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• **Supplementary File** •

Resilient dynamic event-triggered and self-triggered control for Markov jump systems under denial-of-service attacks

Pengyu ZENG, Feiqi DENG*, Xiaohua LIU & Hongyang ZHANG

[1]*School of Automation Science and Engineering, South China University of Technology, Guangzhou* 510640*, China*

Appendix A Preliminaries

Considering that adversary's energy to launch DoS attacks is usually limited, we provides the characteristics of DoS attack based on frequency and duration.

Assumption 1. [1](DoS Frequency) Let $N_D(0,t)$ is the number of attack off/on transitions in the interval $[0,t)$. There exist constants $\nu \geqslant 0$ and $\tau_D > 0$ such that

$$N_D(0,t) \leqslant \nu + \tfrac{t}{\tau_D}. \tag{A1}$$

Assumption 2. [1](DoS Duration) Let $\Delta(0,t)$ stand for the total DoS duration in the interval $[0,t)$. There exist constants $\nu_1 \geqslant 0$ and $T > 1$ such that

$$\Delta(0,t) \leqslant \nu_1 + \tfrac{t}{T}. \tag{A2}$$

To gain the main results, the definition and a lemma are first provided.

Definition 1. The resulting switched Markov jump closed-loop system is stochastically stable if

$$E\big[\int_0^\infty x^T(s)x(s)ds|x_0, r_0\big] < \infty. \tag{A3}$$

Lemma 1. [2] Letting $N_r(t)$ be the total jump number of $r(t)$ on the interval $[0,t)$, then it holds that

$$P(N_r(t)=k) \leqslant e^{-\tilde{\lambda}t}\frac{(\bar{\lambda}t)^k}{k!}, \forall k \geqslant 0,$$

where $\tilde{\lambda} = \max\{\lambda_{ij} : i,j \in \Lambda, i \neq j\}$, $\bar{\lambda} = \max\{|\lambda_{ii}| : i \in \Lambda\}$.

Appendix B The Proof of Theorem 1

Considering that there may exist jumps in the inter-execution interval $[t_k, t_{k+1})$, the following two situations are presented to clarify the relationship between jump instants and triggering instants.

Situation 1: Letting $\{t_\rho\}_{\rho\in\mathbb{N}}$ stands for jump instant and supposing that no jump occurs in the triggering interval $[t_k, t_{k+1})$, then it follows that $t_\rho < t_k < t_{k+1} < t_{\rho+1}$. Furthermore, taking the derivative for $\|e_{t_k}(t)\|$ along with the state trajectories of system (1) and control input (3), we get

$$\begin{aligned}
\frac{d\|e_{t_k}(t)\|}{dt} &= \frac{1}{2}\|e_{t_k}(t)\|^{-\frac{1}{2}}[e^T_{t_k}(t)\dot{e}_{t_k}(t) + \dot{e}^T_{t_k}(t)e_{t_k}(t)] \\
&\leqslant \|\dot{e}_{t_k}(t)\| \\
&= \| - A_{r(t_\rho)}x(t) - \bar{B}_{r(t_\rho)}x(t_k)\| \\
&= \|A_{r(t_\rho)}(x(t_k) - x(t)) - (A_{r(t_\rho)} + \bar{B}_{r(t_\rho)})x(t_k)\| \\
&\leqslant \varphi_1\|e_{t_k}(t)\| + \varphi_2\|x(t_k)\|.
\end{aligned} \tag{B1}$$

Letting $\dot{y}(t) = \varphi_1 y + \varphi_2\|x(t_k)\|$ with the initial condition $y(t_k) = e_{t_k}(t_k) = 0$ and using comparison lemma, we get $\|e_{t_k}(t)\| \leqslant y(t)$ for any $t \geqslant t_k$. Meanwhile, according to the definition of $\dot{y}(t)$, we have

$$y(t) = \frac{\varphi_2\|x(t_k)\|}{\varphi_1}[e^{\varphi_1(t-t_k)} - 1]. \tag{B2}$$

Note that $\|x(t_k)\| = \|x(t_k) - x(t) + x(t)\| \leqslant \|e_{t_k}(t)\| + \|x(t)\|$, which combining with (B2), we have

$$\|e_{t_k}(t)\| \leqslant y(t) \leqslant \frac{\varphi_2(\|e_{t_k}(t)\| + \|x(t)\|)}{\varphi_1}[e^{\varphi_1(t-t_k)} - 1]. \tag{B3}$$

* Corresponding author (email: aufdeng@scut.edu.cn)

Recalling the definition of Φ_1 and Φ_2 in (5), we have

$$e^T_{t_k}(t)M_{r(t_q)}e_{t_k}(t) \leqslant \Phi_1^2\|e_{t_k}(t)\|^2, \Phi_2^2\|x(t)\|^2 \leqslant \kappa_{r(t_q)}x^T(t)M_{r(t_q)}x(t). \tag{B4}$$

Hence, the triggering inequality $e^T_{t_k}(t)M_{r(t_q)}e_{t_k}(t) \leqslant \kappa_{r(t_q)}x^T(t)M_{r(t_q)}x(t)$ can be ensured by

$$\|e_{t_k}(t)\| \leqslant \Psi\|x(t)\|. \tag{B5}$$

In view of this result, combining (B3) and (B5), the lower bound (5) can be obtained with $\tau = t - t_k$.

Situation 2: In this sequel, we assume that some jumps occur in the triggering interval $[t_k, t_{k+1})$, that is $t_\rho \leqslant t_k < t_{\rho+1} < \cdots < t_{\rho+m} \leqslant t_{k+1}$, where $m \in \mathbb{N}$.

As to this situation, we divide the triggering interval $[t_k, t_{k+1})$ into some subintervals according to jump instants. In the first subinterval $[t_k, t_{\rho+1})$, it is clear that the inequality $\frac{d\|e_{t_k}(t)\|}{dt} \leqslant \varphi_1\|e_{t_k}(t)\| + \varphi_2\|x(t_k)\|$ is still satisfied. Recalling the definition of $\dot{y}(t)$ in Situation 1, we also have $\|e_{t_k}(t)\| \leqslant y(t)$ by using comparison lemma, which further implies that $\|e_{t_k}(t^-_{\rho+1})\| \leqslant y(t^-_{\rho+1})$. Considering that jumps do not change the continuity of system state and no sampling data is updated, we obtain $\|e_{t_k}(t^-_{\rho+1})\| = \|e_{t_k}(t_{\rho+1})\|$. Letting $y(t_{q+1}) = y(t^-_{q+1})$, then the initial condition $\|e_{t_k}(t_{\rho+1})\| \leqslant y(t_{q+1})$ is provided. Note that $\frac{d\|e_{t_k}(t)\|}{dt} \leqslant \varphi_1\|e_{t_k}(t)\| + \varphi_2\|x(t_k)\|$ holds on the subsequent interval $[t_{\rho+1}, t_{\rho+2})$. Hence, using comparison lemma again, we get $\|e_{t_k}(t)\| \leqslant y(t)$ with $\|e_{t_k}(t^-_{\rho+2})\| \leqslant y(t^-_{\rho+2})$. Furthermore, similar to the analysis on the interval $[t_{\rho+1}, t_{\rho+2})$, the result $\|e_{t_k}(t)\| \leqslant y(t)$ can be obtained on the subintervals $[t_{\rho+2}, t_{\rho+3}), \cdots, [t_{\rho+m}, t_{k+1})$. Therefore, the inequality $|e_{t_k}(t)| \leqslant y(t)$ is also guaranteed in Situation 2, which combining with (B2)-(B5) further provides the lower bound τ in (5).

In conclusion, the estimation τ in (5) is able to be a lower bound of inter-execution intervals and Zeno behavior is completely excluded.

Appendix C The Proof of Theorem 2

Since there exist DoS attacks and jumps in the switched Markov jump closed-loop system (4), we are ready to choose different Lyapunov function to analyze system performance. First, considering that the closed-loop system (4) in the absence of DoS attacks, we provides the following Lyapunov function

$$V(t) = V_{1r(t)}(x(t), \eta(t)) = x^T(t)P_{1r(t)}x(t) + \eta(t). \tag{C1}$$

Considering that there is no DoS attacks, parameter $\varsigma(t) = 1$ and the corresponding system parameter matrices $\bar{A}_{1r(t)}$ and $\bar{B}_{1r(t)}$ appear. Then for each subsystem $i \in S$, combining closed-loop system (4) and variable $\eta(t)$ in (2), we obtain

$$\begin{aligned}\dot{V}_{1i}(x(t), \eta(t)) = {} & x^T(t)(\bar{A}^T_{1i}P_{1i} + P_{1i}\bar{A}_{1i})x(t) + e^T_{t_k}(t)\bar{B}^T_{1i}P_{1i}x(t) \\ & + x^T(t)P_{1i}\bar{B}_{1i}e_{t_k}(t) - \alpha_i\eta(t) + \beta_i[\kappa_i x^T(t)M_i x(t) \\ & - e^T_{t_k}(t)M_i e_{t_k}(t)].\end{aligned} \tag{C2}$$

Moreover, from dynamic ETS (2) and (C2), we have

$$\begin{aligned}\dot{V}_{1i}(x(t), \eta(t)) + \vartheta_1 V_{1i}(x(t), \eta(t)) \leqslant {} & x^T(t)[\bar{A}^T_{1i}P_{1i} + P_{1i}\bar{A}_{1i} + \vartheta_1 P_{1i} + (\beta_i + 1)\kappa_i M_i]x(t) \\ & + e^T_{t_k}(t)\bar{B}^T_{1i}P_{1i}x(t) + x^T(t)P_{1i}\bar{B}_{1i}e_{t_k}(t) \\ & - (\alpha_i - \vartheta_1 - \theta_i)\eta(t) - (\beta_i + 1)e^T_{t_k}(t)M_i e_{t_k}(t)].\end{aligned} \tag{C3}$$

Combining matrix inequality (6) and condition (C3), we get

$$\dot{V}_{1i}(x(t), \eta(t)) \leqslant -\vartheta_1 V_{1i}(x(t), \eta(t)). \tag{C4}$$

On the other hand, when the switched Markov jump closed-loop system (4) in the presence of DoS attacks, the corresponding Lyapunov function is chosen as

$$V(t) = V_{2r(t)}(x(t), \eta(t)) = x^T(t)P_{2r(t)}x(t) + \eta(t). \tag{C5}$$

It can be observed that parameter $\varsigma(t) = 0$ is obtained when there exist DoS attacks. Then taking the derivative of $V(t)$ along with the system (4) and dynamic ETS (2), we have

$$\begin{aligned}\dot{V}_{2i}(x(t), \eta(t)) = {} & x^T(t)(A_i P_{2i} + P_{2i}A_i)x(t) - \alpha_i\eta(t) \\ & + \beta_i[\kappa_i x^T(t)M_i x(t) - e^T_{t_k}(t)M_i e_{t_k}(t)].\end{aligned} \tag{C6}$$

Then from the equation (C6), we obtain

$$\begin{aligned}\dot{V}_{2i}(x(t), \eta(t)) - \vartheta_2 V_{2i}(x(t), \eta(t)) = {} & x^T(t)(A_i P_{2i} + P_{2i}A_i - \vartheta_2 P_{2i})x(t) \\ & - (\alpha_i + \vartheta_2)\eta(t) + \beta_i[\kappa_i x^T(t)M_i x(t) \\ & - e^T_{t_k}(t)M_i e_{t_k}(t)].\end{aligned} \tag{C7}$$

In the light of matrix inequality (7) and condition (C7), we have

$$\dot{V}_{2i}(x(t), \eta(t)) \leqslant \vartheta_2 V_{2i}(x(t), \eta(t)). \tag{C8}$$

Note that inequalities (C4) and (C8) respectively reflect the dynamic performance of the system under attacks and under no attacks. Next, we are in the position to give the relationship between $V(t)$ and $V(0)$. First, due to parameter $\mu_1 > 1$, the results $V_{1i}(x(t),\eta(t)) \leqslant \mu_1 V_{2i}(x(t),\eta(t))$ and $V_{2i}(x(t),\eta(t)) \leqslant \mu_1 V_{1i}(x(t),\eta(t))$ are obtained from (8). In addition, parameter $\mu_2 > 1$ and condition (9) lead to $V_{1i}(x(t),\eta(t)) \leqslant \mu_2 V_{1j}(x(t),\eta(t))$ and $V_{2i}(x(t),\eta(t)) \leqslant \mu_2 V_{2j}(x(t),\eta(t))$ for any $i,j \in S$.

Based on these results, we continue to estimate the Lyapunov function $V(t)$. Also, for any $t \in [0,\infty)$, there are two situations because of DoS attacks. When the situation that the instant t is in the presence of DoS attacks is considered, we suppose that $T_n^{on} \leqslant t_\rho \leqslant t < T_n^{off}$ and $\Delta_1(0,t)$ is the total duration where there is no DoS attacks. Then, it follows from (C4) and (C8) that

$$\begin{aligned}
V(t) &\leqslant e^{\vartheta_2(t-t_\rho)} V_{2r(t_\rho)}(x(t_\rho),\eta(t_\rho)) \\
&\leqslant \mu_2 e^{\vartheta_2(t-t_\rho)} V_{2r(t_\rho^-)}(x(t_\rho^-),\eta(t_\rho^-)) \\
&\;\;\vdots \\
&\leqslant \mu_2^{N_r(T_n^{on},t)} e^{\vartheta_2(t-T_n^{on})} V_{2r(T_n^{on})}(x(T_n^{on}),\eta(T_n^{on})) \\
&\leqslant \mu_1 \mu_2^{N_r(T_n^{on},t)} e^{\vartheta_2(t-T_n^{on})} V_{1r(T_n^{on-})}(x(T_n^{on-}),\eta(T_n^{on-})) \\
&\;\;\vdots \\
&\leqslant \mu_1 \mu_2^{N_r(T_{n-1},t)} e^{\vartheta_2(t-T_n^{on})-\vartheta_1(T_n^{on}-T_{n-1}^{off})} V_{1r(T_{n-1}^{off})}(x(T_{n-1}^{off}),\eta(T_{n-1}^{off})) \\
&\leqslant \mu_1^2 \mu_2^{N_r(T_{n-1},t)} e^{\vartheta_2\Pi_n-\vartheta_1(T-\Pi_n)} V_{2r(T_n^{off-})}(x(T_n^{off-}),\eta(T_n^{off-})) \\
&\;\;\vdots \\
&\leqslant \mu_1^{N_D(0,t)} \mu_2^{N_r(0,t)} e^{\vartheta_2\Delta(0,t)-\vartheta_1\Delta_1(0,t)} V(0).
\end{aligned} \tag{C9}$$

Considering the situation that the instant t is in the absence of DoS attacks, it is assumed that $T_{n-1} \leqslant t_\rho \leqslant t < T_n^{on} < T_n$ and we have the same result as that in (C9). The detail is omitted here.

Applying Lemma 1 to (C9), we have

$$\begin{aligned}
EV(t) &\leqslant \mu_1^{N_D(0,t)} e^{\vartheta_2\Delta(0,t)-\vartheta_1\Delta_1(0,t)} E(\mu_2^{N_r(0,t)}) V(0) \\
&= \mu_1^{N_D(0,t)} e^{\vartheta_2\Delta(0,t)-\vartheta_1\Delta_1(0,t)} \sum_{k=0}^{\infty} \mu_2^k P(N_r(t)=k) V(0) \\
&\leqslant \mu_1^{N_D(0,t)} e^{\vartheta_2\Delta(0,t)-\vartheta_1\Delta_1(0,t)} \sum_{k=0}^{\infty} \mu_2^k e^{-\bar{\lambda}t} \frac{(\bar{\lambda}t)^k}{k!} V(0) \\
&\leqslant \mu_1^{N_D(0,t)} e^{\vartheta_2\Delta(0,t)-\vartheta_1\Delta_1(0,t)} e^{-(\bar{\lambda}-\mu_2\bar{\lambda})t} V(0).
\end{aligned} \tag{C10}$$

In addition, the condition in (10) and DoS duration in Assumption 2 give that

$$\vartheta_2\Delta(0,t) - \vartheta_1\Delta_1(0,t) \leqslant \vartheta_2(\nu_1 + \frac{t}{T}) - \vartheta_1(t - \nu_1 - \frac{t}{T}) \leqslant -\vartheta_3 t + (\vartheta_1+\vartheta_2)\nu_1. \tag{C11}$$

Combining Assumption 1 and (C10)-(C11), we obtain

$$EV(t) \leqslant e^{(\vartheta_1+\vartheta_2)\nu_1 + \ln\mu_1 \nu} e^{-(\vartheta_3 - \frac{\ln\mu_1}{\tau_D} + \tilde{\lambda} - \mu_2\bar{\lambda})t} V(0). \tag{C12}$$

Besides, from the definition of Lyapunov function $V(t)$ in (C1) and (C5), we get

$$cE\|x(t)\|^2 \leqslant EV(t), V(0) \leqslant d\|x(0)\|^2 + \eta(0), \tag{C13}$$

where $c = \min\left\{\min_{i\in S}\{\lambda_{\min}(P_{1i})\}, \min_{i\in S}\{\lambda_{\min}(P_{2i})\}\right\}$, $d = \max\left\{\max_{i\in S}\{\lambda_{\max}(P_{1i})\}, \max_{i\in S}\{\lambda_{\max}(P_{2i})\}\right\}$, $\lambda_{\max}(\cdot)$ and $\lambda_{\max}(\cdot)$ respectively denote minimum and maximum eigenvalues. Combining (C12) and (C13), we get

$$E\|x(t)\|^2 \leqslant \frac{1}{c} e^{(\vartheta_1+\vartheta_2)\nu_1 + \ln\mu_1\nu} e^{-(\vartheta_3 - \frac{\ln\mu_1}{\tau_D} + \tilde{\lambda} - \mu_2\bar{\lambda})t} (d\|x(0)\|^2 + \eta(0)). \tag{C14}$$

Moreover, it can be obtained from (10) that $\vartheta_3 - \frac{\ln\mu_1}{\tau_D} + \tilde{\lambda} - \mu_2\bar{\lambda} > 0$. Then, integrating both sides of (C14) from 0 to ∞, we have the result (A3) and the stochastic stability of the switched Markov jump closed-loop system (4) is ensured. This completes the proof.

Appendix D The Proof of Theorem 3

Due to the triggering interval of STS is based on the last sampling information, we first use $x(t_k)$ to construct the following triggering inequality

$$e_{t_k}^T(t) M_{r(t)} e_{t_k}(t) \leqslant \varsigma_{r(t)} x^T(t_k) M_{r(t)} x(t_k) + \theta_{r(t)}\eta(t). \tag{D1}$$

Substituting the equation $x(t_k) = x(t) + e_{t_k}(t)$ into (D1) gives

$$e_{t_k}^T(t) M_{r(t)} e_{t_k}(t) \leqslant \alpha_{r(t)} (x(t) + e_{t_k}(t))^T M_{r(t)} (x(t) + e_{t_k}(t)) + \theta_{r(t)}\eta(t)$$

$$\leqslant 2\alpha_{r(t)}x^T(t)M_{r(t)}x(t) + 2\varsigma_{r(t)}e_{t_k}^T(t)M_{r(t)}e_{t_k} + \theta_{r(t)}\eta(t). \tag{D2}$$

In the light of condition (D2), we can observer that the triggering inequality $e_{t_k}^T(t)M_{r(t)}e_{t_k}(t) \leqslant \kappa_{r(t)}x^T(t)M_{r(t)}x(t)+\theta_{r(t)}\eta(t)$ can be ensured by (D1).

Meanwhile, via the definition of Φ_1 in (5) and the fact $\eta(t) \geqslant e^{-\delta t}\eta(0)$, we get

$$e_{t_k}^T(t)M_{r(t)}e_{t_k}(t) \leqslant \Phi_1^2\|e_{t_k}(t)\|^2,$$
$$\min_{i\in S}\{\kappa_i x^T(t_k)M_i x(t_k)\} + e^{-\delta t}\eta(0) \leqslant \varsigma_{r(t)}x^T(t_k)M_{r(t)}x(t_k) + \theta_{r(t)}\eta(t). \tag{D3}$$

By using the inequality $(a+b)^2 \leqslant 2a^2+2b^2$, we obtain that $\Phi_1^2\|e_{t_k}(t)\|^2 \leqslant \min_{i\in S}\{\varsigma_i x^T(t_k)M_i x(t_k)\} + e^{-\delta t}\eta(0)$ is guaranteed by the following inequality

$$\|e_{t_k}(t)\| \leqslant \Psi_1(\min_{i\in S}\{\varsigma_i x^T(t_k)M_i x(t_k)\})^{\frac{1}{2}} + \Psi_1 e^{-\frac{\delta}{2}t}\sqrt{\eta(0)}. \tag{D4}$$

which combining with (D3) indicates that (D4) is able to ensure the triggering inequality $e_{t_k}^T(t)M_{r(t)}e_{t_k}(t) \leqslant \kappa_{r(t)}x^T(t_k)M_{r(t)}x(t_k)+\theta_{r(t)}\eta(t)$.

On the other hand, from the condition (D1) and the fact $e_{t_k}(t) \leqslant y(t)$ in the proof of Theorem 1, we have

$$\|e_{t_k}(t)\| \leqslant \frac{\varphi_2\|x(t_k)\|}{\varphi_1}[e^{\varphi_1(t-t_k)}-1]. \tag{D5}$$

In view of the results (D4) and (D5), we are ready to provide a triggering interval $\tau_k = t_{k+1}-t_k$ such that

$$\frac{\varphi_2\|x(t_k)\|}{\varphi_1}[e^{\varphi_1(t_{k+1}-t_k)}-1] \leqslant \Psi_1(\min_{i\in S}\{\varsigma_i x^T(t_k)M_i x(t_k)\})^{\frac{1}{2}} + \Psi_1 e^{-\frac{\delta}{2}t_k}e^{-\frac{\delta}{2}(t_{k+1}-t_k)}\sqrt{\eta(0)}. \tag{D6}$$

By setting the upper bound $\bar{\tau}$, the STS (11) is derived by (D6). This completes the proof.

Appendix E Parameter design

For any $i,j\in S$, given a set of positive constants τ_D, α_i, β_i, κ_i, θ_i, $\tilde{\lambda}$, $\bar{\lambda}$, ϵ_1, ϵ_2, $T>1$, $\mu_\iota > 1(\iota\in\{1,2\})$ and $\vartheta_l(l\in\{1,2,3\})$, if there exist matrices $X_{1i}>0$, $X_{2i}>0$, $\check{M}_i>0$ and Y_i such that

$$\begin{bmatrix} X_{1i}A_i^T + A_iX_{1i} + B_iY_i + Y_i^TB_i^T + \vartheta_1X_{1i} & B_iY_i & 0 & X_{1i} \\ * & (\beta_i+1)(\epsilon_1^2\check{M}_i - 2\epsilon_1X_{1i}) & 0 & 0 \\ * & * & -(\alpha_i-\vartheta_1-\theta_i)I & 0 \\ * & * & * & -\frac{\check{M}_i}{(\beta_i+1)\kappa_i} \end{bmatrix} < 0, \tag{E1}$$

$$\begin{bmatrix} X_{2i}A_i^T + A_iX_{2i} - \vartheta_2X_{2i} & 0 & 0 & X_{2i} \\ * & \beta_i(\epsilon_2^2\check{M}_i - 2\epsilon_2X_{2i}) & 0 & 0 \\ * & * & -(\alpha_i+\vartheta_2)I & 0 \\ * & * & * & -\frac{\check{M}_i}{\beta_i\kappa_i} \end{bmatrix} < 0, \tag{E2}$$

$$\begin{bmatrix} -\mu_1X_{2i} & X_{2i} \\ * & -X_{1i} \end{bmatrix} < 0, \begin{bmatrix} -\mu_1X_{1i} & X_{1i} \\ * & -X_{2i} \end{bmatrix} < 0, \tag{E3}$$

$$\begin{bmatrix} -\mu_2X_{1j} & X_{1j} \\ * & -X_{1i} \end{bmatrix} < 0, \begin{bmatrix} -\mu_2X_{2j} & X_{2j} \\ * & -X_{2i} \end{bmatrix} < 0, \tag{E4}$$

then the switched Markov jump closed-loop system (4) is stochastically stable under the DoS attacks satisfying (10). Moreover, the parameters of dynamic ETS and controller gains are provided by $M_i = \check{M}_i^{-1}$ and $K_i = Y_iX_{1i}^{-1}$.

Proof.

Define $X_{1i}^{-1} = P_{1i}$ and $X_{2i}^{-1} = P_{2i}$, then pre- and post-multiplying (6) and (7) by diagonal matrices diag$\{X_{1i}, X_{1i}, I\}$, diag$\{X_{2i}, X_{2i}, I\}$ and their transposes, we get

$$\begin{bmatrix} \Theta_i + (\beta_i+1)\kappa_iX_{1i}M_iX_{1i} & B_iY_i & 0 \\ * & -(\beta_i+1)X_{1i}M_iX_{1i} & 0 \\ * & * & -(\alpha_i-\vartheta_1)I \end{bmatrix} < 0, \tag{E5}$$

$$\begin{bmatrix} X_{2i}A_i^T + A_iX_{2i} - \vartheta_2X_{2i} + \beta_i\kappa_iX_{2i}M_iX_{2i} & 0 & 0 \\ * & -\beta_iX_{2i}M_iX_{2i} & 0 \\ * & * & -(\alpha_i+\vartheta_2)I \end{bmatrix} < 0, \tag{E6}$$

where $\Theta_i = X_{1i}A_i^T + A_iX_{1i} + B_iY_i + Y_i^TB_i^T + \vartheta_1X_{1i}$.

In addition, it appears from inequality $-XMX \leqslant \epsilon^2M^{-1} - 2\epsilon X$ that

$$-X_{1i}M_iX_{1i} \leqslant \epsilon_1^2M_i^{-1} - 2\epsilon_1X_{1i}, -X_{2i}M_iX_{2i} \leqslant \epsilon_2^2M_i^{-1} - 2\epsilon_1X_{2i}. \tag{E7}$$

Letting $\check{M}_i = M_i^{-1}$ and substituting (E7) into (E5) and (E6), it can be derived from Schur complement lemma that (E5) and (E6) are able to be ensured by (E1) and (E2), respectively.

On the other hand, as to inequalities $P_{1i} \leqslant \mu_1 P_{2i}$, $P_{2i} \leqslant \mu_1 P_{1i}$ in (8) and $P_{1i} \leqslant \mu_2 P_{1j}$, $P_{2i} \leqslant \mu_2 P_{2j}$ in (9), respectively pre- and post-multiplying both sides of them by X_{2i}, X_{1i} and X_{1j}, X_{2j}, we get

$$\begin{aligned} &-\mu_1 X_{2i} + X_{2i} P_{1i} X_{2i} \leqslant 0, -\mu_1 X_{1i} + X_{1i} P_{2i} X_{1i} \leqslant 0, \\ &-\mu_2 X_{1j} + X_{1j} P_{1i} X_{1j} \leqslant 0, -\mu_2 X_{2j} + X_{2j} P_{2i} X_{2j} \leqslant 0. \end{aligned} \tag{E8}$$

By using Schur complement lemma again, we can see that (E8) can be guaranteed by matrix inequalities (E3) and (E4). This completes the proof.

Appendix F Numerical example

In this section, a numerical example is presented to evaluate our proposed approach. We consider the Markov jump system of the form (1) with

$$A_1 = \begin{bmatrix} -1.4 & 0 \\ 1 & 0.5 \end{bmatrix}, A_2 = \begin{bmatrix} 0.8 & 1 \\ 0 & -1.5 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

The uncertain transition rates is considered and $\lambda_{ij} \in [0.1, 0.2]$ with $|\lambda_{ii}| \in [0.1, 0.2]$ are set, in which $i, j \in \{1, 2\}$ and $i \neq j$. Then the maximum values $\tilde{\lambda} = \bar{\lambda} = 0.2$ follows. To solve the linear matrix inequalities (E1)-(E4), parameters $\vartheta_1 = 1$, $\vartheta_2 = 2$, $\alpha_1 = \alpha_2 = 1.5$, $\theta_1 = \theta_2 = 0.1$, $\beta_1 = \beta_2 = 1$, $\epsilon_1 = \epsilon_2 = 1$, $\kappa_1 = \kappa_2 = 0.5$ and $\mu_1 = \mu_2 = 3$ are chosen. Furthermore, the parameters of dynamic ETS and controller gains are obtained as

$$\begin{aligned} M_1 &= \begin{bmatrix} 0.0131 & -0.0001 \\ -0.0001 & 0.0088 \end{bmatrix}, M_2 = \begin{bmatrix} 0.0073 & 0.0010 \\ 0.0010 & 0.0158 \end{bmatrix}, \\ K_1 &= \begin{bmatrix} -0.3695 & -0.7013 \end{bmatrix}, K_2 = \begin{bmatrix} -2.3299 & -1.0119 \end{bmatrix}. \end{aligned}$$

Meanwhile, from (5), we can calculate the lower bound of inter-execution intervals as $\tau = 0.0466$. On the basis of Assumptions 1-2, the parameters relating to DoS attacks $\nu = \nu_1 = 4$, $\tau_D = 5$, $T = 8$ and $\vartheta_3 = 0.7$ are set so as to ensure the conditions in (10). In order to implement STS, parameters $\varsigma_1 = \varsigma_2 = 0.1$ are set and upper bound of inter-execution intervals $\bar{\tau} = 5$ is chosen.

Table F1 Comparison of sampling numbers for triggering schemes in [3] and this paper

Triggering schemes	Sampling numbers
Static ETS in [3]	132
Dynamic ETS (2)	30
STS in [3]	251
STS (11)	104

By setting the initial system state $x(0) = [-1 \quad 1]^T$ and initial value $\eta(0) = 5$, the simulation results are presented in Figure F1-Figure F8. The evolution of system state under dynamic ETS and STS are respectively plotted in Figure F1 and Figure F4. Meanwhile, Figure F2 and Figure F3 respectively show the inter-execution interval of dynamic ETS (2) and static ETS in [3]. The inter-execution interval of STS (11) and STS in [3] are depicted in Figure F5 and Figure F6. The comparison of sampling numbers for triggering schemes in [3] and this paper is provided in Table 1, it can be found that the dynamic ETS (2) and the corresponding STS (11) are able to greatly reduce the sampling frequency. In addition, the DoS attack signal and Markov jump signal are respectively described in Figure F7 and Figure F8.

In conclusion, under the proposed control method, the desired performance of Markov jump system (1) subject to DoS attacks is ensured and the sampling frequency can be further declined by using dynamic ETS (2) and STS (11).

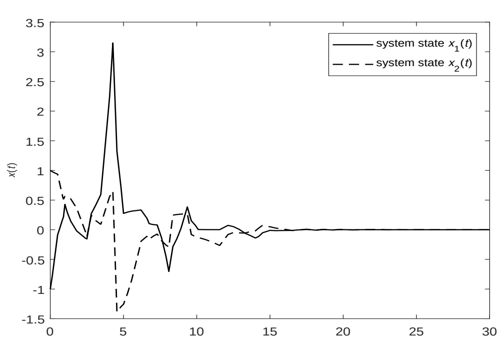

Figure F1 Evolution of system state under dynamic ETS and DoS attacks

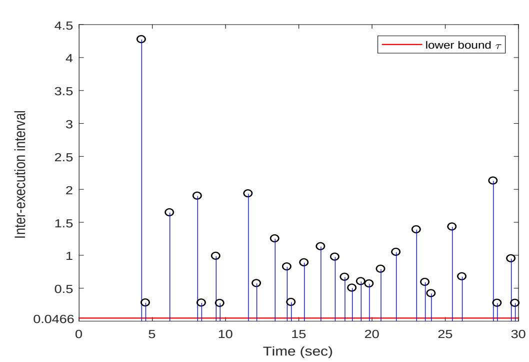

Figure F2 Inter-execution interval of dynamic ETS

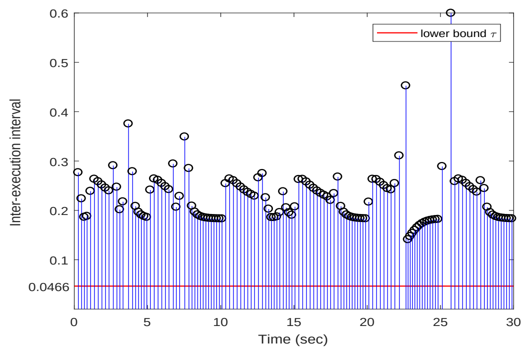

Figure F3 Inter-execution interval of static ETS in [3]

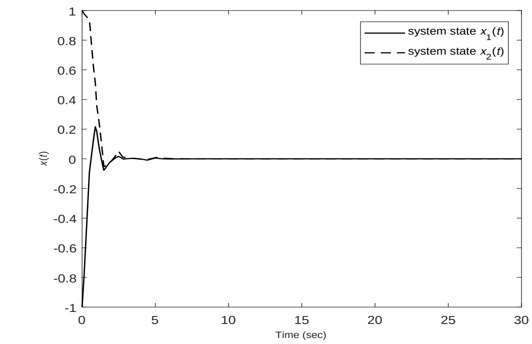

Figure F4 Evolution of system state under STS and DoS attacks

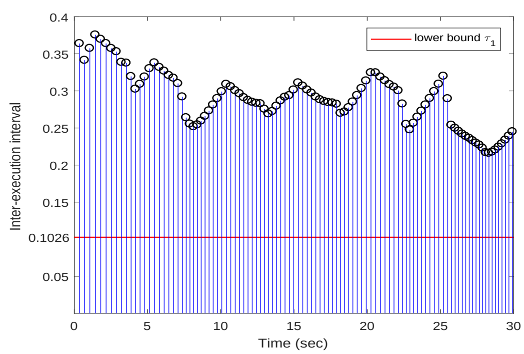

Figure F5 Inter-execution interval of STS (11)

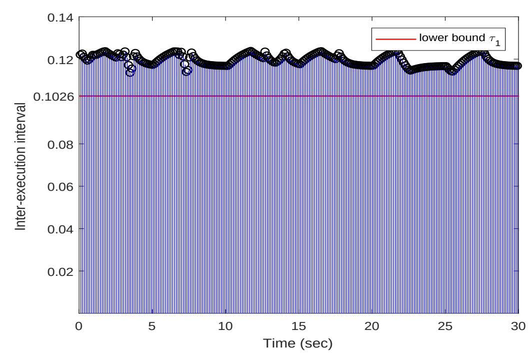

Figure F6 Inter-execution interval of STS in [3]

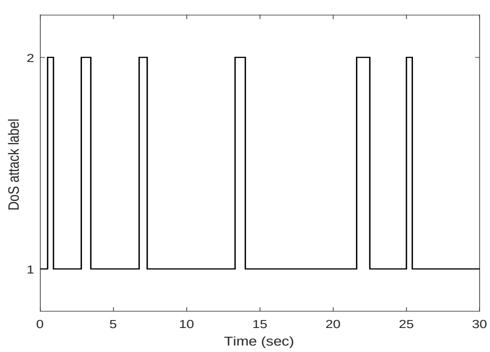

Figure F7 DoS attack signal

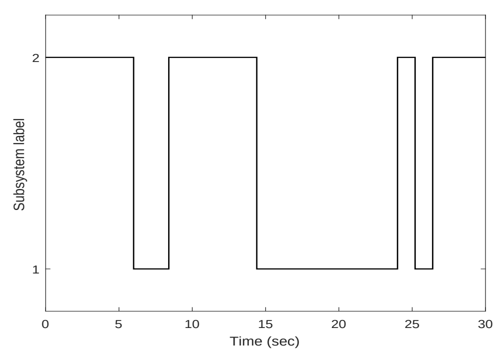

Figure F8 Markov jump signal

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