SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

September 2022, Vol. 65 192203:1–192203:18
https://doi.org/10.1007/s11432-020-3223-6

# Multisensor fusion estimation of nonlinear systems with intermittent observations and heavy-tailed noises

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Received 6 November 2020/Revised 13 January 2021/Accepted 26 February 2021/Published online 29 August 2022

**Abstract** Inspired by the robust student $t$-distribution based nonlinear filter (RSTNF), a student $t$-distribution and unscented transform (UT) based filter for state estimation of heavy-tailed nonlinear dynamic systems, a modified RSTNF for intermittent observations is derived. The fusion estimation for nonlinear multisensor systems with intermittent observations and heavy-tailed measurement and process noises is studied. In this work, the centralized fusion, the sequential fusion, and the naïve distributed fusion algorithms are presented, respectively. Theoretical analysis shows that the presented algorithms are effective, which are the efficient extension of the classical unscented Kalman filter (UKF) or the cubature Kalman filter (CKF) based algorithms with Gaussian noises. Simulation results show that the presented algorithms are effective and feasible.

**Keywords** state fusion estimation, nonlinear systems, heavy-tailed noise, intermittent observations, multivariate $t$-distribution

**Citation** Xiao B, Wu Q M J, Yan L P. Multisensor fusion estimation of nonlinear systems with intermittent observations and heavy-tailed noises. Sci China Inf Sci, 2022, 65(9): 192203, https://doi.org/10.1007/s11432-020-3223-6

## 1 Introduction

Because of the limitation of networks' bandwidth and the influence of unknown disturbances, large deviations appear in real data more frequently than when being modeled by Gaussian distributions. State estimation methods that can handle these "outliers" with a competitive computational complexity of Gaussian models are of interest to many researchers [1, 2].

In the presence of outliers, Kalman filtering (KF) and other least-square-based estimation methods will drastically degrade in estimation performance [1, 3, 4]. Bounded influence M-estimators were used to cope with the effect of outliers for state estimation in [5–8]. Based on the variational Bayesian (VB) technique and strong tracking filter, an adaptive quantized estimation fusion algorithm was presented in [9].

However, the above mentioned techniques are suitable only for linear dynamic systems with uncorrelated corruption. In [10], robust filtering is investigated for linear systems with correlated system and observation noises, where the outlier is explicitly modeled and estimated to reduce its influence on the state estimation. A robust statistical method that intends to remove the negative impact of measurement outliers completely was presented in [11]. However, it is assumed that there is only one outlier-contaminated measurement in each single estimation window. When the algorithm is generalized to the multiple outlier case, the computational time will become very large because of the increased number of combinations for real-time applications.

Multivariate student $t$-distribution is widely used in modeling the heavy-tailed noises, which can accommodate outliers, has low computation complexity, and is easy to implement without the requirement

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of parameter tuning [1, 12]. Therefore, many robust filters were deduced by formulating the noise as multivariate student $t$-distribution and using the VB method to estimate the state [13]. Filtering algorithms for linear systems of heavy-tailed noises were studied on the basis of $t$-distribution in [14–18]. In [2], for a class of heavy-tailed noise linear systems, a robust Kalman filter is presented using Gaussian mixture distributions. The student $t$ based filtering and smoothing of nonlinear systems were studied in [1, 12]. In [19], state estimation of heavy-tailed nonlinear systems was studied, and a fixed-point iterative method was employed to generate the maximum correntropy extended Kalman filter (EKF). In [12], robust student $t$-distribution based nonlinear filter (RSTNF) was derived. Further, in [20], the robust cubature filter based on student $t$-distribution (RSTCF) was proposed. An unscented particle filter for non-Gaussian observation noise was studied using $t$-distribution in [21].

The above mentioned filtering or smoothing algorithms for heavy-tailed systems focused only on the state estimation of single-sensor systems. While in reality, to improve the reliability of the state estimation, multiple sensors are usually employed to observe a target at a time. Multisensor data fusion is a technology that can effectively combine the information from multiple sources to achieve inferences that cannot be generated from any single sensor or source, or whose quality surpasses that of an inference reached from any single source [22]. For multisensors observing a target simultaneously, the fusion estimation problem has been extensively explored in the case of Gaussian noise corruption [23–28]. However, research on multisensor fusion estimation of heavy-tailed systems is inadequate. The centralized batch fusion was studied in [16] for linear systems and [29] for nonlinear heavy-tailed systems. The distributed fusion and the sequential fusion problems were studied for multisensor linear systems with heavy-tailed system and observation noises in [4, 30], respectively. In [31], a robust adaptive distributed fusion algorithm was proposed for nonlinear Gaussian system noise and student's $t$ measurement noise systems using VB methods.

In target tracking, data may be stochastically lost owing to communication problems. In the case where the measurements are assumed to follow Bernoulli distribution, KF for discrete-time intermittent linear systems were studied in [32–34]. Under Gaussian assumption, there are many estimation results for the fusion of intermittent observations. Networked information fusion with packet losses was studied in [35], and an optimal state estimation was achieved. Ref. [36] investigated the optimal filter for systems with random observation losses or delays using an innovation analysis approach. In [37], state estimation for multirate asynchronous multisensor nonlinear systems with data dropout was studied. Ref. [38] investigated the linear minimum mean-square error estimation for Markovian jump linear system subject to unknown Markov chains, multi-channel mode and observation delays, and packet losses. In [39], a non-fragile distributed state estimation algorithm for discrete-time nonlinear systems over sensor networks subjected to denial-of-service attacks was proposed.

In this paper, the state fusion estimation for heavy-tailed systems and measurement noises for nonlinear systems as well as intermittent observations are studied. The RSTNF deduced in [12] is modified, and a filter is derived for handling intermittent observations. Based on the modified RSTNF (MRSTNF), the centralized, the sequential, and the naïve-distributed fusion algorithms for multisensor nonlinear heavy-tailed systems are deduced. The main contributions of this paper are as follows:

(1) A nonlinear filter for heavy-tailed systems with intermittent observations is proposed.

(2) A centralized fusion algorithm for heavy-tailed intermittent nonlinear systems is presented.

(3) A sequential fusion algorithm for heavy-tailed intermittent nonlinear systems is deduced.

(4) A distributed fusion estimation method is given for heavy-tailed multisensor intermittent nonlinear systems.

(5) The proposed algorithms are a generalization of the traditional unscented Kalman filter (UKF)/cubature Kalman filter (CKF)-based algorithms.

The rest of this paper is organized as follows. Section 2 describes the problem. Section 3 derives the MRSTNF. Section 4 presents the centralized, the sequential and the naïve-distributed fusion algorithms in sequence. Section 5 presents a numerical example, and Section 6 concludes this paper.

## 2 Problem formulation

Consider the following multisensor nonlinear dynamic systems where there are $N$ sensors observing a single target at the same time [32, 33, 37]:

$$x_{u+1} = f_u(x_u) + w_u, \quad u = 0, 1, \dots, \tag{1}$$

$$z_{p,u} = \gamma_{p,u}[h_{p,u}(x_u) + v_{p,u}], \quad p = 1, 2, \ldots, N, \tag{2}$$

where $x_u \in \mathbb{R}^n$ is the system state, $f_u$ is the nonlinear function of $x_u$, $z_{p,u} \in \mathbb{R}^{n_p}$ is the measurement of sensor $p$ at time $u$, $h_{p,u}$ is the nonlinear function of $x_u$ corresponding to sensor $p$, and $N$ is the number of sensors. The process noise $w_u$ and the observation noise $v_{p,u}$ are all heavy-tailed noises. They satisfy the following multivariate student $t$-distribution:

$$p(w_u) = \mathrm{St}(w_u; 0, Q_u, \nu_w), \tag{3}$$

$$p(v_{p,u}) = \mathrm{St}(v_{p,u}; 0, R_{p,u}, \nu_p), \tag{4}$$

where $\mathrm{St}(\cdot; \mu, \Sigma, \nu)$ is the multivariate $t$-distribution, whose mean, scale matrix, and degrees of freedom (dof) are $\mu$, $\Sigma$, and $\nu$, respectively [40, 41]. $\gamma_{p,u} \in \mathbb{R}$ is a variable of random sequence, which is Bernoulli distribution that takes values on 1 and 0, respectively, whose expectation is $\bar{\gamma}_p$. It is used to formulate the percentage of data dropout.

Similarly, suppose the system state initial value $x_0$ is heavy-tailed too, which satisfies the student $t$-distribution, whose mean, scale matrix, and the dof are $\hat{x}_0$, $P_0$, and $\nu_0$, respectively, i.e.,

$$p(x_0) = \mathrm{St}(x_0; \hat{x}_0, P_0, \nu_0). \tag{5}$$

It is assumed that $x_0$, $w_u$, $v_{p,u}$, and $\gamma_{p,u}$ are mutually independent, $p = 1, 2, \ldots, N$.

The goal of this paper is to derive the state fusion estimation of $x_u$ by combining the observations of all sensors up to and including time $u$, i.e., $Z_u = \{z_{p,t}, t = 1, 2, \ldots, u; p = 1, 2, \ldots, N\}$.

## 3 The modified robust student $t$-distribution based nonlinear filter

In the sequel, we will introduce the state estimation algorithm under the assumption that the initial state, the system noise, and the observation noises are all student distributed with the same dof. Namely, $\nu_w = \nu_p = \nu$ for $p = 0, 1, 2, \ldots, N$. When the dofs are different, we will state the modification of the algorithm in Remark 2.

For simplicity, let $z_u = z_{1,u}$, $\gamma_u = \gamma_{1,u}$, $R_u = R_{1,u}$ in this section. For systems (1) and (2), in case of $N = 1$, the modified robust student $t$-distribution based nonlinear filter (MRSTNF) can be deduced as follows.

**Step 1: time update.**

$$\begin{aligned}
\hat{x}_{u|u-1} &= E[x_u|Z_{u-1}] = \int x_u p(x_u|Z_{u-1}) \mathrm{d}x_u \\
&= \int \left[ \int x_u \mathrm{St}(x_u; f_{u-1}(x_{u-1}), Q_{u-1}, \nu) \mathrm{d}x_u \right] \times \mathrm{St}(x_{u-1}; \hat{x}_{u-1|u-1}, P_{u-1|u-1}, \nu) \mathrm{d}x_{u-1} \\
&= \int f_{u-1}(x_{u-1}) \mathrm{St}(x_{u-1}; \hat{x}_{u-1|u-1}, P_{u-1|u-1}, \nu) \mathrm{d}x_{u-1} \\
&\approx \sum_{j=0}^{2n} \omega_j f_{u-1}(\hat{x}^j_{u-1|u-1}),
\end{aligned} \tag{6}$$

where $\hat{x}^j_{u-1|u-1}$ are the sigma points of $\hat{x}_{u-1|u-1}$, i.e., $\hat{x}^j_{u-1|u-1} = \hat{x}_{u-1|u-1}$ when $j = 0$, $\hat{x}^j_{u-1|u-1} = \hat{x}_{u-1|u-1} + \eta(\sqrt{P_{u-1|u-1}})_j$ for $j = 1, \ldots, n$, and $\hat{x}^j_{u-1|u-1} = \hat{x}_{u-1|u-1} - \eta(\sqrt{P_{u-1|u-1}})_{j-n}$ for $j = n+1, \ldots, 2n$. The weights of the sigma points $\omega_j$ are computed by

$$\omega_j = \begin{cases} \kappa/(n+\kappa), & j = 0, \\ 0.5/(n+\kappa), & j = 1, 2, \ldots, 2n, \end{cases} \tag{7}$$

where $\kappa$ is the free parameter of the unscented transform (UT), and is usually taken as zero. $\eta = \sqrt{\frac{\nu}{\nu-2}(n+\lambda)}$, $\lambda = \alpha^2(n+\kappa) - n$, $10^{-3} < \alpha \leqslant 1$.

Let

$$\tilde{x}_{u|u-1} = x_u - \hat{x}_{u|u-1}. \tag{8}$$

It can be easily derived that

$$\begin{aligned}
P_{u|u-1} &= \frac{\nu-2}{\nu}E\big[\tilde{x}_{u|u-1}\tilde{x}_{u|u-1}^{\mathrm{T}}|Z_{u-1}\big] \\
&= \frac{\nu-2}{\nu}\int(x_u-\hat{x}_{u|u-1})(x_u-\hat{x}_{u|u-1})^{\mathrm{T}},p(x_u|Z_{u-1})\mathrm{d}x_u \\
&= \frac{\nu-2}{\nu}\int x_u x_u^{\mathrm{T}}p(x_u|Z_{u-1})\mathrm{d}x_u - \frac{\nu-2}{\nu}\hat{x}_{u|u-1}\hat{x}_{u|u-1}^{\mathrm{T}} \\
&= \frac{\nu-2}{\nu}\Bigg\{\int\left[\int x_u x_u^{\mathrm{T}}\mathrm{St}(x_u;f_{u-1}(x_{u-1}),Q_{u-1},\nu_1)\mathrm{d}x_u\right] \\
&\quad \cdot\mathrm{St}(x_{u-1};\hat{x}_{u-1|u-1},P_{u-1|u-1},\nu)\mathrm{d}x_{u-1} - \hat{x}_{u|u-1}\hat{x}_{u|u-1}^{\mathrm{T}}\Bigg\} \\
&= \frac{\nu-2}{\nu}\Bigg\{\int f_{u-1}(x_{u-1})f_{u-1}^{\mathrm{T}}(x_{u-1})\times\mathrm{St}(x_{u-1};\hat{x}_{u-1|u-1},P_{u-1|u-1},\nu)\mathrm{d}x_{u-1} \\
&\quad -\hat{x}_{u|u-1}\hat{x}_{u|u-1}^{\mathrm{T}}\Bigg\}+Q_{u-1} \\
&\approx \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j f_{u-1}(\hat{x}_{u-1|u-1}^j)f_{u-1}^{\mathrm{T}}(\hat{x}_{u-1|u-1}^j)-\hat{x}_{u|u-1}\hat{x}_{u|u-1}^{\mathrm{T}}\right\}+Q_{u-1}.
\end{aligned} \tag{9}$$

**Step 2: measurement update.**

$$\begin{aligned}
\hat{z}_{u|u-1} &= E[z_u|Z_{u-1}] = \int z_u p(z_u|Z_{u-1})\mathrm{d}z_u \\
&= \int\left[\int z_u\mathrm{St}(z_u;h_u(x_u),R_u,\nu)\mathrm{d}z_u\right]\times\mathrm{St}(x_u;\hat{x}_{u|u-1},P_{u|u-1},\nu)\mathrm{d}x_u \\
&= \int h_u(x_u)\mathrm{St}(x_u;\hat{x}_{u|u-1},P_{u|u-1},\nu)\mathrm{d}x_u \\
&\approx \sum_{j=0}^{2n}\omega_j h_u(\hat{x}_{u|u-1}^j),
\end{aligned} \tag{10}$$

where $\hat{x}_{u|u-1}^j$ are the sigma points of $\hat{x}_{u|u-1}$ and $\omega_j$ are the corresponding weights.
Let

$$\tilde{z}_{u|u-1} = \gamma_u[z_u-\hat{z}_{u|u-1}], \tag{11}$$

and then

$$\begin{aligned}
P_{u|u-1}^{\tilde{x}\tilde{z}} &= \frac{\nu-2}{\nu}E\big[\tilde{x}_{u|u-1}\tilde{z}_{u|u-1}^{\mathrm{T}}|Z_{u-1}\big] \\
&= \frac{\nu-2}{\nu}\iint\gamma_u[x_u-\hat{x}_{u|u-1}][z_u-\hat{z}_{u|u-1}]^{\mathrm{T}}p(x_u,z_u|Z_{u-1})\mathrm{d}x_u\mathrm{d}z_u \\
&= \frac{(\nu-2)\gamma_u}{\nu}\left\{\iint x_u z_u^{\mathrm{T}}P(x_u,z_u|Z_{u-1})\mathrm{d}x_u\mathrm{d}z_u-\hat{x}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\right\} \\
&= \frac{(\nu-2)\gamma_u}{\nu}\left\{\int x_u\left[\int z_u^{\mathrm{T}}p(z_u|x_u)\mathrm{d}z_u\right]p(x_u|Z_{u-1})\mathrm{d}x_u-\hat{x}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\right\} \\
&= \frac{(\nu-2)\gamma_u}{\nu}\Bigg\{\int x_u\left[\int z_u^{\mathrm{T}}\mathrm{St}(z_u;h_u(x_u),R_u,\nu_2)\mathrm{d}z_u\right]\mathrm{St}(x_u;\hat{x}_{u|u-1},P_{u|u-1},\nu)\mathrm{d}x_u \\
&\quad -\hat{x}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\Bigg\}
\end{aligned}$$

$$= \frac{(\nu-2)\gamma_u}{\nu}\left\{\int x_u h_u^{\mathrm{T}}(x_u)\mathrm{St}(x_u;\hat{x}_{u|u-1},P_{u|u-1},\nu)\mathrm{d}x_u - \hat{x}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\right\}$$

$$\approx \frac{(\nu-2)\gamma_u}{\nu}\left\{\sum_{j=0}^{2n}\omega_j\hat{x}_{u|u-1}^{j}h_u^{\mathrm{T}}(\hat{x}_{u|u-1}^{j}) - \hat{x}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\right\}, \tag{12}$$

$$\begin{aligned}
P_{u|u-1}^{\tilde{z}\tilde{z}} &= \frac{\nu-2}{\nu}E\big[\tilde{z}_{u|u-1}\tilde{z}_{u|u-1}^{\mathrm{T}}|Z_{u-1}\big]\\
&= \frac{\nu-2}{\nu}\int \gamma_u^2(z_u-\hat{z}_{u|u-1})(z_u-\hat{z}_{u|u-1})^{\mathrm{T}}p(z_u|Z_{u-1})\mathrm{d}z_u\\
&= \frac{(\nu-2)\gamma_u}{\nu}\left\{\int z_u z_u^{\mathrm{T}}p(z_u|Z_{u-1})\mathrm{d}z_u - \hat{z}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\right\}\\
&= \frac{(\nu-2)\gamma_u}{\nu}\Bigg\{\int\left[\int z_u z_u^{\mathrm{T}}\mathrm{St}(z_u;h_u(x_u),R_u,\nu)\mathrm{d}z_u\right]\mathrm{St}(x_u;\hat{x}_{u|u-1},P_{u|u-1},\nu)\mathrm{d}x_u\\
&\quad - \hat{z}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\Bigg\}\\
&= \frac{(\nu-2)\gamma_u}{\nu}\left\{\int h_u(x_u)h_u^{\mathrm{T}}(x_u)\mathrm{St}(x_u;\hat{x}_{u|u-1},P_{u|u-1},\nu)\mathrm{d}x_u - \hat{z}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\right\} + \gamma_u R_u\\
&\approx \frac{(\nu-2)\gamma_u}{\nu}\left\{\sum_{j=0}^{2n}\omega_j h_u(\hat{x}_{u|u-1}^{j})h_u^{\mathrm{T}}(\hat{x}_{u|u-1}^{j}) - \hat{z}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\right\} + \gamma_u R_u.
\end{aligned} \tag{13}$$

Let

$$P_{u|u-1}^{\tilde{z}\tilde{z},*} = \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j h_u(\hat{x}_{u|u-1}^{j})h_u^{\mathrm{T}}(\hat{x}_{u|u-1}^{j}) - \hat{z}_{u|u-1}\hat{z}_{u|u-1}^{\mathrm{T}}\right\} + R_u, \tag{14}$$

and then

$$P_{u|u-1}^{\tilde{z}\tilde{z}} = \gamma_u P_{u|u-1}^{\tilde{z}\tilde{z},*}. \tag{15}$$

The state estimation is computed by [12, 17, 41]

$$\hat{x}_{u|u} = \hat{x}_{u|u-1} + K_u\tilde{z}_{u|u-1}, \tag{16}$$

$$P_{u|u} = \frac{(\nu-2)(\nu+\Delta_u^2)\gamma_u}{\nu(\nu+n_1-2)}\big[P_{u|u-1} - K_u(P_{u|u-1}^{\tilde{z}\tilde{z},*})^{-1}K_u^{\mathrm{T}}\big] + (1-\gamma_u)P_{u|u-1}, \tag{17}$$

$$K_u = P_{u|u-1}^{\tilde{x}\tilde{z}}(P_{u|u-1}^{\tilde{z}\tilde{z},*})^{-1}, \tag{18}$$

$$\Delta_u^2 = \tilde{z}_{u|u-1}^{\mathrm{T}}(P_{u|u-1}^{\tilde{z}\tilde{z},*})^{-1}\tilde{z}_{u|u-1}. \tag{19}$$

**Remark 1.** The general idea for filter of intermittent observations is that: in the measurement update step, in case of $\gamma_u = 1$, the measurement is received and being applied to generate the state estimation; while in case of $\gamma_u = 0$, the state estimation should be equal to the state prediction. It can be seen from (16)–(19) that when $\gamma_u = 1$, one has $\hat{x}_{u|u} = \hat{x}_{u|u-1} + K_u\tilde{z}_{u|u-1}$, $P_{u|u} = \frac{(\nu-2)(\nu+\Delta_u^2)}{\nu(\nu+n_1-2)}(P_{u|u-1} - K_u(P_{u|u-1}^{\tilde{z}\tilde{z}})^{-1}K_u^{\mathrm{T}})$, which is the same as that presented in [12]. When the measurement is missing, i.e., $\gamma_u = 0$, then $K_u = 0$, and $\hat{x}_{u|u} = \hat{x}_{u|u-1}$, $P_{u|u} = P_{u|u-1}$. In computing of $K_u$ and $\Delta_u$, $P_{u|u-1}^{\tilde{z}\tilde{z},*}$ is used to replace $P_{u|u-1}^{\tilde{z}\tilde{z}}$ to avoid matrix singularity. The filter presented in this section is the generalization of RSTNF in [12].

## 4 The information fusion algorithms

Let

$$Z_{p,1:u} = \{z_{p,l}; l = 1, 2, \ldots, u\}, \tag{20}$$

$$Z_{1:p,u} = \{z_{j,u}; j = 1, 2, \ldots, p\}, \tag{21}$$

$$\begin{aligned} Z_{1:p,1:u} &= \{z_{j,l}; j = 1, 2, \ldots, p; l = 1, 2, \ldots, u\} \\ &= \{Z_{j,1:u}; j = 1, 2, \ldots, p\} \\ &= \{Z_{1:p,l}; l = 1, 2, \ldots, u\}, \end{aligned} \tag{22}$$

where $Z_{p,1:u}$ denotes the measurements of sensor $p$ up to time $u$, $Z_{1:p,u}$ denotes the measurements of sensors 1 to $p$ at time $u$, and $Z_{1:p,1:u}$ then denotes the measurements of sensor 1 to $p$ from the initial time to time $u$. It can be easily shown that $Z_u = Z_{1:N,1:u}$, which denotes all the measurements before and including time $u$.

### 4.1 The centralized batch fusion

Let

$$z_u^a = [z_{1,u}^{\mathrm{T}}, z_{2,u}^{\mathrm{T}}, \ldots, z_{N,u}^{\mathrm{T}}]^{\mathrm{T}}, \tag{23}$$

$$h_u^a(x_u) = [h_{1,u}^{\mathrm{T}}(x_u), h_{2,u}^{\mathrm{T}}(x_u), \ldots, h_{N,u}^{\mathrm{T}}(x_u)]^{\mathrm{T}}, \tag{24}$$

$$v_u^a = [v_{1,u}^{\mathrm{T}}, v_{2,u}^{\mathrm{T}}, \ldots, v_{N,u}^{\mathrm{T}}]^{\mathrm{T}}, \tag{25}$$

$$\gamma_u^a = \mathrm{diag}\{\gamma_{1,u} I_{n_1}, \gamma_{2,u} I_{n_2}, \ldots, \gamma_{N,u} I_{n_N}\}, \tag{26}$$

where $I_{n_p}$ denotes the identity matrix with dimension $n_p$, $p = 1, 2, \ldots, N$. Then Eq. (2) can be rewritten as follows:

$$z_u^a = \gamma_u^a [h_u^a(x_u) + v_u^a]. \tag{27}$$

From the property of student $t$-distribution [40], we have

$$p(v_u^a) = \mathrm{St}(v_u^a; 0, R_u^a, \nu), \tag{28}$$

where

$$R_u^a = \mathrm{diag}\{R_{1,u}, R_{2,u}, \ldots, R_{N,u}\}. \tag{29}$$

For system (1) and (27), similar to the MRSTNF, the centralized fusion state estimation can be deduced based on information of sensors 1 to $N$.

**Step 1: time update.**

$$\begin{aligned} \hat{x}_{c,u|u-1} &= E[x_u|Z_{u-1}] = \int x_u p(x_u|Z_{u-1}) \mathrm{d}x_u \\ &= \int f_{u-1}(x_{u-1}) \mathrm{St}(x_{u-1}; \hat{x}_{c,u-1|u-1}, P_{c,u-1|u-1}, \nu) \mathrm{d}x_{u-1} \\ &\approx \sum_{j=0}^{2n} \omega_j f_{u-1}(\hat{x}_{c,u-1|u-1}^j), \end{aligned} \tag{30}$$

where $\hat{x}_{c,u-1|u-1}^j$ are the sigma points of $\hat{x}_{c,u-1|u-1}$ and $\omega_j$ are the corresponding weights.

Let $\tilde{x}_{c,u|u-1} = x_u - \hat{x}_{c,u|u-1}$, and then

$$\begin{aligned} P_{c,u|u-1} &= \frac{\nu-2}{\nu} E\left[\tilde{x}_{c,u|u-1} \tilde{x}_{c,u|u-1}^{\mathrm{T}} | Z_{u-1}\right] \\ &= \frac{\nu-2}{\nu} \Bigg\{ \int f_{u-1}(x_{u-1}) f_{u-1}^{\mathrm{T}}(x_{u-1}) \times \mathrm{St}(x_{u-1}; \hat{x}_{u-1|u-1}, P_{u-1|u-1}, \nu) \mathrm{d}x_{u-1} \\ &\quad - \hat{x}_{c,u|u-1} \hat{x}_{c,u|u-1}^{\mathrm{T}} \Bigg\} + Q_{u-1} \\ &\approx \frac{\nu-2}{\nu} \left\{ -\hat{x}_{c,u|u-1} \hat{x}_{c,u|u-1}^{\mathrm{T}} \sum_{j=0}^{2n} \omega_j f_{u-1}(\hat{x}_{c,u-1|u-1}^j) f_{u-1}^{\mathrm{T}}(\hat{x}_{c,u-1|u-1}^j) \right\} + Q_{u-1}. \end{aligned} \tag{31}$$

**Step 2: measurement update.**

$$\hat{z}_{c,u|u-1} = E[z_u^a|Z_{u-1}] = \int h_u^a(x_u)\mathrm{St}(x_u;\hat{x}_{c,u|u-1},P_{c,u|u-1},\nu_3)\mathrm{d}x_u \approx \sum_{j=0}^{2n}\omega_j h_u^a(\hat{x}^j_{c,u|u-1}), \quad (32)$$

where $\hat{x}^j_{c,u|u-1}$ are the sigma points of $\hat{x}_{c,u|u-1}$ and $\omega_j$ are the corresponding weights.

Let

$$\tilde{z}_{c,u|u-1} = \gamma_u^a[z_u^a - \hat{z}_{c,u|u-1}], \quad (33)$$

and then

$$\begin{aligned} P^{\tilde{x}\tilde{z}}_{c,u|u-1} &= \frac{\nu-2}{\nu}E\big[\tilde{x}_{c,u|u-1}\tilde{z}^{\mathrm{T}}_{c,u|u-1}|Z_{u-1}\big] \\ &= \frac{\nu-2}{\nu}\left\{\int x_u\gamma_u^a h_u^{a,\mathrm{T}}(x_u)\mathrm{St}(x_u;\hat{x}_{c,u|u-1},P_{c,u|u-1},\nu)\mathrm{d}x_u - \hat{x}_{c,u|u-1}\gamma_u^a\hat{z}^{\mathrm{T}}_{c,u|u-1}\right\} \\ &\approx \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j\hat{x}^j_{c,u|u-1}\gamma_u^a h_u^{a,\mathrm{T}}(\hat{x}^j_{c,u|u-1}) - \hat{x}_{c,u|u-1}\gamma_u^a\hat{z}^{\mathrm{T}}_{c,u|u-1}\right\}, \end{aligned} \quad (34)$$

$$\begin{aligned} P^{\tilde{z}\tilde{z}}_{c,u|u-1} &= \frac{\nu-2}{\nu}E\big[\tilde{z}_{c,u|u-1}\tilde{z}^{\mathrm{T}}_{c,u|u-1}|Z_{u-1}\big] \\ &= \frac{\nu-2}{\nu}\gamma_u^a\bigg\{\int h_u^a(x_u)h_u^{a,\mathrm{T}}(x_u)\times\mathrm{St}(x_u;\hat{x}_{c,u|u-1},P_{c,u|u-1},\nu)\mathrm{d}x_u \\ &\quad - \hat{z}_{c,u|u-1}\hat{z}^{\mathrm{T}}_{c,u|u-1}\bigg\}\gamma_u^{a,\mathrm{T}} + \gamma_u^a R_u\gamma_u^{a,\mathrm{T}} \\ &\approx \frac{\nu-2}{\nu}\gamma_u^a\left\{\sum_{j=0}^{2n}\omega_j h_u^a(\hat{x}^j_{c,u|u-1})h_u^{a,\mathrm{T}}(\hat{x}^j_{c,u|u-1}) - \hat{z}_{c,u|u-1}\hat{z}^{\mathrm{T}}_{c,u|u-1}\right\}\gamma_u^{a,\mathrm{T}} + \gamma_u^a R_u\gamma_u^{a,\mathrm{T}}. \end{aligned} \quad (35)$$

Let

$$P^{\tilde{z}\tilde{z},*}_{c,u|u-1} = \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j h_u^a(\hat{x}^j_{c,u|u-1})h_u^{a,\mathrm{T}}(\hat{x}^j_{c,u|u-1}) - \hat{z}_{c,u|u-1}\hat{z}^{\mathrm{T}}_{c,u|u-1}\right\} + R_u, \quad (36)$$

and then

$$P^{\tilde{z}\tilde{z}}_{c,u|u-1} = \gamma_u^a P^{\tilde{z}\tilde{z},*}_{c,u|u-1}\gamma_u^{a,\mathrm{T}}. \quad (37)$$

It can be easily shown that $\gamma_u^a$ is a diagonal matrix. Therefore, $\gamma_u^{a,\mathrm{T}} = \gamma_u^a$. We have

$$\hat{x}_{c,u|u} = \hat{x}_{c,u|u-1} + K_{c,u}\tilde{z}_{c,u|u-1}, \quad (38)$$

$$\begin{aligned} P_{c,u|u} &= \max_{p=1,2,\ldots,N}\gamma_{p,u}\frac{(\nu-2)(\nu+\Delta^2_{c,u})}{\nu(\nu+n_{(z)}-2)}\times\left(P_{c,u|u-1} - K_{c,u}(P^{\tilde{z}\tilde{z},*}_{c,u|u-1})^{-1}K^{\mathrm{T}}_{c,u}\right) \\ &\quad + \left(1 - \max_{p=1,2,\ldots,N}\gamma_{p,u}\right)P_{c,u|u-1}, \end{aligned} \quad (39)$$

$$K_{c,u} = P^{\tilde{x}\tilde{z}}_{c,u|u-1}\gamma_u^a(P^{\tilde{z}\tilde{z},*}_{c,u|u-1})^{-1}\gamma_u^a, \quad (40)$$

$$\Delta^2_{c,u} = \tilde{z}^{\mathrm{T}}_{c,u|u-1}\gamma_u^a(P^{\tilde{z}\tilde{z},*}_{c,u|u-1})^{-1}\gamma_u^a\tilde{z}_{c,u|u-1}, \quad (41)$$

where $n_{(z)} = \sum_{p=1}^N\gamma_{p,u}n_p$. The subscript $c$ in (30)–(41) denotes the centralized fusion.

### 4.2 Sequential fusion estimation

To avoid augmentation of matrices and vectors, and to improve the efficiency of fusion estimation, we will deduce the sequential fusion algorithm in this subsection.

**Theorem 1** (Sequential fusion of complete measurements)**.** For system (1)–(5), suppose $\gamma_{p,u} = 1$ for all $p = 1, 2, \ldots, N$, and then by sequential fusion of the information of sensor 1 to sensor $N$, the state fusion estimation can be computed by

$$\begin{cases} \hat{x}_{s,u|u} = \hat{x}_{s,u|u-1} + \sum_{p=1}^{N} K_{p,u}\tilde{z}_{p,u}, \\ P_{s,u|u} = \left(\frac{\nu-2}{\nu}\right)^{N} \prod_{p=1}^{N} \left(\frac{\nu+\Delta_{p,u}^2}{\nu+n_p-2}\right) P_{s,u|u-1} - \sum_{p=1}^{N} \Bigg\{ \left(\frac{\nu-2}{\nu}\right)^{N-p+1} \prod_{i=p}^{N} \left(\frac{\nu+\Delta_{i,u}^2}{v+n_i-2}\right) \\ \qquad \times K_{p,u}(P_u^{\tilde{z}_p\tilde{z}_p})^{-1} K_{p,u}^{\mathrm{T}} \Bigg\}, \end{cases} \tag{42}$$

where subscript $s$ denotes the sequential fusion, $p$ denotes the $p$-th sensor, and for $p = 1, 2, \ldots, N$,

$$\begin{cases} K_{p,u} = P_u^{\tilde{x}\tilde{z}_p}(P_u^{\tilde{z}_p\tilde{z}_p})^{-1}, \\ \Delta_{p,u}^2 = \tilde{z}_{p,u}^{\mathrm{T}}(P_u^{\tilde{z}_p\tilde{z}_p})^{-1}\tilde{z}_{p,u} \\ \tilde{z}_{p,u} = z_{p,u} - \hat{z}_{p,u}, \\ P_u^{\tilde{x}\tilde{z}_p} = \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j\hat{x}_{p-1,u|u}^{j}h_u^{\mathrm{T}}(\hat{x}_{p-1,u|u}^{j}) - \hat{x}_{p-1,u|u}\hat{z}_{p,u}^{\mathrm{T}}\right\}, \\ P_u^{\tilde{z}_p\tilde{z}_p} = \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j h_u(\hat{x}_{p-1,u|u}^{j})h_u^{\mathrm{T}}(\hat{x}_{p-1,u|u}^{j}) - \hat{z}_{p,u}\hat{z}_{p,u}^{\mathrm{T}}\right\} + R_{p,u}, \\ \hat{z}_{p,u} = \sum_{j=0}^{2n}\omega_j h_u(\hat{x}_{p-1,u|u}^{j}), \\ \hat{x}_{p,u|u} = \hat{x}_{p-1,u|u} + K_{p,u}\tilde{z}_{p,u}, \\ \hat{x}_{0,u|u} = \hat{x}_{s,u|u-1} = \sum_{j=0}^{2n}\omega_j f_{u-1}(\hat{x}_{s,u-1|u-1}^{j}), \\ P_{0,u|u} = P_{s,u|u-1} = \frac{\nu-2}{\nu}\Bigg\{\sum_{j=0}^{2n}\left[\omega_j f_{u-1}(\hat{x}_{s,u-1|u-1}^{j}) \times f_{u-1}^{\mathrm{T}}(\hat{x}_{s,u-1|u-1}^{j})\right] \\ \qquad - \hat{x}_{s,u|u-1}\hat{x}_{s,u|u-1}^{\mathrm{T}}\Bigg\} + Q_{u-1}, \end{cases} \tag{43}$$

where $\hat{x}_{s,u|u-1}^{j}$ and $\hat{x}_{p-1,u|u}^{j}$ are the sigma points of $\hat{x}_{s,u|u-1}$ and $\hat{x}_{p-1,u|u}$, respectively, and $\omega_j$ are the corresponding weights.

*Proof.* **Step 1: time update.** Similar to the centralized fusion, we have

$$\hat{x}_{s,u|u-1} = \int x_u P(x_u|Z_{u-1})\mathrm{d}x_u \approx \sum_{j=0}^{2n}\omega_j f_{u-1}(\hat{x}_{s,u-1|u-1}^{j}), \tag{44}$$

where $\hat{x}_{s,u-1|u-1}^{j}$ are the sigma points of $\hat{x}_{s,u-1|u-1}$ and $\omega_j$ are the corresponding weights.

Let $\tilde{x}_{s,u|u-1} = x_u - \hat{x}_{s,u|u-1}$, and then

$$P_{s,u|u-1} = \frac{\nu-2}{\nu}\int \tilde{x}_{s,u|u-1}\tilde{x}_{s,u|u-1}^{\mathrm{T}} p(x_u|Z_{u-1})\mathrm{d}x_u$$

$$\approx \frac{\nu-2}{\nu}\left\{\left[\sum_{j=0}^{2n}\omega_j f_{u-1}(\hat{x}^j_{s,u-1|u-1})\times f^{\mathrm{T}}_{u-1}(\hat{x}^j_{s,u-1|u-1})\right] - \hat{x}_{s,u|u-1}\hat{x}^{\mathrm{T}}_{s,u|u-1}\right\} + Q_{u-1}. \tag{45}$$

**Step 2: measurement update step by step.** From the assumptions in [12], we have

$$p(x_u, z_{1,u}|Z_{u-1}) = \mathrm{St}\left(\begin{bmatrix} x_u \\ z_{1,u}\end{bmatrix}; \begin{bmatrix}\hat{x}_{s,u|u-1} \\ \hat{z}_{1,u}\end{bmatrix}, \begin{bmatrix} P_{s,u|u-1} & P^{\tilde{x}\tilde{z}_1}_u \\ P^{\tilde{x}\tilde{z}_1,T}_u & P^{\tilde{z}_1\tilde{z}_1}_u\end{bmatrix}, \nu\right), \tag{46}$$

where

$$\begin{aligned}\hat{z}_{1,u} &= E[z_{1,u}|Z_{u-1}] = \int z_{1,u}p(z_{1,u}|Z_{u-1})\mathrm{d}z_{1,u} \\ &= \int h_u(x_u)\mathrm{St}(x_u;\hat{x}_{s,u|u-1},P_{s,u|u-1},\nu)\mathrm{d}x_u \\ &\approx \sum_{j=0}^{2n}\omega_j h_u(\hat{x}^j_{s,u|u-1}),\end{aligned} \tag{47}$$

$\hat{x}^j_{s,u|u-1}$ are the sigma points of $\hat{x}_{s,u|u-1}$ and $\omega_j$ are the corresponding weights.

Let $\tilde{z}_{1,u} = z_{1,u} - \hat{z}_{1,u}$, and then

$$\begin{aligned}P^{\tilde{x}\tilde{z}_1}_u &= \frac{\nu-2}{\nu}E\big[\tilde{x}_{s,u|u-1}\tilde{z}^{\mathrm{T}}_{1,u}|Z_{u-1}\big] \\ &= \frac{\nu-2}{\nu}\left\{\int x_u h^{\mathrm{T}}_u(x_u)\mathrm{St}(x_u;\hat{x}_{s,u|u-1},P_{s,u|u-1},\nu)\mathrm{d}x_u - \hat{x}_{s,u|u-1}\hat{z}^{\mathrm{T}}_{1,u}\right\} \\ &\approx \frac{\nu-2}{\nu}\left\{\left(\sum_{j=0}^{2n}\omega_j\hat{x}^j_{s,u|u-1}h^{\mathrm{T}}_u(\hat{x}^j_{s,u|u-1})\right) - \hat{x}_{s,u|u-1}\hat{z}^{\mathrm{T}}_{1,u}\right\},\end{aligned} \tag{48}$$

$$\begin{aligned}P^{\tilde{z}_1\tilde{z}_1}_u &= \frac{\nu-2}{\nu}E\big[\tilde{z}_{1,u}\tilde{z}^{\mathrm{T}}_{1,u}|Z_{u-1}\big] \\ &= \frac{\nu-2}{\nu}\left\{\int h_u(x_u)h^{\mathrm{T}}_u(x_u)\mathrm{St}(x_u;\hat{x}_{s,u|u-1},P_{s,u|u-1},\nu)\mathrm{d}x_u - \hat{z}_{1,u}\hat{z}^{\mathrm{T}}_{1,u}\right\} + R_{1,u} \\ &\approx \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j h_u(\hat{x}^j_{s,u|u-1})h^{\mathrm{T}}_u(\hat{x}^j_{s,u|u-1}) - \hat{z}_{1,u}\hat{z}^{\mathrm{T}}_{1,u}\right\} + R_{1,u}.\end{aligned} \tag{49}$$

The conditional probability

$$p(x_u|Z_{u-1}, z_{1,u}) = \mathrm{St}(x_u;\hat{x}'_{1,u|u},P'_{1,u|u},\nu^{(1)}) \tag{50}$$

can be obtained by

$$\nu^{(1)} = \nu + n_1, \tag{51}$$

$$\hat{x}'_{1,u|u} = \hat{x}_{s,u|u-1} + K_{1,u}\tilde{z}_{1,u}, \tag{52}$$

$$K_{1,u} = P^{\tilde{x}\tilde{z}_1}_u(P^{\tilde{z}_1\tilde{z}_1}_u)^{-1}, \tag{53}$$

$$P'_{1,u|u} = \frac{\nu+\Delta^2_{1,u}}{\nu+n_1}\big[P_{s,u|u-1} - K_{1,u}(P^{\tilde{z}_1\tilde{z}_1}_u)^{-1}K^{\mathrm{T}}_{1,u}\big], \tag{54}$$

$$\Delta^2_{1,u} = \tilde{z}^{\mathrm{T}}_{1,u}(P^{\tilde{z}_1\tilde{z}_1}_u)^{-1}\tilde{z}_{1,u}, \tag{55}$$

$$\tilde{z}_{1,u} = z_{1,u} - \hat{z}_{1,u}. \tag{56}$$

To keep the heavy-tailed property, by the moment matching method, we have the approximate $t$-distribution [12]

$$p(x_u|Z_{u-1},z_{1,u}) \approx \mathrm{St}(x_u;\hat{x}_{1,u|u},P_{1,u|u},\nu), \tag{57}$$

where

$$\hat{x}_{1,u|u} = \hat{x}'_{1,u|u} = \hat{x}_{s,u|u-1} + K_{1,u}\tilde{z}_{1,u}, \tag{58}$$

$$\begin{aligned} P_{1,u|u} &= \frac{\nu^{(1)}(\nu-2)}{\nu(\nu^{(1)}-2)}P'_{1,u|u} \\ &= \frac{(\nu-2)(\nu+\Delta^2_{1,u})}{\nu(\nu+n_1-2)}[P_{s,u|u-1} - K_{1,u}(P_u^{\tilde{z}_1\tilde{z}_1})^{-1}K^{\mathrm{T}}_{1,u}]. \end{aligned} \tag{59}$$

Similarly, we have

$$p(x_u,z_{2,u}|Z_{u-1},z_{1,u}) = \mathrm{St}\left(\begin{bmatrix} x_u \\ z_{2,u}\end{bmatrix};\begin{bmatrix}\hat{x}_{1,u|u} \\ \hat{z}_{2,u}\end{bmatrix},\begin{bmatrix} P_{1,u|u} & P_u^{\tilde{x}\tilde{z}_2} \\ P_u^{\tilde{x}\tilde{z}_2,T} & P_u^{\tilde{z}_2\tilde{z}_2}\end{bmatrix},\nu\right), \tag{60}$$

where

$$\begin{aligned} \hat{z}_{2,u} &= E[z_{2,u}|Z_{u-1},z_{1,u}] \\ &= \int h_u(x_u)\mathrm{St}(x_u;\hat{x}_{1,u|u},P_{1,u|u},\nu)\mathrm{d}x_u \\ &\approx \sum_{j=0}^{2n}\omega_j h_u(\hat{x}^j_{1,u|u}), \end{aligned} \tag{61}$$

$\hat{x}^j_{1,u|u}$ are the sigma points of $\hat{x}_{1,u|u}$ and $\omega_j$ are the corresponding weights.

$$\begin{aligned} P_u^{\tilde{x}\tilde{z}_2} &= \frac{\nu-2}{\nu}E\big[\tilde{x}_{1,u|u}\tilde{z}^{\mathrm{T}}_{2,u}|Z_{u-1},z_{1,u}\big] \\ &= \frac{\nu-2}{\nu}\left\{\int x_u h^{\mathrm{T}}_u(x_u)\mathrm{St}(x_u;\hat{x}_{1,u|u},P_{1,u|u},\nu)\mathrm{d}x_u - \hat{x}_{1,u|u}\hat{z}^{\mathrm{T}}_{2,u}\right\} \\ &\approx \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j\hat{x}^j_{1,u|u}h^{\mathrm{T}}_u(\hat{x}^j_{1,u|u}) - \hat{x}_{1,u|u}\hat{z}^{\mathrm{T}}_{2,u}\right\}, \end{aligned} \tag{62}$$

$$\begin{aligned} P_u^{\tilde{z}_2\tilde{z}_2} &= \frac{\nu-2}{\nu}E\big[\tilde{z}_{2,u}\tilde{z}^{\mathrm{T}}_{2,u}|Z_{u-1},z_{1,u}\big] \\ &= \frac{\nu-2}{\nu}\int\left\{h_u(x_u)h^{\mathrm{T}}_u(x_u)\mathrm{St}(x_u;\hat{x}_{1,u|u},P_{1,u|u},\nu)\mathrm{d}x_u - \hat{z}_{2,u}\hat{z}^{\mathrm{T}}_{2,u}\right\} + R_{2,u} \\ &\approx \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j h_u(\hat{x}^j_{1,u|u})h^{\mathrm{T}}_u(\hat{x}^j_{1,u|u}) - \hat{z}_{2,u}\hat{z}^{\mathrm{T}}_{2,u}\right\} + R_{2,u}, \end{aligned} \tag{63}$$

$$\hat{x}_{2,u|u} = \hat{x}_{1,u|u} + K_{2,u}\tilde{z}_{2,u}, \tag{64}$$

$$K_{2,u} = P_u^{\tilde{x}\tilde{z}_2}(P_u^{\tilde{z}_2\tilde{z}_2})^{-1}, \tag{65}$$

$$P_{2,u|u} = \frac{(\nu-2)(\nu+\Delta^2_{2,u})}{\nu(\nu+n_2-2)}\big[P_{1,u|u} - K_{2,u}(P_u^{\tilde{z}_2\tilde{z}_2})^{-1}K^{\mathrm{T}}_{2,u}\big], \tag{66}$$

$$\Delta^2_{2,u} = \tilde{z}^{\mathrm{T}}_{2,u}(P_u^{\tilde{z}_2\tilde{z}_2})^{-1}\tilde{z}_{2,u}, \tag{67}$$

$$\tilde{z}_{2,u} = z_{2,u} - \hat{z}_{2,u}. \tag{68}$$

Generally speaking, for $2 \leqslant p \leqslant N$, we have the approximate $t$-distribution

$$p(x_u|Z_{u-1}, Z_{1:p,u}) \approx \mathrm{St}(x_u; \hat{x}_{p,u|u}, P_{p,u|u}, \nu), \tag{69}$$

where

$$\hat{x}_{p,u|u} = \hat{x}_{p-1,u|u} + K_{p,u}\tilde{z}_{p,u}, \tag{70}$$

$$P_{p,u|u} = \frac{(\nu-2)(\nu+\Delta^2_{p,u})}{\nu(\nu+n_p-2)}\big(P_{p-1,u|u} - K_{p,u}(P_u^{\tilde{z}_p\tilde{z}_p})^{-1}K^{\mathrm{T}}_{p,u}\big). \tag{71}$$

In the above equations,

$$K_{p,u} = P_u^{\tilde{x}\tilde{z}_p}(P_u^{\tilde{z}_p\tilde{z}_p})^{-1}, \tag{72}$$

$$\Delta^2_{p,u} = \tilde{z}^{\mathrm{T}}_{p,u}(P_u^{\tilde{z}_p\tilde{z}_p})^{-1}\tilde{z}_{p,u}, \tag{73}$$

$$\tilde{z}_{p,u} = z_{p,u} - \hat{z}_{p,u}, \tag{74}$$

$$\begin{aligned}\hat{z}_{p,u} &= E[z_{p,u}|Z_{u-1}, Z_{1:p-1,u}]\\ &= \int h_u(x_u)\mathrm{St}(x_u; \hat{x}_{p-1,u|u}, P_{p-1,u|u}, \nu)\mathrm{d}x_u\\ &\approx \sum_{j=0}^{2n}\omega_j h_u\big(\hat{x}^j_{p-1,u|u}\big),\end{aligned} \tag{75}$$

$$\begin{aligned}P_u^{\tilde{x}\tilde{z}_p} &= \frac{\nu-2}{\nu}E\big[\tilde{x}_{p-1,u|u}\tilde{z}^{\mathrm{T}}_{p,u}|Z_{u-1}, Z_{1:p-1,u}\big]\\ &= \frac{\nu-2}{\nu}\left\{\int x_u h_u^{\mathrm{T}}(x_u)\mathrm{St}(x_u; \hat{x}_{p-1,u|u}, P_{p-1,u|u}, \nu)\mathrm{d}x_u - \hat{x}_{p-1,u|u}\hat{z}^{\mathrm{T}}_{p,u}\right\}\\ &\approx \frac{\nu-2}{\nu}\left\{\left(\sum_{j=0}^{2n}\omega_j\hat{x}^j_{p-1,u|u}h_u^{\mathrm{T}}(\hat{x}^j_{p-1,u|u})\right) - \hat{x}_{p-1,u|u}\hat{z}^{\mathrm{T}}_{p,u}\right\},\end{aligned} \tag{76}$$

$$\begin{aligned}P_u^{\tilde{z}_p\tilde{z}_p} &= \frac{\nu-2}{\nu}E\big[\tilde{z}_{p,u}\tilde{z}^{\mathrm{T}}_{p,u}|Z_{u-1}, Z_{1:p-1,u}\big]\\ &= \frac{\nu-2}{\nu}\left\{\int h_u(x_u)h_u^{\mathrm{T}}(x_u)\mathrm{St}(x_u; \hat{x}_{p-1,u|u}, P_{p-1,u|u}, \nu)\mathrm{d}x_u - \hat{z}_{p,u}\hat{z}^{\mathrm{T}}_{p,u}\right\} + R_{p,u}\\ &\approx \frac{\nu-2}{\nu}\left\{\sum_{j=0}^{2n}\omega_j h_u\big(\hat{x}^j_{p-1,u|u}\big)h_u^{\mathrm{T}}(\hat{x}^j_{p-1,u|u}) - \hat{z}_{p,u}\hat{z}^{\mathrm{T}}_{p,u}\right\} + R_{p,u},\end{aligned} \tag{77}$$

where $\hat{x}^j_{p-1,u|u}$ are the sigma points of $\hat{x}_{p-1,u|u}$.

When $p = N$, the state estimation by sequentially fusing of sensors 1 to $N$ can be deduced,

$$\begin{aligned}\hat{x}_{s,u|u} &= \hat{x}_{N,u|u}\\ &= \hat{x}_{N-1,u|u} + K_{N,u}\tilde{z}_{N,u}\\ &= \hat{x}_{s,u|u-1} + \sum_{p=1}^{N}K_{p,u}\tilde{z}_{p,u},\end{aligned} \tag{78}$$

$$\begin{aligned}P_{s,u|u} &= P_{N,u|u}\\ &= \frac{(\nu-2)(\nu+\Delta^2_{N,u})}{\nu(\nu+n_N-2)}\big(P_{N-1,u|u} - K_{N,u}(P_u^{\tilde{z}_N\tilde{z}_N})^{-1}K^{\mathrm{T}}_{N,u}\big)\\ &= \left(\frac{\nu-2}{\nu}\right)^N\prod_{p=1}^{N}\left(\frac{\nu+\Delta^2_{p,u}}{\nu+n_p-2}\right)P_{s,u|u-1} - \sum_{p=1}^{N}\left\{\left(\frac{\nu-2}{\nu}\right)^{N-p+1}\prod_{j=p}^{N}\left(\frac{\nu+\Delta^2_{j,u}}{v+n_j-2}\right)\right.\end{aligned}$$

$$\times K_{p,u}(P_u^{\tilde{z}_p\tilde{z}_p})^{-1}K_{p,u}^{\mathrm{T}}\Bigg\},\tag{79}$$

where for $p=1,2,\ldots,N$, $\tilde{x}_{p,u|u}=x_u-\hat{x}_{p,u|u}$, and $\hat{x}_{0,u|u}=\hat{x}_{s,u|u-1}$, $P_{0,u|u}=P_{s,u|u-1}$. The subscript $s$ in (44)–(79) denotes the sequential fusion. This completes the proof.

**Theorem 2** (Sequential fusion of intermittent measurements)**.** For system (1)–(5), by sequential fusion of sensors 1 to $N$, the state fusion estimation can be computed by

$$\begin{cases}\hat{x}^*_{s,u|u}=\hat{x}^*_{s,u|u-1}+\displaystyle\sum_{p=1}^{N}K^*_{p,u}\tilde{z}^*_{p,u},\\ P^*_{s,u|u}=P^*_{N,u|u},\end{cases}\tag{80}$$

where

$$\begin{cases}\hat{x}^*_{0,u|u}=\hat{x}^*_{s,u|u-1}=\displaystyle\sum_{j=0}^{2n}\omega_j f_{u-1}(\hat{x}^{j,*}_{s,u-1|u-1}),\\ P^*_{0,u|u}=P^*_{s,u|u-1}\\ \qquad=\dfrac{\nu-2}{\nu}\Bigg\{\displaystyle\sum_{j=0}^{2n}\left[\omega_j f_{u-1}(\hat{x}^{j,*}_{s,u-1|u-1})f^{\mathrm{T}}_{u-1}(\hat{x}^{j,*}_{s,u-1|u-1})\right]\\ \qquad\quad-\hat{x}^*_{s,u|u-1}\hat{x}^{*,\mathrm{T}}_{s,u|u-1}\Bigg\}+Q_{u-1},\end{cases}\tag{81}$$

and for $p=1,2,\ldots,N$,

$$\begin{cases}\hat{x}^*_{p,u|u}=\hat{x}^*_{p-1,u|u}+K^*_{p,u}\tilde{z}^*_{p,u},\\ P^*_{p,u|u}=(1-\gamma_{p,u})P^*_{p-1,u|u}+\dfrac{(\nu-2)(\nu+\Delta^{*,2}_{p,u})\gamma_{p,u}}{\nu(\nu+n_p-2)}\left(P^*_{p-1,u|u}-K^*_{p,u}(P_u^{\tilde{z}_p\tilde{z}_p,**})^{-1}K^{*,\mathrm{T}}_{p,u}\right),\\ K^*_{p,u}=P_u^{\tilde{x}\tilde{z}_p,*}(P_u^{\tilde{z}_p\tilde{z}_p,**})^{-1},\\ (\Delta^*_{p,u})^2=\tilde{z}^{*,\mathrm{T}}_{p,u}(P_u^{\tilde{z}_p\tilde{z}_p,**})^{-1}\tilde{z}^*_{p,u},\\ \tilde{z}^*_{p,u}=\gamma_{p,u}[z_{p,u}-\hat{z}^*_{p,u}],\\ P_u^{\tilde{x}\tilde{z}_p,*}=\dfrac{(\nu-2)\gamma_{p,u}}{\nu}\left\{\left(\displaystyle\sum_{j=0}^{2n}\omega_j\hat{x}^{j,*}_{p-1,u|u}h_u^{\mathrm{T}}(\hat{x}^{j,*}_{p-1,u|u})\right)-\hat{x}^*_{p-1,u|u}\hat{z}^{*,\mathrm{T}}_{p,u}\right\},\\ P_u^{\tilde{z}_p\tilde{z}_p,**}=\dfrac{\nu-2}{\nu}\left\{\displaystyle\sum_{j=0}^{2n}\left[\omega_j h_u(\hat{x}^{j,*}_{p-1,u|u})h_u^{\mathrm{T}}(\hat{x}^{j,*}_{p-1,u|u})\right]-\hat{z}^*_{p,u}\hat{z}^{*,\mathrm{T}}_{p,u}\right\}+R_{p,u},\\ \hat{z}^*_{p,u}=\displaystyle\sum_{j=0}^{2n}\omega_j h_u(\hat{x}^{j,*}_{p-1,u|u}),\end{cases}\tag{82}$$

where the joint subscript $s$ and superscript $*$ denotes the sequential fusion of intermittent observations. $\hat{x}^{j,*}_{s,u|u-1}$ and $\hat{x}^{j,*}_{i-1,u|u}$ are the sigma points of $\hat{x}^*_{s,u|u-1}$ and $\hat{x}^*_{p-1,u|u}$, respectively, for $j=0,1,2,\ldots,2n$.

*Proof.* By iteratively use of the MRTNF presented in Section 3, this theorem can be proved. The detailed proof is similar to that of Theorem 2.

### 4.3 Naïve distributed fusion

Distributed fusion structure means every sensor gets its local estimation and then the local estimations from different sensors are sent to the fusion center to get the fusion estimation. For systems (1) and (2), the naïve fusion (NF) estimation of distributed structure $\hat{x}_{d,m|m}$ can be computed by [42, 43]

$$\begin{cases} \hat{x}_{d,u|u} = \sum_{p=1}^{N} \alpha_{p,u} \hat{x}'_{p,u|u}, \\ \alpha_{p,u} = \bar{P}_{d,u|u} \bar{P}'^{-1}_{p,u|u}, \\ \bar{P}_{d,u|u} = \left[ \sum_{p=1}^{N} \bar{P}'^{-1}_{p,u|u} \right]^{-1}, \\ \bar{P}'_{p,u|u} = \frac{\nu}{\nu - 2} P'_{p,u|u}, \end{cases} \tag{83}$$

where the subscript $d$ denotes the distributed fusion. $\hat{x}'_{p,u|u}$, $P'_{p,u|u}$, and $\bar{P}'_{p,u|u}$ are the local estimation, the local scale matrix, and the local estimation error covariance by filtering of sensor $p$ by use of the MRSTNF presented in Section 3, respectively, $p = 1, 2, \ldots, N$. $\hat{x}_{d,u|u}$ and $\bar{P}_{d,u|u}$ denote the distributed fusion estimation and the corresponding estimation error covariances.

One can see that the naïve distributed fusion is very simple, that is why it is called naïve fusion. Compared to the centralized fusion, it is the suboptimal algorithm. While, because of its high efficiency in computing and good robustness, it is widely used in many applications where the accuracy of the estimator is not highly concerned.

**Remark 2.** If the dof for the initial state, the measurement noises and the process noise are not equal, one may use the moment matching method to generate the modified filter, the centralized fusion, the sequential fusion and the distributed fusion algorithms, respectively. For example, for system (3)–(5), to best keep the heavy-tailed property, let $\nu = \min\{\nu_p, p = w, 0, 1, 2, \ldots, N\}$ [17]. By applying the moment matching method, $p(x_0)$, $p(w_u)$, and $p(v_{p,u})$ can be approximated by $p(x_0^*) = \mathrm{St}(x_0^*; \hat{x}_0, P_0^*, \nu)$, $p(w_u^*) = \mathrm{St}(w_u^*; 0, Q_u^*, \nu)$, and $p(v_{p,u}^*) = \mathrm{St}(v_{p,u}^*; 0, R_{p,u}^*, \nu)$, respectively, where $P_0^* = \frac{(\nu-2)\nu_0}{(\nu_0-2)\nu} P_0$, $Q_u^* = \frac{(\nu-2)\nu_w}{(\nu_w-2)\nu} Q_u$, and $R_{p,u}^* = \frac{(\nu-2)\nu_p}{(\nu_p-2)\nu} R_{p,u}$.

Then, $w_u^*$ and $w_u$, $v_{p,u}^*$ and $v_{p,u}$, $x_0^*$ and $x_0$ have the same mean and covariance, respectively [40]. By use of $Q_u^*$, $P_0^*$, and $R_{p,u}^*$ to replace $Q_u$, $P_0$, and $R_{p,u}$, respectively, in Sections 3 and 4, one can obtain the $t$-distribution based algorithms for heavy-tailed systems with unequal dofs.

**Remark 3.** It can be easily proved that when $\nu_w$ and $\nu_p, p = 0, 1, \ldots, N$ approach to infinity, the deduced algorithms reduce to the classical UKF/CKF based filter [37, 44, 45], centralized fusion, sequential fusion, and distributed fusion, respectively. Hence, the algorithms deduced in this paper based on multivariate $t$-distribution are the extension of the classical methods based on Gaussian distributions. In fact, $t$-distribution is approaching to the Gaussian distribution when the dof is large enough. So, in order to better describe the heavy-tailed noise, a smaller value for the dof of the $t$-distribution is better.

## 5 Simulation

Consider a target moving with the nearly constant turn (NCT) in 2D with unknown angular velocity. The state of the target is

$$x_u = [\xi \;\; \dot{\xi} \;\; \eta \;\; \dot{\eta} \;\; \Theta]^{\mathrm{T}}, \tag{84}$$

where $\xi$ and $\eta$ denote the target position in horizontal ($x$) and vertical ($y$) directions, respectively, $\dot{\xi}$ and $\dot{\eta}$ denote velocity of the target in $x$ and $y$ directions, respectively, and $\Theta$ is the turn rate.

The dynamic model for the NCT motion can be formulated by

$$x_u = \begin{bmatrix} 1 & \frac{\sin\Theta T}{\Theta} & 0 & \frac{\cos\Theta T - 1}{\Theta} & 0 \\ 0 & \cos\Theta T & 0 & -\sin\Theta T & 0 \\ 0 & \frac{1 - \cos\Theta T}{\Theta} & 1 & \frac{\sin\Theta T}{\Theta} & 0 \\ 0 & \sin\Theta T & 0 & \cos\Theta T & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_{u-1} + w_{u-1}, \tag{85}$$

where $T$ means the fixed time-interval between two consecutive measurement times.

We consider a radar (treated as sensor 1) located at $(\xi_{\rm s1}, \eta_{\rm s1})$ that measures the range and azimuth of the target. A ground moving target indication (GMTI) radar (treated as the sensor 2) located at $(\xi_{\rm s2}, \eta_{\rm s2})$ measures the range, azimuth, and radial velocity of the target. In this simulation, we choose the location (1500, 1000) for sensor 1, and location (0, 1000) for sensor 2.

The measurement model of the first radar is given by

$$z_{1,u} = \begin{bmatrix} \sqrt{(\xi-\xi_{\rm s1})^2+(\eta-\eta_{\rm s1})^2} \\ \tan^{-1}(\xi-\xi_{\rm s1}, \eta-\eta_{\rm s1}) \end{bmatrix} + v_{1,u}. \tag{86}$$

The measurement model of the GMTI radar is given by

$$z_{2,u} = \begin{bmatrix} \sqrt{(\xi-\xi_{\rm s2})^2+(\eta-\eta_{\rm s2})^2} \\ \tan^{-1}(\xi-\xi_{\rm s2}, \eta-\eta_{\rm s2}) \\ \dfrac{(\xi-\xi_{\rm s2})\dot{\xi}+(\eta-\eta_{\rm s2})\dot{\eta}}{\sqrt{(\xi-\xi_{\rm s2})^2+(\eta-\eta_{\rm s2})^2}} \end{bmatrix} + v_{2,u}. \tag{87}$$

The heavy-tailed system noise $w_u$ and observation noises $v_{1,u}$ and $v_{2,u}$ are generated by zero mean $t$-distributions with dof $\nu = 3$, and the scale matrices $Q$ and $R_i$, $i = 1, 2$ meet

$$\begin{aligned} &Q = \text{diag}\left[\sigma_{1w}M \ \sigma_{1w}M \ \sigma_{2w}T\right], \quad M = \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix}, \\ &\sigma_{1w} = 0.1 \ \text{m}^2\text{s}^{-3}, \quad \sigma_{2w} = 6.25 \times 10^{-4} \ \text{s}^{-3}, \\ &R_1 = \text{diag}\left[\sigma_{1,r}^2 \ \sigma_{1,\theta}^2\right], \quad R_2 = \text{diag}\left[\sigma_{2,r}^2 \ \sigma_{2,\theta}^2 \ \sigma_{2,v_r}^2\right], \end{aligned}$$

where $\sigma_{1,r} = 25$ m, $\sigma_{1,\theta} = 16$ mrad, $\sigma_{2,r} = 30$ m, $\sigma_{2,\theta} = 25$ mrad, $\sigma_{2,v_r} = 2.5$ m/s.

The measurement time intervals of the first radar and the GMTI radar are both $T = 1$ s. The system initial values of the state meet

$$p(x_0) = \text{St}(x_u; \hat{x}_0, P_0, \nu_0), \tag{88}$$

where $\nu_0 = 3$, and

$$\hat{x}_0 = [1000 \ \text{m} \ \ 8 \ \text{ms}^{-1} \ \ 1000 \ \text{m} \ \ 5 \ \text{ms}^{-1} \ 6^\circ \ \text{s}^{-1}]^{\rm T}, \tag{89}$$

$$P_0 = \text{diag}[100 \ \text{m}^2 \ 9 \ \text{m}^2\text{s}^{-2} \ 100 \ \text{m}^2 \ 9 \ \text{m}^2\text{s}^{-2} \ 3.25 \ \text{mrad}^2\text{s}^{-2}]. \tag{90}$$

The RMSE (root mean square error) of the position and the velocity at time $u$ will be used to evaluate the fusion performance, which is computed by (91) and (92).

$$\text{RMSE}_{\rm p}(u) = \sqrt{\frac{1}{M_c}\sum_{i=1}^{M_c}[(\xi_u^i - \hat{\xi}_u^i)^2 + (\eta_u^i - \hat{\eta}_u^i)^2]}, \tag{91}$$

$$\text{RMSE}_{\rm v}(u) = \sqrt{\frac{1}{M_c}\sum_{i=1}^{M_c}[(\dot{\xi}_u^i - \hat{\dot{\xi}}_u^i)^2 + (\dot{\eta}_u^i - \hat{\dot{\eta}}_u^i)^2]}. \tag{92}$$

For each simulation, $M_c = 100$ Monte Carlo simulations are run to get the RMSEs of the target position or velocity.

Suppose the measurements are randomly missing with the probability of 10% for each sensor. Namely, for each sensor $p = 1, 2$, 10% of $\gamma_{p,u}$ are randomly set to be 0, and the remaining are equal to 1. To show the effectiveness of fusion algorithms of Section 4, we compare them with the classical CKF based centralized fusion, in which the heavy-tailed system noise and observation noises are treated as Gaussian noises, whose covariances are $Q_g = \frac{\nu}{\nu-2}Q$ and $R_{p,g} = \frac{\nu}{\nu-2}R_p$, respectively. For the proposed algorithms, the UT parameters are set as $\kappa = 0$, $\alpha = 1$.

Figure 1(a) draws the RMSEs of the position for different algorithms, including the MRSTNF of sensor 1 (S1) and sensor 2 (S2), the centralized fusion result (CF), the sequential fusion result (SF), and the

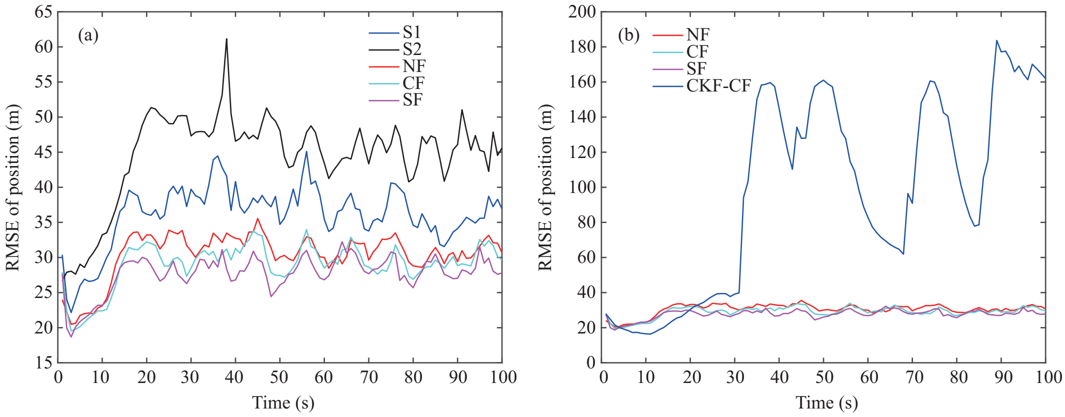


**Figure 1** (Color online) RMSEs of the position by (a) different algorithms and (b) different fusion algorithms.

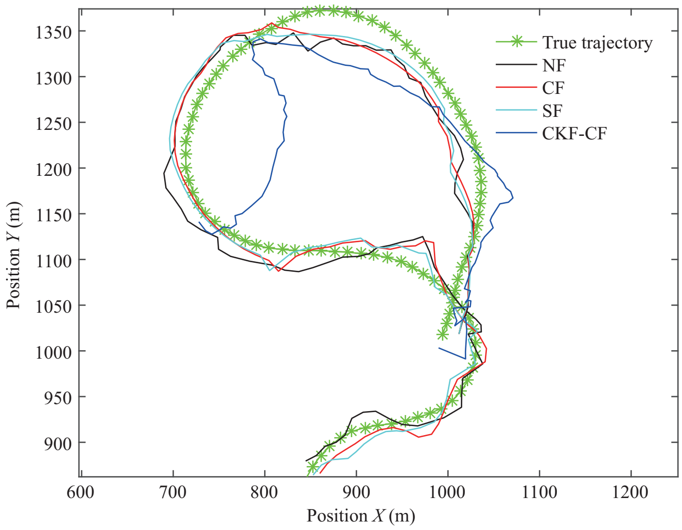


**Figure 2** (Color online) Trajectories of the true target and the estimates.

RMSE by the NF. From Figure 1(a), it can be seen that in case of package drop out, all the presented heavy-tailed algorithms are effective and can get reliable and convergent estimations. All fusion algorithms are effective due to the fact that the RMSEs of which are smaller than the single sensor's. The SF has the highest tracking accuracy in position estimation, followed by the CF and then the NF.

To assess the performance of the presented fusion algorithms for heavy-tailed noises, we compare them with the classical CKF based centralized fusion in Figure 1(b), from which one can notice that the CKF based centralized fusion (CKF-CF) has a much larger RMSE compared to those obtained by the presented CF, SF, and NF. Figure 1(b) shows that the classical CKF based centralized fusion, which is well known to be effective in state estimation of Gaussian noises does not work well in the estimation of nonlinear systems with heavy-tailed noises. Therefore, the presented algorithms are more robust in handling the estimation problem in the case of outliers.

In order to observe the tracking effect intuitively, we show the tracking trajectories of different algorithms in Figure 2. From the trajectories of the target and the estimates obtained by different algorithms, one can see that all the fusion algorithms for heavy-tailed noises (CF, SF, NF) can track the target effectively, while the CKF based centralized fusion (CKF-CF) fail to track. Therefore, from Figure 2, one may reach the same conclusion as those from Figure 1(b), the presented algorithms are more effective in handling heavy-tailed noises than the classical CKF based fusion.

The RMSEs of velocity from different algorithms are drawn in Figure 3. One can observe that all the presented algorithms are effective in estimating the velocity. Besides, the velocity RMSEs in Figure 3(a) illustrate the similar trend as the RMSEs of position in Figure 1(a), which means the fusion algorithms are superior to the MRSTNF of a single sensor in velocity estimation. Figure 3(b) shows the same trend

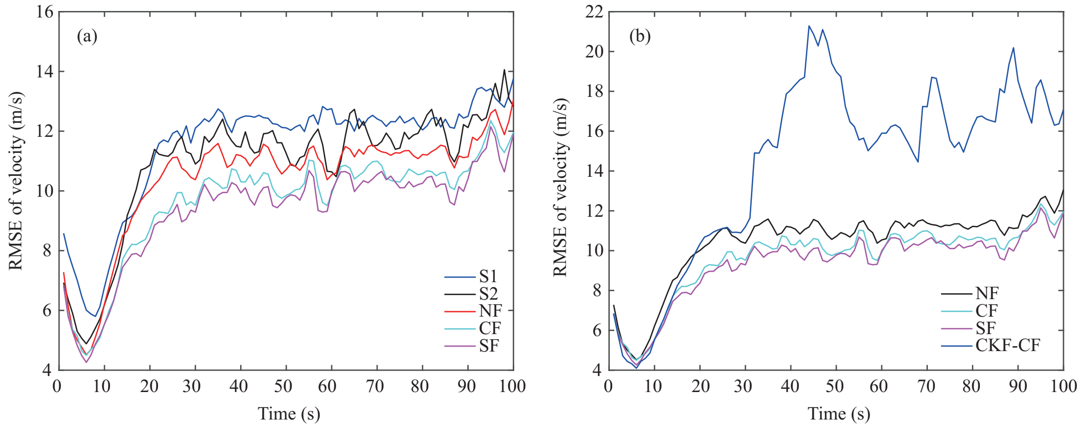


**Figure 3** (Color online) RMSEs of the velocity by (a) different algorithms and (b) different fusion algorithms.

**Table 1** Average RMSEs of position and velocity by different algorithms

| | S1 | S2 | NF | CF | SF | CKF-CF |
|---|---|---|---|---|---|---|
| $RMSE_p$ (m) | 36.0867 | 44.5847 | 30.4353 | 29.0117 | 27.7376 | 96.8804 |
| $RMSE_v$ (m/s) | 11.4976 | 10.9172 | 10.3956 | 9.6463 | 9.3494 | 14.3077 |

**Table 2** Average CPU time per Monte Carlo run of various algorithms

| | S1 | S2 | NF | CF | SF | CKF-CF |
|---|---|---|---|---|---|---|
| CPU time (ms) | 33.94 | 35.44 | 43.58 | 40.81 | 53.76 | 29.22 |

as Figure 1(b), which means the classical CKF based fusion (CKF-CF) performs with big deviation when estimating the target velocity and the proposed fusion algorithms show superior performance.

To show more specific numerical analysis, we present the average position and velocity RMSEs of sensor 1, sensor 2, and different fusion algorithms mentioned above in Table 1, in which S1 and S2 are the average RMSEs by using the MRSTNF derived in Section 3, CF, SF, and NF are the average RMSEs of the centralized, the sequential and the naïve fusion algorithms of Section 4, while CKF-CF denotes the average RMSEs by using the classical CKF based centralized fusion. From Table 1, one could reach the same result as those from Figures 1–3: (1) In case of package drop out, the heavy-tailed filtering algorithm and fusion algorithms are effective; (2) the proposed fusion algorithms show higher accuracy than the single sensor filtering; (3) the SF shows the best performance among all fusion algorithms, followed by the CF and then the NF; (4) the CKF-CF for Gaussian noises is no longer fit for state estimation of nonlinear heavy-tailed noise systems, which shows effectiveness and application value of proposed algorithms.

In Table 2, the CPU time of different algorithms is listed. Combining the results of Tables 1 and 2, one can see that the CKF-CF has the fastest computation speed but lowest accuracy among all algorithms, which is consistent with the theoretical analysis that the computation complexity of heavy-tailed algorithms is a little bit higher than the Gaussian distribution based methods. Therefore, it is reasonable to take more computation time. For all algorithms based on heavy-tailed noises, including single sensor filtering of S1 and S2, fusion algorithms NF, CF, and SF, there is no doubt that the CPU time of a single sensor is smaller than all fusion methods'. For the three heavy-tailed fusion algorithms, the CF shows the fastest computation speed and middle level of accuracy. Comparing with the NF in lowest accuracy and middle level of computation speed and the SF with the best accuracy and lowest computation speed, the CF seems more cost-effective. But in reality, the observations of different sensors are hardly received by the fusion center at exactly the same time, so the SF and the NF are more valuable and meaningful in practical applications.

**Remark 4.** For the state estimation of Gaussian driven linear systems, it is well known that the Kalman filter based optimal centralized fusion and the optimal sequential fusion are equivalent in the sense of least mean-square error (LMSE) [22, 46, 47]. While, for the student $t$-distribution based filter, to keep the heavy-tailed property, the moment matching method is used in generating the estimation, so it is in fact an approximate filter. Therefore, unlike the Gaussian driven system, for the heavy-tailed system, the approximate $t$-filter based centralized fusion and the sequential fusion are not equivalent. In fact, we have proved in [3] that either of them could be better. In estimating the nonlinear system state of heavy-tailed systems, we have the same conclusion. Therefore, it is not strange that the SF shows better result than the CF in this example.

## 6 Conclusion

The state estimation of a class of nonlinear heavy-tailed multisensor system is studied. Based on multivariate $t$-distribution, a robust nonlinear filter is presented when the measurements are obtained intermittently. Based on the presented robust nonlinear filter, the centralized, the sequential, and the naïve-distributed fusion algorithms are proposed, respectively. Through theoretical analysis and simulation results, one can conclude the following: (1) the CKF-based centralized fusion algorithm for Gaussian noise is no longer suitable to handle the state estimation problem of nonlinear heavy-tailed systems, while all the presented algorithms are effective; (2) the centralized and the sequential fusion algorithms are both superior to the naïve-distributed fusion algorithm, which shows better result than the local estimates; (3) the presented student $t$-based algorithms are extensions of the classical UKF/CKF-based algorithms; (4) the computation complexity of the heavy-tailed methods is slightly higher (in an acceptable range) than the Gaussian distribution-based fusion methods. Based on the algorithms presented in [36–39, 48], the developed algorithms can be further used to solve various engineering problems such as distribution filtering or state estimation under time-delay, event-triggered strategy, or cyber attacks. Based on the above analysis, the proposed algorithms have potential application in many practical problems, such as target tracking and navigation.

**Acknowledgements** This work was supported by the National Natural Science Foundation of China (Grant Nos. 62076031, 62073036) and Beijing Natural Science Foundation (Grant No. 4202071).

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