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Resilient dynamic event-triggered and self-triggered control for Markov jump systems under denial-of-service attacks

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Dear editor,

As a kind of hybrid systems, Markov jump systems have made remarkable progress by the researchers from the various fields in the past few decades [1]. Unlike switched systems with a definite jump signal [2], a random signal adjusts the working sequence of subsystems in Markov jump systems. Meanwhile, for networks with limited bandwidth and energy, the time-triggering scheme accelerates resource consumption. One of the effective ways of tackling this problem is to adopt event-triggering scheme (ETS) [3,4]. On another frontier, various cyber attacks exist owing to the nature of networks, and the issue of event-triggered resilient control under attacks has been concerned [5]. Although there exist some researches on event-triggered control (ETC) of Markov jump systems, resilient dynamic ETC of Markov jump systems under cyber attacks remains an open issue; this is the first motivation. In addition, the existing self-triggering schemes (STSs) are provided via static ETS, an STS based on dynamic ETC needs to be proposed to further reduce sampling frequency; this is the second motivation.

The main contributions of this study are as follows: (1) A dynamic ETS is developed to investigate stochastic stability of Markov jump systems under aperiodic denial-of-service (DoS) attacks; (2) In order to avoid real-time measurement of triggering conditions in ETC, an STS is constructed according to the dynamic ETS, which allows the controller to compute the next triggering moment based on current information.

Problem description. In this study, we focus on a class of Markov jump systems in the following form:

$$\dot{x}(t) \ = \ A_{r(t)}x(t) + B_{r(t)}u(t), \tag{1}$$

where $x \in \mathbb{R}^n$ represents the system state and $u \in \mathbb{R}^{n_u}$ is the control input. $A_{r(t)}$ and $B_{r(t)}$ are real constant matrices with appropriate dimensions. $\{r(t), t \geqslant 0\}$ denotes a homogeneous right-continuous Markov process in a finite space $S = \{1, 2, \ldots, N\}$. In addition, $r(t)$ is provided by an infinitesimal generator $\Lambda = [\lambda_{ij}]$, where $\lambda_{ij} \geqslant 0$ for $i \neq j$ and $\lambda_{ii} = -\sum_{j=1,i\neq j}^{N} \lambda_{ij}$.

To reduce system sampling and save network resources, an ETS is adopted for the Markov jump system. Letting $\{x(t_k)\}_{k\in\mathbb{N}}$ be the triggering data, the dynamic ETS is provided by

$$\begin{cases} t_{k+1} = \inf\{t > t_k |\ e_{t_k}^{\mathrm{T}}(t)M_{r(t)}e_{t_k}(t) \geqslant \kappa_{r(t)}x^{\mathrm{T}}(t) \\ \qquad \times M_{r(t)}x(t) + \theta_{r(t)}\eta(t)\}, \\ \dot{\eta}(t) = -\alpha_{r(t)}\eta(t) + \beta_{r(t)}[\kappa_{r(t)}x^{\mathrm{T}}(t)M_{r(t)}x(t) \\ \qquad - e_{t_k}^{\mathrm{T}}(t)M_{r(t)}e_{t_k}(t)], \end{cases} \tag{2}$$

where $e_{t_k}(t) = x(t_k) - x(t)$, scalars $\eta(0) > 0$, $\kappa_{r(t)} > 0$, $\theta_{r(t)} > 0$, $\alpha_{r(t)} > 0$, $\beta_{r(t)} > 0$, and $M_{r(t)} > 0$ is the weighting matrix of appropriate dimension.

Since networks are vulnerable to cyber attacks and security issues are increasingly important, DoS attacks are taken into account in this study. According to [6], it is supposed that $[T_n^{\mathrm{on}}, T_n^{\mathrm{off}})$ is the interval during which the system suffers from DoS attacks. By using a full feedback controller, we obtain

$$u(t) = \varsigma(t)K_{r(t)}x(t_k), \quad t \in [t_k, t_{k+1}), \tag{3}$$

where

$$\varsigma(t) = \begin{cases} 0, \ t \in [t_k, t_{k+1}) \cap [T_n^{\mathrm{on}}, T_n^{\mathrm{off}}), \\ 1, \ \text{others}. \end{cases}$$

Combining original system (1) and control input (3), we have the following switched Markov jump closed-loop system:

$$\dot{x}(t) = \bar{A}_{r(t)}x(t) + \bar{B}_{r(t)}e_{t_k}(t), \tag{4}$$

where

$$\bar{A}_{r(t)} = A_{r(t)} + \varsigma(t)B_{r(t)}K_{r(t)},\ \bar{B}_{r(t)} = \varsigma(t)B_{r(t)}K_{r(t)}.$$

Some preliminaries are provided in Appendix A.

Main result. First, we provide a lower bound of inter-execution intervals to avoid Zeno behavior in the following theorem.

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Theorem 1. For the Markov jump system (1) with dynamic ETS (2) and control input (3), a lower bound τ is given by

$$\tau = \frac{1}{\varphi_1} \ln\left(\frac{\Psi\varphi_1}{\varphi_2(\Psi+1)} + 1\right), \tag{5}$$

where

$$\Psi = \frac{\Phi_2}{\Phi_1},\ \varphi_1 = \max_{i\in S}\{\|A_i\|\},\ \varphi_2 = \max\left\{\max_{i\in S}\{\|\bar{A}_{1i}\|\}, \varphi_1\right\},$$
$$\Phi_1 = \max_{i\in S}\{\|\bar{M}_i\|\},\ \Phi_2 = \min_{i\in S}\{\|\hat{M}_i\|\},\ \bar{M}_i^2 = M_i,$$
$$\hat{M}_i^2 = \kappa_i M_i.$$

The proof of Theorem 1 is given in Appendix B.

Based on the switched Markov jump closed-loop system (4) and dynamic ETS (2) in Theorem 1, the piecewise Lyapunov function method is utilized and some sufficient conditions are provided to guarantee the stochastic stability in the following theorem.

Theorem 2. For any $i, j \in S$, given a set of positive constants τ_D, α_i, β_i, κ_i, θ_i, $\tilde{\lambda}$, $\bar{\lambda}$, $T > 1$, $\mu_\iota > 1$ ($\iota \in \{1,2\}$) and ϑ_l ($l \in \{1,2,3\}$), if there exist matrices $P_{1i} > 0$, $P_{2i} > 0$ and $M_i > 0$ such that

$$\begin{bmatrix} \Xi_1 & P_{1i}\bar{B}_{1i} & 0 \\ * & -(\beta_i+1)M_i & 0 \\ * & * & -(\alpha_i - \vartheta_1 - \theta_i)I \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} \Xi_2 & 0 & 0 \\ * & -\beta_i M_i & 0 \\ * & * & -(\alpha_i + \vartheta_2)I \end{bmatrix} < 0, \tag{7}$$

$$P_{1i} \leqslant \mu_1 P_{2i},\ P_{2i} \leqslant \mu_1 P_{1i}, \tag{8}$$

$$P_{1i} \leqslant \mu_2 P_{1j},\ P_{2i} \leqslant \mu_2 P_{2j}, \tag{9}$$

$$\frac{\vartheta_2+\vartheta_3}{\vartheta_1-\vartheta_3} \leqslant T-1,\ \vartheta_3 - \frac{\ln\mu_1}{\tau_D} + \tilde{\lambda} - \mu_2\bar{\lambda} > 0, \tag{10}$$

where

$$\Xi_1 = \bar{A}_{1i}^{\mathrm{T}} P_{1i} + P_{1i}\bar{A}_{1i} + \vartheta_1 P_{1i} + (\beta_i+1)\kappa_i M_i,$$
$$\Xi_2 = A_i P_{2i} + P_{2i} A_i - \vartheta_2 P_{2i} + \beta_i\kappa_i M_i,$$

then the switched Markov jump closed-loop system (4) is stochastically stable under the DoS attacks satisfying (10).

The proof of Theorem 2 is presented in Appendix C.

Note that the triggering condition (2) needs to be continuously detected, which is not easy to implement in practical systems. To overcome this difficulty, we are ready to develop an STS in the following theorem.

Theorem 3. Under the control gain in Theorem 1, if the inter-execution interval is implemented as

$$\begin{aligned} t_{k+1} &= t_k \\ &+ \min\left\{\frac{1}{\varphi_1}\ln\left[1 + \frac{\varphi_1\Psi_1(\min_{i\in S}\{\varsigma_i x^{\mathrm{T}}(t_k)M_i x(t_k)\})^{\frac{1}{2}}}{\varphi_2\|x(t_k)\|}\right.\right. \\ &+ \left.\left.\frac{\varphi_1\Psi_1 \mathrm{e}^{-\frac{\delta}{2}t_k}\mathrm{e}^{-\frac{\delta}{2}\bar{\tau}}\sqrt{\eta(0)}}{\varphi_2\|x(t_k)\|}\right], \bar{\tau}\right\}, \end{aligned} \tag{11}$$

where $\bar{\tau}$ is a positive scalar and given in advance, $\Psi_1 = \frac{1}{\sqrt{2}\Phi_1}$, Φ_1, φ_1 and φ_2 are defined in (5), and ς_i satisfies

$$1 - 2\varsigma_i > 0,\ \frac{2\varsigma_i}{1-2\varsigma_i} \leqslant \kappa_i, \tag{12}$$

then when the condition (10) of the DoS attacks is met, the switched Markov jump closed-loop system (4) is stochastically stable.

The proof of Theorem 3 is provided in Appendix D. In addition, the parameters' design of dynamic ETS and controller gains and a numerical example are shown in Appendixes E and F, respectively.

Conclusion. In this study, we have concerned the problems of resilient event-triggered and self-triggered control for a class of Markov jump systems under DoS attacks. A dynamic ETS with the discussion of Zeno behavior has been proposed to further cut down triggering frequency. By using a full state feedback controller, the switched Markov jump closed-loop system has been constructed to describe the effect of DoS attacks. Via using the piecewise Lyapunov function method, sufficient conditions have been established for guaranteeing stochastic stability of the resulting closed-loop system. Considering that the implementation of the dynamic ETS requires real-time detection of triggering conditions, an STS has been developed. According to the proposed results, there exist many possible directions to be extended, such as Markov jump systems with asynchronous modes, fuzzy systems, and input-saturated systems [7].

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Dong S, Liu M, Wu Z G, et al. Observer-based sliding mode control for Markov jump systems with actuator failures and asynchronous modes. IEEE Trans Circ Syst II, 2020. doi: 10.1109/TCSII.2020.3030703
2 Wang Y, Chang Y, Alkhateeb A F, et al. Adaptive fuzzy output-feedback tracking control for switched nonstrict-feedback nonlinear systems with prescribed performance. Circ Syst Signal Process, 2021, 40: 88–113
3 Cai J, Yu R, Wang B, et al. Decentralized event-triggered control for interconnected systems with unknown disturbances. J Franklin Institute, 2020, 357: 1494–1515
4 Chen Z Y, Han Q-L, Yan Y M, et al. How often should one update control and estimation: review of networked triggering techniques. Sci China Inf Sci, 2020, 63: 150201
5 Xu Y, Fang M, Pan Y J, et al. Event-triggered output synchronization for nonhomogeneous agent systems with periodic denial-of-service attacks. Int J Robust Nonlin Control, 2021, 31: 1851–1865
6 Hu S, Yue D, Cheng Z, et al. Co-design of dynamic event-triggered communication scheme and resilient observer-based control under aperiodic DoS attacks. IEEE Trans Cybern, 2020. doi: 10.1109/TCYB.2020.3001187
7 Wang Q, Wu Z G. Robust output feedback control for input-saturated systems based on a sliding mode observer. Circ Syst Signal Process, 2021, 40: 2267–2281