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Adaptive synchronization control of uncertain multiple USVs with prescribed performance and preserved connectivity

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Dear editor,

Cooperative control of multi-vehicle systems has recently received increasing attention from system and control engineering communities owing to its broad applications, such as coordinated exploration, cooperative carrying and cooperative patrolling [1]. A typical topic in this field is synchronization control, whose objective is to design control protocol for each vehicle using the local interaction with its neighbors to synchronize to the (virtual) leader. The unmanned vehicles are generally equipped with onboard communication devices to exchange information with their neighboring vehicles, while the communication ranges are often limited [2]. It is challenging to design synchronization control approach for each vehicle using the local information to achieve the global goal (e.g., synchronization motion) in the presence of communication range constraints.

This study presents an adaptive synchronization tracking control algorithm for multiple unmanned surface vehicles (USVs) with prescribed performance under a directed communication graph. The USVs are subject to limited communication capability and external time-varying disturbances. The proposed synchronization control algorithm could guarantee that (i) the prescribed transient and steady-state performance of synchronization tracking errors are satisfied, and (ii) each USV synchronizes to the leader while maintaining the initial connectivity among USVs.

USV model. Consider a multi-vehicle system consisting of a leader ϑ_0 and N USVs with limited communication ranges. The trajectory of the leader is generated by a given virtual USV. The mathematical model of the ith USV is [2]

$$\begin{aligned}&\dot{\boldsymbol{\eta}}_i = \boldsymbol{J}(\psi_i)\boldsymbol{\nu}_i,\\&\boldsymbol{M}\dot{\boldsymbol{\nu}}_i = -\boldsymbol{C}(\boldsymbol{\nu}_i)\boldsymbol{\nu}_i - \boldsymbol{D}(\boldsymbol{\nu}_i)\boldsymbol{\nu}_i + \boldsymbol{\tau}_i + \boldsymbol{d}_i\end{aligned} \tag{1}$$

with $i \in \mathcal{V} = \{1,\ldots,N\}$, where $\boldsymbol{\eta}_i = [x_i, y_i, \psi_i]^{\mathrm{T}}$ denotes the vehicle outputs that contain the position (x_i, y_i) and yaw angle ψ_i; $\boldsymbol{\nu}_i = [u_i, v_i, r_i]^{\mathrm{T}}$ is the velocity vector; $\boldsymbol{\tau}_i$ is the control input vector; $\boldsymbol{J}(\psi_i)$ is the rotation matrix; $\boldsymbol{d}_i = [d_{i1}, d_{i2}, d_{i3}]^{\mathrm{T}}$ is the unknown external time-varying disturbance; $\boldsymbol{M}$ is the unknown inertia matrix satisfying $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} > 0$; $\boldsymbol{C}(\boldsymbol{\nu}_i)$ is the uncertain total Coriolis and centripetal acceleration matrix; and $\boldsymbol{D}(\boldsymbol{\nu}_i)$ is the uncertain hydrodynamic damping matrix.

The directed graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ is employed to describe the communication topology of the vehicle group, where $\bar{\mathcal{V}} = \{0,1,\ldots,N\}$ denotes the set of vertices, $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ is the set of edge between two vertices. An edge $(j,i) \in \bar{\mathcal{E}}$ means that the ith USV can receive information from vehicle j but not vice versa. Vehicle i ($i \in \mathcal{V}$) can obtain the information from vehicle j ($j \in \bar{\mathcal{V}}$) if the following inequality

$$\|\boldsymbol{\eta}_{i,L}(t) - \boldsymbol{\eta}_{j,L}(t)\| < \bar{L}_i, \quad i \neq j, \tag{2}$$

holds with $\boldsymbol{\eta}_{j,L} = [x_j, y_j]^{\mathrm{T}}$, where $\bar{L}_i$ denotes the allowed maximal communication radius of vehicle i. A sufficient condition for $(j,i) \in \bar{\mathcal{E}}$ is that the distance between vehicle i and vehicle j satisfies (2). Then, the neighbor set is defined as $\mathcal{N}_i(t) = \{j|(j,i) \in \bar{\mathcal{E}}\}$. If $(j,i) \in \bar{\mathcal{E}}$, then there exists an arrow with tail at j and head at i. The in-degree of $i \in \mathcal{V}$ is the number of edges having i as a head. A path in a directed graph from vertex $i \in \mathcal{V}$ to vertex $j \in \bar{\mathcal{V}}$ is an ordered sequence of distinct edges starting with i and ending with j. A directed graph has a directed spanning tree if at least one vertex has directed paths to all other vertices.

Assumption 1. At least one vehicle in the group can obtain the leader's trajectory $\boldsymbol{\eta}_0$.

Assumption 2. The directed graph $\bar{\mathcal{G}}(t)$ has a spanning tree with in-degree of $i \in \mathcal{V}$ no more than one at initial time.

Assumption 3. The external time-varying disturbance $\boldsymbol{d}_i = [d_{i,1}, d_{i,2}, d_{i,3}]^{\mathrm{T}}$ is bounded, i.e., $|d_{i,l}| \leqslant \bar{d}_{i,l}$ with $\bar{d}_{i,l}$ being unknown positive constants, where $i \in \mathcal{V}$, $l = 1, 2, 3$.

Control objective. The objective of this study is to design distributed adaptive synchronization tracking control laws $\boldsymbol{\tau}_i$ for system (1) such that (i) each follower can synchronously track the leader's trajectory and perform in a

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desired synchronization shape, (ii) the velocities of each vehicle converge to a small neighbourhood around the leader's velocities, and (iii) the initial connectivity among all vehicles is maintained.

Adaptive synchronization control design. Define the synchronization tracking errors:

$$\boldsymbol{p}_{ij} = \boldsymbol{\eta}_i - \boldsymbol{\eta}_j + \boldsymbol{c}_{ij}, \quad i = 1, \ldots, N, \ j \in \mathcal{N}_i, \tag{3}$$

where $\boldsymbol{p}_{ij} = [p_{ij,1}, p_{ij,2}, p_{ij,3}]^{\mathrm{T}}$ with $p_{ij,1} = x_i - x_j + c_{ij,1}$, $p_{ij,2} = y_i - y_j + c_{ij,2}$, $p_{ij,3} = \psi_i - \psi_j + c_{ij,3}$, and $\boldsymbol{c}_{ij} = \boldsymbol{c}_i - \boldsymbol{c}_j = [c_{ij,1}, c_{ij,2}, c_{ij,3}]^{\mathrm{T}}$ is the relative desired offset of the ith vehicle relative to the jth vehicle with $\boldsymbol{c}_i = [c_{i1}, c_{i2}, c_{i3}]^{\mathrm{T}}$ denoting the desired offset of the ith vehicle to the leader. Let $\xi_{ij,l} = p_{ij,l}/\rho_{ij,l}$, where $\rho_{ij,l}$ denote the performance functions. Consider the following barrier functions [3]:

$$e_{ij,l} = \ln \frac{1 + \xi_{ij,l}}{1 - \xi_{ij,l}}, \qquad l = 1, 2, 3 \tag{4}$$

with $\boldsymbol{e}_{ij} = [e_{ij,1}, e_{ij,2}, e_{ij,3}]^{\mathrm{T}}$, where $\rho_{ij,l}$ are chosen as

$$\rho_{ij,l} = (\rho_{ij,l,0} - \rho_{ij,l,\infty})\mathrm{e}^{-k_{ij,l}t} + \rho_{ij,l,\infty} \tag{5}$$

with initial values satisfying $\rho_{ij,1,0} = \min\{\frac{\bar{L}_i}{\sqrt{2}} - c_{ij,1}, \frac{\bar{L}_i}{\sqrt{2}} + c_{ij,1}\}$, $\rho_{ij,2,0} = \min\{\frac{\bar{L}_i}{\sqrt{2}} - c_{ij,2}, \frac{\bar{L}_i}{\sqrt{2}} + c_{ij,2}\}$, $\rho_{ij,3,0} = \min\{\frac{\pi}{2} - c_{ij,3}, \frac{\pi}{2} + c_{ij,3}\}$, and the design parameters satisfying $|c_{ij,1}| < \frac{\bar{L}_i}{2\sqrt{2}}$, $|c_{ij,2}| < \frac{\bar{L}_i}{2\sqrt{2}}$, $|c_{ij,3}| < \frac{\pi}{4}$. Define the local neighborhood error vector:

$$\boldsymbol{s}_i = \sum_{j=0}^{N} a_{ij}\boldsymbol{q}_{ij} = \sum_{j=1}^{N} a_{ij}\boldsymbol{q}_{ij} + a_{i0}\boldsymbol{q}_{i0}, \tag{6}$$

where $\boldsymbol{s}_i = [s_{i,1}, s_{i,2}, s_{i,3}]^{\mathrm{T}}$, $\boldsymbol{q}_{ij} = [e^2_{ij,1}, e^2_{ij,2}, e^2_{ij,3}]^{\mathrm{T}}$, and a_{ij} are defined by $a_{ij} = 1$ if $j \in \mathcal{N}_i(0)$ and $j \in \mathcal{N}_i(t)$, otherwise $a_{ij} = 0$.

Step 1. From (3)–(5), we have

$$\dot{e}_{ij,l} = 2\dot{\xi}_{ij,l}/(1 - \xi^2_{ij,l}), \tag{7}$$

$$\dot{\boldsymbol{\xi}}_{ij} = \boldsymbol{\rho}^{-1}_{ij}(\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{\eta}}_j - \dot{\boldsymbol{\rho}}_{ij}\boldsymbol{\xi}_{ij}), \tag{8}$$

where $\boldsymbol{\xi}_{ij} = [\xi_{ij,1}, \xi_{ij,2}, \xi_{ij,3}]^{\mathrm{T}}$. Let $\Pi_{ij,l} = 1/(1 - \xi^2_{ij,l})$, $\boldsymbol{\Pi}_{ij} = \mathrm{diag}[\Pi_{ij,1}, \Pi_{ij,2}, \Pi_{ij,3}]$, $\boldsymbol{E}_{ij} = \mathrm{diag}[e_{ij,1}, e_{ij,2}, e_{ij,3}]$, $\boldsymbol{\rho}_{ij} = \mathrm{diag}[\rho_{ij,1}, \rho_{ij,2}, \rho_{ij,3}]$. Using (6)–(8), we have

$$\dot{\boldsymbol{s}}_i = 2(\boldsymbol{E}^{\mathrm{T}}_i \boldsymbol{F}_i \dot{\boldsymbol{\eta}}_i - \boldsymbol{E}^{\mathrm{T}}_i \boldsymbol{\Pi}_i \boldsymbol{\rho}^{-1}_i \boldsymbol{Q}_i), \tag{9}$$

where $\boldsymbol{E}_i = [\boldsymbol{E}_{in_{i,1}}, \ldots, \boldsymbol{E}_{in_{i,j}}]^{\mathrm{T}}$, $\boldsymbol{F}_i = [\boldsymbol{\Pi}_{in_{i,1}}\boldsymbol{\rho}^{-1}_{in_{i,1}}, \ldots, \boldsymbol{\Pi}_{in_{i,j}}\boldsymbol{\rho}^{-1}_{in_{i,j}}]^{\mathrm{T}}$, $\boldsymbol{\Pi}_i = \mathrm{diag}[\boldsymbol{\Pi}_{in_{i,1}}, \ldots, \boldsymbol{\Pi}_{in_{i,j}}]$, $\boldsymbol{\rho}^{-1}_i = \mathrm{diag}[\boldsymbol{\rho}^{-1}_{in_{i,1}}, \ldots, \boldsymbol{\rho}^{-1}_{in_{i,j}}]$, $\boldsymbol{Q}_i = [[\dot{\boldsymbol{\eta}}_{n_{i,1}} + \dot{\boldsymbol{\rho}}_{in_{i,1}}\boldsymbol{\xi}_{in_{i,1}}]^{\mathrm{T}}, \ldots, [\dot{\boldsymbol{\eta}}_{n_{i,j}} + \dot{\boldsymbol{\rho}}_{in_{i,j}}\boldsymbol{\xi}_{in_{i,j}}]^{\mathrm{T}}]^{\mathrm{T}}$, in which $n_{i,1}, \ldots, n_{i,j}$ denote the elements of the neighbor set $\mathcal{N}_i(t)$. Define the error variables:

$$\boldsymbol{z}_i = \boldsymbol{\nu}_i - \bar{\boldsymbol{\alpha}}_i, \tag{10}$$

$$\boldsymbol{e}_{\alpha i} = \bar{\boldsymbol{\alpha}}_i - \boldsymbol{\alpha}_i, \tag{11}$$

where $\boldsymbol{z}_i = [z_{i,1}, z_{i,2}, z_{i,3}]^{\mathrm{T}}$, $\boldsymbol{e}_{\alpha i} = [e_{\alpha i,1}, e_{\alpha i,2}, e_{\alpha i,3}]^{\mathrm{T}}$, $\boldsymbol{\alpha}_i = [\alpha_{i,1}, \alpha_{i,2}, \alpha_{i,3}]^{\mathrm{T}}$ is the virtual control input, and $\bar{\boldsymbol{\alpha}}_i = [\bar{\alpha}_{i,1}, \bar{\alpha}_{i,2}, \bar{\alpha}_{i,3}]^{\mathrm{T}}$ is the filtered virtual control input.

Under Assumption 2, the neighbor set satisfies $|\mathcal{N}_i| = 1$ with $|\mathcal{N}_i|$ being the number of element of $\mathcal{N}_i$, which ensures that $\boldsymbol{F}_i$ in (9) is invertible. Then, $\boldsymbol{\alpha}_i$ is designed as

$$\boldsymbol{\alpha}_i = \boldsymbol{J}(\psi_i)^{-1}\boldsymbol{F}^{-1}_i(-\boldsymbol{k}_{1i}\boldsymbol{E}_i\boldsymbol{1}_3 + \boldsymbol{\Pi}_i\boldsymbol{\rho}^{-1}_i\boldsymbol{Q}_i), \tag{12}$$

where $\boldsymbol{k}_{1i}$ is a positive design parameter and $\boldsymbol{1}_3 = [1, 1, 1]^{\mathrm{T}}$. The dynamic surface control technique is introduced by

$$\boldsymbol{\mu}_i\dot{\bar{\boldsymbol{\alpha}}}_i + \bar{\boldsymbol{\alpha}}_i = \boldsymbol{\alpha}_{f,i} \tag{13}$$

with $\bar{\boldsymbol{\alpha}}_i(0) = \boldsymbol{\alpha}_{f,i}(0)$ and $\boldsymbol{\alpha}_{f,i} = \boldsymbol{\alpha}_i - \boldsymbol{\mu}_i\boldsymbol{J}^{\mathrm{T}}(\psi_i)\boldsymbol{F}^{\mathrm{T}}_i\boldsymbol{E}_i\boldsymbol{s}_i$, where $\boldsymbol{\mu}_i = \mathrm{diag}[\mu_{i,1}, \mu_{i,2}, \mu_{i,3}]$ are filter time constants.

Step 2. Differentiating the error $\boldsymbol{z}_i$ in (10) along (1), (11), (13) and premultiplying $\boldsymbol{M}$ yields

$$\boldsymbol{M}\dot{\boldsymbol{z}}_i = -\boldsymbol{\Phi}_i\boldsymbol{m} - \boldsymbol{D}(\boldsymbol{\nu}_i)\boldsymbol{\nu}_i + \boldsymbol{d}_i + \boldsymbol{\tau}_i, \tag{14}$$

where $\boldsymbol{\Phi}_i$ is a known matrix, $\boldsymbol{m}$ is an unknown parameter vector, $\boldsymbol{D}(\boldsymbol{\nu}_i)\boldsymbol{\nu}_i$ is the vector of hydrodynamic damping dynamics, and $\boldsymbol{d}_i$ is the vector of unknown disturbance. The distributed connectivity-preserving synchronization control laws $\boldsymbol{\tau}_i$ can be taken as

$$\boldsymbol{\tau}_i = -\boldsymbol{k}_{2i}\boldsymbol{z}_i - \boldsymbol{J}^{\mathrm{T}}(\psi_i)\boldsymbol{F}^{\mathrm{T}}_i\boldsymbol{E}_i\boldsymbol{s}_i + \boldsymbol{\Phi}_i\hat{\boldsymbol{m}}_i + \hat{\boldsymbol{W}}^{\mathrm{T}}_i\boldsymbol{S}_i(\boldsymbol{Z}_i) - \hat{\bar{\boldsymbol{d}}}_i \tag{15}$$

with

$$\dot{\hat{\boldsymbol{m}}}_i = \boldsymbol{\Gamma}_m(-\boldsymbol{\Phi}^{\mathrm{T}}_i\boldsymbol{z}_i - \sigma_m\hat{\boldsymbol{m}}_i - \beta_m\boldsymbol{\Psi}_{m,i}), \tag{16}$$

$$\dot{\hat{\boldsymbol{W}}}_{i,l} = \boldsymbol{\Gamma}_{i,l}(-\boldsymbol{S}_{i,l}(\boldsymbol{Z}_i)z_{i,l} - \sigma_{i,l}|z_{i,l}|\hat{\boldsymbol{W}}_{i,l}), \tag{17}$$

$$\dot{\hat{\bar{d}}}_{i,l} = \gamma_{di,l}(|z_{i,l}| - \sigma_{di,l}\hat{\bar{d}}_{i,l}), \ l = 1, 2, 3, \tag{18}$$

where $\boldsymbol{\Psi}_{m,i} = \sum_{j=1}^{N} a_{ij}(\hat{\boldsymbol{m}}_i - \hat{\boldsymbol{m}}_j)$; $\boldsymbol{\Gamma}_m = \boldsymbol{\Gamma}^{\mathrm{T}}_m$, $\boldsymbol{\Gamma}_{i,l} = \boldsymbol{\Gamma}^{\mathrm{T}}_{i,l}$, $\boldsymbol{k}_{2i}$, $\gamma_{di,l}$, $\sigma_{i,l}$, σ_m, and $\sigma_{di,l}$ are positive design parameters; β_m is a nonnegative design parameter; $\hat{\boldsymbol{m}}_i$ is the estimate of $\boldsymbol{m}$; $\hat{\boldsymbol{W}}^{\mathrm{T}}_i\boldsymbol{S}_i(\boldsymbol{Z}_i)$ is the estimate of $\boldsymbol{D}(\boldsymbol{\nu}_i)\boldsymbol{\nu}_i$ [2]; and the estimate of the upper of the external disturbance is $\hat{\bar{\boldsymbol{d}}}_i = [\hat{\bar{d}}_{i,1}\tanh(\frac{z_{i,1}\hat{\bar{d}}_{i,1}}{\zeta}), \hat{\bar{d}}_{i,2}\tanh(\frac{z_{i,2}\hat{\bar{d}}_{i,2}}{\zeta}), \hat{\bar{d}}_{i,3}\tanh(\frac{z_{i,3}\hat{\bar{d}}_{i,3}}{\zeta})]^{\mathrm{T}}$ with ζ being a positive design parameter.

Theorem 1. Under Assumptions 1–3, consider the networked uncertain USVs (1) with synchronization tracking control laws (15) and adaptation laws (16)–(18). Then, we have that (i) each USV synchronizes to the leader with bounded residual error, and the velocities of each vehicle converge to a small neighbourhood around the leader's velocities; and (ii) the initial connectivity among USVs is preserved.

The proof of Theorem 1 can be found in Appendix A.

Conclusion. This study has proposed a connectivity-preserving synchronization tracking control algorithm for multiple uncertain USVs with prescribed performance under time-varying disturbances and limited communication ranges. A simulation is performed as shown in Appendix B, demonstrating the effectiveness of the proposed method.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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