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Is fully distributed adaptive protocol applicable to graphs containing a directed spanning tree?

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Dear editor,

As is mentioned in [1], most of the existing protocols for cooperative control of multiagent systems under directed graphs require global connectivity information of the entire communication graph, which may not be able to accommodate the network size fluctuation, impeding the application on large-scaled systems. Fortunately, such limitations can be overcome by introducing the adaptive control methods to estimate such global graph connectivity and generate fully distributed adaptive protocols [1]. The fully distributed adaptive protocols for linear multiagent systems were presented in [2] to realize the leader-follower consensus tracking under directed graphs. Novel simplified distributed adaptive protocols, which use an additive term, were put forward in [3] for linear multiagent systems. The fully distributed adaptive formation tracking of multiple leaders was studied in [4]. The bipartite output consensus tracking was considered in [5] with fully distributed adaptive protocols. Fully distributed adaptive anti-windup protocols were further investigated in [6] to achieve consensus with input saturation constraints, and Ref. [7] proposed the fully distributed containment control of multiagent systems under directed graphs.

Yet despite all that, a basic problem remains unsolved, that is, whether the fully distributed adaptive protocol is applicable to general directed graphs containing a spanning tree. Note that the communication topologies in the above literature are either leader-follower graphs containing the directed spanning tree with the leader as the root node, or the strongly connected graphs. Although the requirement on communication topologies is relaxed compared with undirected graphs, a gap still exists between the leader-follower graphs (or, strongly connected graphs) and the general directed graphs containing the spanning tree. To be specific, the former two types of directed graphs are special cases of the latter.

This study intends to present a unified framework of consensus for linear multiagent systems with fully distributed adaptive protocols. It is revealed that the sufficient and necessary condition to realize consensus of linear multiagent systems under fully distributed adaptive protocols is that the communication graph contains a directed spanning tree.

Problem statements. Consider a group of N agents with the dynamics described by

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^p$ represent the state and control input of the i-th agent, respectively. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$ are the constant dynamic matrix and the input matrix.

In this study, we intend to realize consensus of multiagent systems under fixed communication graphs in the sense that $\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0, \forall i, j$ with arbitrary initial values $x_i(0)$. Let $\xi_i = \sum_{j=1}^N a_{ij}(x_i - x_j)$ denote the consensus error of each agent. Then, the consensus is realized if and only if ξ_i converges to zero.

Protocol development. The distributed linear controller was proposed in [8],

$$u_i = cK\xi_i, \tag{2}$$

where $K = -B^{\mathrm{T}}P$ with $Q = P^{-1} > 0$ being the solution of LMI $AQ + QA^{\mathrm{T}} - 2BB^{\mathrm{T}} < 0$, and the common constant gain $c > \frac{1}{\mathrm{Re}(\lambda_2(\mathcal{L}))}$ is designed to strengthen the graph connectivity. It was revealed in [8] that the consensus can be realized under protocol (2) if and only if the communication graph contains a directed spanning tree.

By employing the adaptive gain to estimate such a common constant gain, fully distributed adaptive protocols were proposed by introducing an extra multiplicative gain to tackle the asymmetric Laplacian matrix [2],

$$\begin{aligned} u_i &= d_i(1 + \xi_i^{\mathrm{T}}P\xi_i)^3K\xi_i, \\ \dot{d}_i &= \xi_i^{\mathrm{T}}PBB^{\mathrm{T}}P\xi_i, \end{aligned} \tag{3}$$

or introducing an extra additive gain to tackle the asymmetric Laplacian matrix [3],

$$\begin{aligned} u_i &= (d_i + \xi_i^{\mathrm{T}}P\xi_i)K\xi_i, \\ \dot{d}_i &= \xi_i^{\mathrm{T}}PBB^{\mathrm{T}}P\xi_i. \end{aligned} \tag{4}$$

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It should be pointed out that the communication graphs in [2, 3] are assumed to be leader-follower directed graphs containing the directed spanning tree or the leaderless case with strongly connected graphs. While graphs satisfying neither of these two conditions exist, but such graphs contain only a directed spanning tree; take Figure 1 as an example.

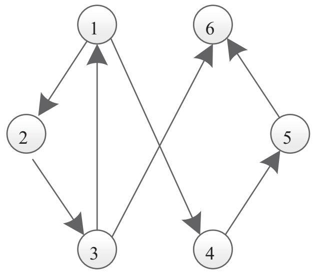

Figure 1 The graph contains a directed spanning tree, which is neither strongly connected nor a leader-follower graph.

It is of interest to consider whether the fully distributed adaptive protocols are applicable to general directed graphs containing a spanning tree. This study demonstrates that the answer to this question is yes!

Main results. We first introduce a property of the directed graph containing a directed spanning tree, which would play a key role in consensus demonstration.

Lemma 1 ([9])**.** Suppose the communication graph $\mathcal{G}$ contains a directed spanning tree. Then, the node set $\mathcal{V}$ can be divided into two disjoint subsets $\mathcal{V}_1$ and $\mathcal{V}_2$ such that the following two statements hold:

(i) The subgraph among $\mathcal{V}_1$ is strongly connected;

(ii) Each node of $\mathcal{V}_1$ has no neighbors in $\mathcal{V}_2$, i.e., $(i, j) \notin \mathcal{E}, \forall i \in \mathcal{V}_1, j \in \mathcal{V}_2$.

Remark 1. The proof of Lemma 1 can be found in Appendix A. For the strongly connected graphs, we have $\mathcal{V}_1 = \mathcal{V}$ and $\mathcal{V}_2 = \emptyset$; while for the leader-follower graph containing the directed spanning tree, $\mathcal{V}_1$ is the set of the leader and $\mathcal{V}_2$ is the set of followers. For the graph given in Figure 1, we have $\mathcal{V}_1 = \{1, 2, 3\}$ and $\mathcal{V}_2 = \{4, 5, 6\}$.

Considering Lemma 1, we are ready to present the main result of this study.

Theorem 1. The consensus of linear multiagent system (1) can be achieved under the fully distributed adaptive protocol (4) if and only if the communication graph $\mathcal{G}$ contains a directed spanning tree.

Remark 2. The proof of Theorem 1 is derived in Appendix B. Theorem 1 presents a unified framework of both leaderless consensus and leader-follower consensus tracking, because both the leader-follower graph containing the directed spanning tree and the strongly connected directed graph can be viewed as the special cases of the graphs containing a spanning tree. Note that the final consensus trajectory depends on all the nodes in $\mathcal{V}_1$, while the nodes in $\mathcal{V}_2$ track this trajectory. This is also coincident with the leader-follower case that all the followers track the trajectory of the leader.

Corollary 1. The consensus of linear multiagent system (1) can be achieved under the fully distributed adaptive protocol (3) if and only if the communication graph $\mathcal{G}$ contains a directed spanning tree.

Remark 3. Following the similar construction of the Lyapunov function, the results can be extended to the output feedback control, the output consensus of heterogeneous multiagent systems, or the consensus of multiagent systems with nonlinearities.

Remark 4. The fully distributed adaptive protocols (3) and (4) are nonlinear controllers. To realize the fully distributed manner, the adaptive control gain d_i is introduced to estimate the global eigenvalue information of the Laplacian matrix. Further, to conquer the asymmetry of the Laplacian matrix associated with a directed graph, an extra term is also added to the controller; see $(1 + \xi_i^{\mathrm{T}} P \xi_i)^3$ in (3) and $\xi_i^{\mathrm{T}} P \xi_i$ in (4).

Conclusion. This study presented a unified framework of consensus with fully distributed adaptive protocols. It is revealed that the sufficient and necessary condition on consensus realization is that the communication graph must contain a directed spanning tree, which fills in the gap between leaderless consensus with strongly connected graphs and consensus tracking with leader-follower graphs.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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