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Simplified prescribed performance tracking control of uncertain nonlinear systems

Quan-Yong FAN[1], Shuoheng XU[1], Bin XU[1] & Jianlong QIU[2,3*]

[1]*School of Automation, Northwestern Polytechnical University, Xi'an* 710129*, China;*
[2]*School of Automation and Electrical Engineering, Linyi University, Linyi* 276005*, China;*
[3]*Key Laboratory of Complex Systems and Intelligent Computing in Universities of Shandong, Linyi University, Linyi* 276005*, China*

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Dear editor,

In practical engineering, some certain transient and steady performance, such as overshoot, adjustment time, and steady-state error, need to meet specific requirements. In order to ensure that the system state or tracking error remains within a specific time-varying boundary, the prescribed performance control (PPC) method is firstly proposed in [1]. Although the PPC method is widely investigated and used at present, most of the existing methods need to derive the transformation function many times [2, 3], which increases the design complexity.

In this study, a new PPC design method based on the sum of squares (SOS) technique is given to make the tracking error meet a desired time-varying constraint condition, which has a simpler analysis process.

Problem formulation and main result. Consider a class of nth order nonlinear systems given by the following form:

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t), \quad 1 \leqslant i \leqslant n-1, \\ \dot{x}_n(t) = f(x(t)) + g(x)u(t) + d(t), \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the system state vector. $u(t)$ is the control input. $y(t) = x_1(t)$ is the output. $f(x(t))$ is an unknown smooth nonlinear function, $g(x(t))$ is a nonzero nonlinear function, and $d(t)$ is the bounded disturbance.

The reference trajectory is given by the following model:

$$\begin{cases} \dot{x}_{r,i}(t) = x_{r,i+1}(t), \quad 1 \leqslant i \leqslant n-1, \\ \dot{x}_{r,n}(t) = f_r(x_r(t)) + v_r(t), \end{cases} \tag{2}$$

where $x_r(t) = [x_{r,1}(t), x_{r,2}(t), \ldots, x_{r,n}(t)] \in \mathbb{R}^n$ is the state vector of the reference model. $y_r(t) = x_{r,1}(t)$ is the reference output. $f_r(x_r(t))$ is an unknown smooth nonlinear function. $v_r(t)$ is the bounded input, whose derivative is bounded.

The main objective of this study is to propose a new controller design and analysis method so that y can track y_r with the desired transient and steady performance.

Define the tracking error $e_r(t)$ as follows:

$$e_r(t) = x(t) - x_r(t) = [e_{r1}, e_{r2}, \ldots, e_{rn}]^{\mathrm{T}}. \tag{3}$$

The derivative of $e_r(t)$ is obtained as

$$\dot{e}_r(t) = Ae_r + Bg(x)u + BD(t) + B(f(x(t)) - f_r(x_r(t))), \tag{4}$$

where $D(t) = d(t) - v_r(t)$,

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ 1 \end{pmatrix},$$

$a_{11} \in \mathbb{R}^{n-1}$ is a zero vector, $a_{12} \in \mathbb{R}^{(n-1)\times(n-1)}$ is an identity matrix, $a_{21} = 0$, $a_{22} = a_{11}^{\mathrm{T}}$, and $b_1 = a_{11}$.

The control policy u is designed as follows:

$$u = g^{-1}(x)(K_p e_r + u_f + u_d + u_s + u_b), \tag{5}$$

where $K_p = [r_1, r_2, \ldots, r_{n-1}, r_n]$. u_f and u_d are used to compensate $f(x(t)) - f_r(x_r(t))$ and $D(t)$, respectively. u_s is a robust control input, which together with u_f and u_d will be designed in the following part. u_b is designed to guarantee that the tracking error e_{r1} meets the prescribed performance, $-F_b < e_{r1} < F_b$. The time-varying boundary function F_b is given as

$$F_b = (F_0 - F_\infty)e^{-\zeta t} + F_\infty, \tag{6}$$

where $\zeta > 0$, $F_0 > 0$, and $F_\infty > 0$. It is assumed that $-F_0 < e_{r1}(0) < F_0$, which means that there exists one positive constant c_e such that $F_0 - |e_{r1}(0)| > c_e$. In this study, it is assumed that $c_e \nrightarrow 0$.

Then, u_b is designed as $u_b = -T(e_{r1})$, where

$$T(e_{r1}) = \frac{e_{r1}^2}{F_b - e_{r1}} q(e_{r1}) - \frac{e_{r1}^2}{F_b + e_{r1}} (1 - q(e_{r1})) \tag{7}$$

with

$$q(e_{r1}) = \begin{cases} 1, & e_{r1} \geqslant 0, \\ 0, & e_{r1} < 0. \end{cases}$$

* Corresponding author (email: qiujianlong@lyu.edu.cn)

It can be found that $T_{e_{r1}} \to +\infty$ and $T_{e_{r1}} \to -\infty$ when $e_{r1} \to F_b$ and $e_{r1} \to -F_b$, respectively.

Substituting (5) into (4) , it is obtained that

$$\dot{e}_r = Ae_r + BK_p e_r + D_q e_r + \Delta, \tag{8}$$

where

$$D_q = \begin{pmatrix} a_{11} & \mathbf{0} \\ d_q & a_{22} \end{pmatrix},$$
$$d_q = -\frac{e_{r1}}{F_b - e_{r1}} q(e_{r1}) + \frac{e_{r1}}{F_b + e_{r1}}(1 - q(e_{r1})),$$
$$\Delta = B(u_f + u_d + D(t) + f(x) - f_r(x_r) + u_s).$$

Suppose that there exists one time instant $t_l > 0$ such that $|e_{r1}(t_l)| \geqslant F_b(t_l)$. Then, define t_β as the first time instant when $-F_b < e_{r1} < F_b$ is violated, which means that $0 < t_\beta \leqslant t_l$. Therefore, we have $|e_{r1}(t)| < F_b(t)$ with $t < t_\beta$. It can be found that $e_{r1}(t)$ is continuous. Therefore, it is known that $\lim_{t \to t_\beta^-} |e_{r1}(t)| = F_b(t_\beta)$.

For $t \in [0, t_\beta)$, Theorem 1 can be obtained based on the SOS technique.

Theorem 1. For the dynamics $\dot{e}_r = Ae_r + BK_p e_r + D_q e_r$, if there exist a matrix Y, a symmetric positive definite matrix Q and a positive constant κ_1, such that the following condition holds:

$$\begin{pmatrix} -N_{be_1} Q_{be} - He(D_{p1} Q) & -Q(F_b - e_{r1}) \\ -Q(F_b - e_{r1}) & \kappa_1 I \end{pmatrix} \text{ is SOS}, \tag{9}$$

$$\begin{pmatrix} -N_{be_2} Q_{be} - He(D_{p2} Q) & -Q(F_b + e_{r1}) \\ -Q(F_b + e_{r1}) & \kappa_1 I \end{pmatrix} \text{ is SOS}, \tag{10}$$

where

$$N_{be_1} = (F_b - e_{r1})^2, \quad N_{be_2} = (F_b + e_{r1})^2,$$
$$Q_{be} = AQ + QA^{\mathrm{T}} + BY + Y^{\mathrm{T}} B^{\mathrm{T}},$$
$$D_{p1} = \begin{pmatrix} a_{11} & \mathbf{0} \\ d_{p1} & a_{22} \end{pmatrix}, \quad D_{p2} = \begin{pmatrix} a_{11} & \mathbf{0} \\ d_{p2} & a_{22} \end{pmatrix},$$
$$d_{p1} = -(F_b - e_{r1}) e_{r1}, \quad d_{p2} = (F_b + e_{r1}) e_{r1},$$

then we have

$$PA + A^{\mathrm{T}} P + PBK_p + K_p^{\mathrm{T}} B^{\mathrm{T}} P + PD_q + D_q^{\mathrm{T}} P < -\frac{1}{\kappa_1} I, \tag{11}$$

where $P = Q^{-1}$, $K_p = YQ^{-1}$.

Remark 1. The SOS conditions (9) and (10) are given in Theorem 1 to solve K_p. F_b acts as an independent variable just like e_{r1}, which means that we need to declare the independent variables e_{r1} and F_b based on the SOS technique.

Based on the method given in [4–6], the neural network approximation technology and the disturbance observer method can be used to estimate $f(x(t)) - f_r(x_r(t))$ and $D(t)$ with bounded estimation errors. The estimation of unknown nonlinear function is defined as $\hat{W}_a \sigma(V\bar{x})$. The estimation of disturbance is defined as $\hat{D}(t)$.

The control input u_f, u_d and u_s are designed as follows:

$$\begin{aligned} & u_f = -\hat{W}_a^{\mathrm{T}} \sigma(V\bar{x}), \quad u_d = -\hat{D}(t), \\ & u_s = -\nu_s \mathrm{sgn}(e_r^{\mathrm{T}} PB), \end{aligned} \tag{12}$$

where $P = Q^{-1}$, Q is obtained in Theorem 1, and ν_s is a positive constant.

Then, Theorem 2 is obtained.

Theorem 2. For the tracking error dynamics (8), the control policy (5) can guarantee that $\lim_{t\to\infty} \|e_r(t)\| = 0$ with the desired transient and steady tracking error constraint $-F_b < e_{r1} < F_b$.

Proof. Choose $V_r = e_r^{\mathrm{T}} P e_r$ as the Lyapunov function. If ν_s is large enough, we can easily prove the boundedness of tracking error e_r for $t \in [0, t_\beta)$. Therefore, it is known that u_s and Δ are bounded.

Define the filter tracking error as $e_f(t) = [\Upsilon, 1] e_r(t)$, where $\Upsilon = [Z_1, Z_2, \ldots, Z_{n-1}]$. Since e_r is bounded and continuous, it is known that e_{r1}, e_f and $\dot{e}_f$ are bounded for $t \in [0, t_\beta)$. Choose one appropriate positive constant m_z to ensure that $Z_{i-1} = -m_z Z_i - r_i$ and $Z_{n-1} = -m_z - r_n$ hold for $2 \leqslant i \leqslant n-1$.

Then, it is obtained that $\dot{e}_f(t) = -m_z e_f(t) + \Delta_{e_f}$, where $\Delta_{e_f} = (m_z Z_1 + r_1) e_{r1} - T_{e_{r1}} + \Delta$. The dynamics $\dot{e}_f(t) = -m_z e_f(t)$ is input-to-state stable. If $e_{r1} \to -F_b$ or $e_{r1} \to F_b$, it is known that $T_{e_{r1}} \to -\infty$ or $T_{e_{r1}} \to \infty$ based on (7). Then, we have $\dot{e}_f \to -\infty$ or $\dot{e}_f \to \infty$, which is in contradiction with the boundedness of $\dot{e}_f$. Therefore, the tracking error $|e_{r1}(t)|$ cannot approach the boundary F_b for $t \in [0, t_\beta)$, which means that t_l does not exist [7].

Since e_r is uniformly continuous, the conclusion $\lim_{t\to\infty} \|e_r(t)\| = 0$ can be obtained based on Barbalat Lemma. Moreover, the tracking error constraint $-F_b < e_{r1} < F_b$ can be guaranteed by the control policy (5).

Conclusion. For a class of uncertain nonlinear systems, a new adaptive PPC method is proposed in this study. Based on the SOS technique, one new transformation function and the corresponding analysis method are given to guarantee that the output tracking error remains within a prescribed time-varying boundary.

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