_a E_\epsilon \hat{x}(k), \\ u(k) = K_{a\xi(k)} E_\epsilon \hat{x}(k), \end{cases} \quad (5)$$

where $H_{a\xi(k)}$ and $K_{a\xi(k)}$ are matrices to be designed.

Then, the closed-loop system (CLS) can be given as

$$\begin{cases} \bar{\eta}(k+1) = \bar{E}_\epsilon[\bar{A}_{a\xi(k)}\bar{\eta}(k) + \bar{D}_a\omega(k)], \\ z(k) = \bar{G}_a\bar{\eta}(k), \end{cases} \quad (6)$$

where $\eta(k) \triangleq [\, \bar{y}^{\mathrm{T}}(k-1) \ x^{\mathrm{T}}(k)E_\epsilon \ e^{\mathrm{T}}(k)E_\epsilon \,]^{\mathrm{T}}$, $e(k) \triangleq x(k) - \hat{x}(k)$, $\bar{\eta}(k) = E_{I34}\eta(k)$, $E_{I34} \triangleq \mathrm{diag}\{I_{Nn+n_1}, \bar{I}_{12}\}$, and other specific forms of the matrices involved are given in Appendix A.

The following Lyapunov functions are constructed:

$$V_{\xi(k)}(\bar{\eta}(k), \sigma(k), \tau) \triangleq \check{y}(k)^{\mathrm{T}}\check{Q}_{\xi(k)}\check{y}(k) + \bar{\eta}^{\mathrm{T}}(k)P_{\sigma(k)}(\tau)\bar{\eta}(k), \quad (7)$$

where $\check{Q}_{\xi(k)} \triangleq \hat{Q} - \hat{Q}\Phi(\xi(k)), \check{y}(k) \triangleq y(k) - \bar{y}(k-1)$, $P_{\sigma(k)}(\tau) > 0$ with $k \in \mathbb{Z}_{[k_m, k_{m+1})}$, $\sigma(k) \in \bar{M}$, $\tau = k - k_m \in \mathbb{Z}_{[0, \bar{d}_{\sigma(k)}]}$ and $\xi(k) \in \tilde{N} \triangleq \{0, 1, 2, \ldots, N\}$.

Theorem 1. For given scalars $\gamma \in (0, \infty)$, $\bar{d}_{\sigma(k)} \in \mathbb{Z}_{>0}$, if the constructed Lyapunov functions $V_{\xi(k)}(\bar{\eta}(k), \sigma(k), \tau)$ $(k \in \mathbb{Z}_{[k_m, k_{m+1})})$ satisfy the following condition for $\forall \sigma(k) \in \bar{M}$, $\xi(k) \in \tilde{N}$, $\tau \in \mathbb{Z}_{[0, \bar{d}_{\sigma(k)})}$, $T_m \in \mathbb{Z}_{[1, \bar{d}_{\sigma(k)}]}$:

$$\begin{aligned} &V_{\xi(k+1)}(\bar{\eta}(k+1), \sigma(k_m), \tau+1) \\ &\leqslant V_{\xi(k)}(\bar{\eta}(k), \sigma(k), \tau) + \omega^{\mathrm{T}}(k)\omega(k), \end{aligned} \quad (8)$$

$$\begin{aligned} &\mathcal{E}\Big\{V_{\xi(k_{m+1})}(\bar{\eta}(k_{m+1}), \sigma(k_{m+1}), 0)\Big\}|_{\bar{\eta}(k_m), \sigma(k_m)} \\ &\leqslant \mathcal{E}\Big\{V_{\xi(k_{m+1})}(\bar{\eta}(k_{m+1}), \sigma(k_m), T_m)\Big\}|_{\bar{\eta}(k_m), \sigma(k_m)}, \end{aligned} \quad (9)$$

$$0 \geqslant \mathcal{E}\Big\{z^{\mathrm{T}}(k)z(k) - \gamma^2 V_{\xi(k)}(\bar{\eta}(k), \sigma(k), \tau)\Big\}|_{\bar{\eta}(k_0), \sigma(k_0)}, \quad (10)$$

then, the CLS (6) is δ-error mean-square stability (δ-EMSS) with a prescribed l_2-l_∞ performance level γ.

Proof. See Appendix D.

Remark 1. For the improved WTODP, the following constraint conditions are critical for obtaining Theorem 2.

(i) For $\forall \lambda_{\bar{\xi}j} > 0$ satisfying $\sum_{j=1}^{N} \lambda_{\bar{\xi}j} = 1, \bar{\xi} \in \bar{N}$, $\check{y}^{\mathrm{T}}(k+1)\check{Q}_{\xi(k+1)}\check{y}(k+1) \leqslant \check{y}^{\mathrm{T}}(k+1)\sum_{j=1}^{N} \lambda_{\bar{\xi}j}\check{Q}_j\check{y}(k+1)$.

(ii) For $\forall \bar{l}_j$ $(j \in \bar{N})$, $\sum_{j=1}^{N}\vartheta(\xi(k)-0)\check{y}^{\mathrm{T}}(k)\bar{l}_j\hat{Q}\Phi(j)\check{y}(k) = 0$.

Theorem 2. For given scalars $\gamma, \bar{\epsilon} \in (0, \infty)$, $\bar{d}_a \in \mathbb{Z}_{>0}$, if there exist scalars $\bar{l}_j$ $(j \in \bar{N})$, $\varrho_a > 0$, positive scalars $\lambda_{\bar{\xi}j}$ with $\bar{\xi} \in \bar{N}$, $j \in \{1, 2, \ldots, N-1\}$, positive matrices $\Omega_{\xi(k)a}(\tau)$ $(\xi(k) \in \bar{N}, \tau \in \mathbb{Z}_{[1, \bar{d}_a]})$ and symmetric positive definite matrices $P_a(\tau)$ $(\tau \in \mathbb{Z}_{[0, \bar{d}_a]})$ satisfying the following conditions for $\forall a \in \bar{M}$, $\xi(k) \in \tilde{N}$, $\tau \in \mathbb{Z}_{[0, \bar{d}_a)}$, $d \in \mathbb{Z}_{[1, \bar{d}_a]}$:

$$\begin{bmatrix} \phi^1_{\xi(k)a}(\tau) & 0 & \phi^2_{\xi(k)a} & 0 \\ * & -I & \bar{D}_a^{\mathrm{T}} & 0 \\ * & * & -\Omega_{\xi(k)a}(\tau) & E_{I34}\mathcal{H}_{a\xi(k)} \\ * & * & * & -\varrho_a I \end{bmatrix} < 0, \quad (11)$$

$$\bar{E}^h \bar{\mathcal{P}}_{a\xi(k)}(\tau)\bar{E}^h - 2I + \Omega_{\xi(k)a}(\tau) < 0, \ h = 1, 2, \quad (12)$$

$$0 \leqslant \bar{\lambda}_{\bar{\xi}j} \leqslant 1, 0 \leqslant \lambda_{\bar{\xi}j} \leqslant 1, \ j \in \{1, 2, \ldots, N-1\}, \quad (13)$$

$$\hat{P}_a(d) \triangleq \sum_{b \in \bar{M}} \frac{\bar{\theta}_{ab}(d)}{\varsigma_a}(P_b(0) - P_a(d)) < 0, \quad (14)$$

$$\bar{G}_a^{\mathrm{T}}\bar{G}_a - \gamma^2(P_a(\tau) + \bar{C}_a^{\mathrm{T}}\check{Q}_{\xi(k)}\bar{C}_a) < 0, \quad (15)$$

where $\mathcal{A}_{a\xi(k)} \triangleq \tilde{A}_{a\xi(k)} - \mathcal{H}_{a\xi(k)}\Delta(k)\mathcal{C}_a$, $\bar{\lambda}_{\bar{\xi}j} \triangleq 1 - \sum_{j=1}^{N-1}\lambda_{\bar{\xi}j}$, $\varsigma_a \triangleq \sum_{d=1}^{\bar{d}_a}\sum_{b \in \bar{M}}\bar{\theta}_{ab}(d)$ and

$$\begin{aligned} \phi^1_{\xi(k)a}(\tau) &\triangleq \sum\nolimits_{j=1}^{N} \bar{l}_j\vartheta(\xi(k)-0)\bar{C}_a^{\mathrm{T}}\hat{Q}\Phi(j)\bar{C}_a - P_a(\tau) \\ &\quad -\bar{C}_a^{\mathrm{T}}\check{Q}_{\xi(k)}\bar{C}_a + \varrho_a E_{I34}^{\mathrm{T}}\mathcal{C}_a^{\mathrm{T}}\Upsilon^{\mathrm{T}}\Upsilon_a\mathcal{C}_a E_{I43}, \\ \bar{\mathcal{P}}_{a\xi(k)}(\tau) &\triangleq P_a(\tau+1) + \sum\nolimits_{j=1}^{N-1} \lambda_{\bar{\xi}j}\bar{C}_a^{\mathrm{T}}\check{Q}_j\bar{C}_a \\ &\quad +\bar{\lambda}_{\bar{\xi}j}\bar{C}_a^{\mathrm{T}}\check{Q}_N\bar{C}_a, \\ \phi^2_{\xi(k)a} &\triangleq E_{I34}\mathcal{A}^{\mathrm{T}}_{a\xi(k)}E_{I43}, \bar{E}^1 \triangleq \mathrm{diag}\{I_{nN+2n_1}, 0_{2n_2}\}, \\ \mathcal{C}_a &\triangleq \begin{bmatrix} 0 & C_a & 0 \end{bmatrix}, \bar{E}^2 \triangleq \mathrm{diag}\{I_{nN+2n_1}, \bar{\epsilon}I_{2n_1}\}, \\ \mathcal{H}_{a\xi(k)} &\triangleq \begin{bmatrix} \Phi^{\mathrm{T}}(\xi(k)) & 0 & -\Phi^{\mathrm{T}}(\xi(k))H^{\mathrm{T}}_{a\xi(k)} \end{bmatrix}^{\mathrm{T}}, \end{aligned}$$

then, the CLS (6) is δ-EMSS with a prescribed l_2-l_∞ performance level γ for $\forall \epsilon \in (0, \bar{\epsilon}]$.

Proof. See Appendix E.

Conclusion. The observer-based l_2-l_∞ controller design issue has been discussed for networked SPSMJSs subjected to the logarithmic quantization effects and improved try-once-discard scheduling protocol. By introducing two constraint conditions related to the improved transmission scheme and constructing a novel Lyapunov function, an observer-based controller, which can ensure the δ-EMSS and enhance the l_2-l_∞ performance of the investigated system, has been designed. Simulation results have been provided in Appendix F, validating the effectiveness of the devised strategy. Future research will entail the extension of our work to the situation of the sliding mode control.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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