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HOSM controller design with asymmetric output constraints

Keqi MEI & Shihong DING*

School of Electrical and Information Engineering, Jiangsu University, Zhenjiang 212013, China

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Dear editor,

In many control systems, the output constraints are imposed for security reasons and intrinsic physical limitations [1]. For instance, the vibrations of moving strings should be kept within the constrained region to avoid serious hazards [2]. Handling output constraints for a controller design is undoubtedly an intractable problem. However, the barrier Lyapunov function (BLF) introduced in [1] provides a reliable approach for dealing with output constraints.

BLF has been recently used for characterization of classical first-order sliding mode (SM) [3] and second-order SM [4]. To the best of our knowledge, asymmetric output constraint problem for the high-order SM (HOSM) with an arbitrary relative degree of the sliding variable has never been considered in the literature.

In this study, we investigate HOSM with asymmetric output constraints for a class of systems governed by

$$\dot{x} = \Gamma(t,x) + \Xi(t,x)u,\ s = s(t,x), \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}$ is the control input, $\Gamma(t,x)$ and $\Xi(t,x)$ are smooth functions, s is the output (i.e., the sliding variable) and its relative degree is equal to n with respect to the control input u, i.e.,

$$s^{(n)} = A(t,x) + B(t,x)u \tag{2}$$

with $A(t,x)$, $B(t,x)$ being not exactly known but satisfying the following assumption.

Assumption 1. For system (2), one can find a positive function $\overline{A}(x)$ and a positive constant $\underline{B}$ such that

$$|A(t,x)| \leqslant \overline{A}(x),\ B(t,x) \geqslant \underline{B} > 0.$$

In addition, the output s should satisfy

$$-\underline{\delta} < s < \overline{\delta} \text{ for all } t \geqslant 0 \tag{3}$$

with two pre-defined positive constants $\underline{\delta}$ and $\overline{\delta}$.

Throughout the article, we use the following notations. For three positive real numbers $\underline{\delta}$, $\overline{\delta}$ and ϑ, $\overline{s}_n := (s_1,\ldots,s_n)^{\mathrm{T}}$, $\Omega_1(\underline{\delta},\overline{\delta}) := \{s_1 \mid s_1 \in \mathbb{R} \text{ with } -\underline{\delta} < s_1 < \overline{\delta}\}$, $\Omega_n(\underline{\delta},\overline{\delta}) := \{\overline{s}_n \mid \overline{s}_n \in \mathbb{R}^n \text{ with } -\underline{\delta} < s_1 < \overline{\delta}\}$, and $\lfloor z \rceil^{\vartheta} := \mathrm{sign}(z)|z|^{\vartheta}$ for any $z \in \mathbb{R}$.

Theorem 1. Consider system (1) under Assumption 1 with the variable s being initially constrained by (3). If the HOSM controller is constructed as

$$u = -\frac{\overline{A}(x)}{\underline{B}}\,\mathrm{sign}(\xi_n) - \beta_n \cdot \phi(s) \cdot \lfloor \xi_n \rceil^{\frac{r_{n+1}}{a}}, \tag{4}$$

where

$$\phi(s) = \frac{\underline{\delta}^{\frac{2\rho+\tau}{r_1}}\overline{\delta}^{\frac{2\rho+\tau}{r_1}}(\underline{\delta}\overline{\delta}+s^2)}{(\overline{\delta}-s)^{\frac{2\rho+\tau}{r_1}+1}(\underline{\delta}+s)^{\frac{2\rho+\tau}{r_1}+1}},$$

$\xi_n = \lfloor s^{(n-1)} \rceil^{\frac{a}{r_n}} - \lfloor \tilde{s}^{(n-1)} \rceil^{\frac{a}{r_n}}$, $\rho \geqslant a \geqslant r_1 > 0$, $\tau > 0$, $r_{j+1} = r_j - \tau \geqslant 0$, $j = 1,\ldots,n$, $\tilde{s} = 0$, $\dot{\tilde{s}} = -\beta_1 \lfloor s \rceil^{\frac{r_2}{r_1}}$, $\tilde{s}^{(i-1)} = -\beta_{i-1}\phi(s)\big\lfloor \lfloor s^{(i-2)} \rceil^{\frac{a}{r_{i-1}}} - \lfloor \tilde{s}^{(i-2)} \rceil^{\frac{a}{r_{i-1}}} \big\rceil^{\frac{r_i}{a}}$ for $i = 3,\ldots,n$, and β_i, $i = 1,\ldots,n$ are proper positive constants, then HOSM $s = \dot{s} = \cdots = s^{(n-1)} = 0$ in system (1) will be finite-time established by controller (4), while the variable s satisfies constraint (3) at all time.

Proof. First, let us assume that $s_i = s^{(i-1)}, i = 1,\ldots,n$. Then, system (2) can be rewritten as

$$\dot{s}_i = s_{i+1},\ i = 1,\ldots,n-1,\ \dot{s}_n = A(t,x) + B(t,x)u. \tag{5}$$

Thus, we only need to verify that system (5) with constraint (3) is finite-time stable when controller (4) is applied. Here, we assume that variable s_1 is initially inside the constraint (3).

We consider function $V_1 : \Omega_1(\underline{\delta},\overline{\delta}) \to \mathbb{R}$ with $V_1(s_1)$ given by

$$V_1(s_1) = \frac{r_1\underline{\delta}^{\frac{2\rho+\tau}{r_1}}\overline{\delta}^{\frac{2\rho+\tau}{r_1}}|s_1|^{\frac{2\rho+\tau}{r_1}}}{(2\rho+\tau)(\overline{\delta}-s_1)^{\frac{2\rho+\tau}{r_1}}(\underline{\delta}+s_1)^{\frac{2\rho+\tau}{r_1}}}. \tag{6}$$

Its time derivative reads as

$$\dot{V}_1(s_1) \leqslant -n\phi(s_1)|\xi_1|^{\frac{2\rho}{a}} + \phi(s_1)\lfloor \xi_1 \rceil^{\frac{2\rho-r_2}{a}}(s_2 - \tilde{s}_2), \tag{7}$$

where $\tilde{s}_2 = \dot{\tilde{s}}$ with $\beta_1 \geqslant n$ and $\xi_1 = \lfloor s_1 \rceil^{\frac{a}{r_1}}$.

* Corresponding author (email: dsh@mail.ujs.edu.cn)

Next, consider the following Lyapunov function:

$$V(\bar{s}_n)=V_1(s_1)+\sum_{j=2}^{n}\int_{\tilde{s}_j}^{s_j}\lfloor\lfloor\mu\rceil^{\frac{a}{r_j}}-\lfloor\tilde{s}_j\rceil^{\frac{a}{r_j}}\rceil^{\frac{2\rho-r_{j+1}}{a}}\mathrm{d}\mu.$$

Here we follow the approach proposed in [4] and differentiate $V(\bar{s}_n)$ along with system (5). We end up with

$$\begin{aligned}\dot{V}(\bar{s}_n)\leqslant&-\phi(s_1)\sum_{j=1}^{n-1}|\xi_j|^{\frac{2\rho}{a}}+c_n\phi(s_1)|\xi_n|^{\frac{2\rho}{a}}\\&+\lfloor\xi_n\rceil^{\frac{2\rho-r_{n+1}}{a}}(A(t,x)+B(t,x)u),\end{aligned}\quad(8)$$

where $c_n>0$ is a proper constant, $\xi_k=\lfloor s_k\rceil^{\frac{a}{r_k}}-\lfloor\tilde{s}_k\rceil^{\frac{a}{r_k}}$, and $\tilde{s}_k=\tilde{s}^{(k-1)}$, $k=1,\ldots,n$. Plugging controller (4) with $\beta_n\geqslant\frac{1+c_n}{\underline{B}}$ into (8), we get

$$\dot{V}(\bar{s}_n)\leqslant-\phi(s_1)\sum_{j=1}^{n}|\xi_j|^{\frac{2\rho}{a}}.\quad(9)$$

From Lemma 1 proposed in [1], it is evident that for $\bar{s}_n(0)\in\Omega_n(\underline{\delta},\bar{\delta})$, the trajectories of $\bar{s}_n(t)$ are well defined on $[0,\infty)$ and satisfy $-\underline{\delta}<s_1(t)<\bar{\delta}$, for all $t\geqslant 0$.

Otherwise, there exists a neighborhood of the origin:

$$\Pi=\left\{\bar{s}_n:-\frac{1}{2}\underline{\delta}\leqslant s_1\leqslant\frac{1}{2}\bar{\delta},\ s_2\in\mathbb{R},\ldots,\ s_n\in\mathbb{R}\right\},$$

such that for all $\bar{s}_n\in\Pi\subset\Omega_n(\underline{\delta},\bar{\delta})$,

$$V(\bar{s}_n)\leqslant 4^{\frac{2\rho+\tau}{r_1}}\sum_{j=1}^{n}|\xi_j|^{\frac{2\rho+\tau}{a}}.\quad(10)$$

Note that there exists a positive constant ε: $\varepsilon=(\frac{4\underline{\delta}\bar{\delta}}{(\underline{\delta}+\bar{\delta})^2})^{\frac{2\rho+\tau+r_1}{r_1}}$, such that $\phi(s_1)>\varepsilon$ for $\forall s_1\in\Omega_1(\underline{\delta},\bar{\delta})$. By choosing $\alpha=4^{-\frac{2\rho}{r_1}}\varepsilon$, it can be verified that for all $\bar{s}_n\in\Pi\subset\Omega_n(\underline{\delta},\bar{\delta})$,

$$\dot{V}(\bar{s}_n)+\alpha\cdot(V(\bar{s}_n))^{\frac{2\rho}{2\rho+\tau}}\leqslant 0.\quad(11)$$

Next, from the finite-time Lyapunov theory, it can be deduced [5] that system (5) can be finite-time stabilized by the controller (4).

Numerical simulations. Consider an electromechanical system of the form [6]:

$$\begin{cases}\dot{x}_1=x_2,\\\dot{x}_2=15.625x_3-0.281x_2-35.656\sin(x_1),\\\dot{x}_3=40u-36x_2-200x_3,\end{cases}\quad(12)$$

where x_1,x_2,x_3 are the system states and u is the control input. Here, we aim to design an HOSM controller such that x_1 converges to zero under constraint $-0.1<x_1<0.2$.

Let $s=x_1$ and apply (12). Then for SM dynamics, we get

$$\dddot{s}=A(t,x)+B(t,x)u,\quad(13)$$

where $A(t,x)=-562.421x_2-3129.395x_3+10.028\sin(x_1)-35.656x_2\cos(x_1)$ and $B(t,x)=625$. It can be easily verified that $\overline{A}(x)=598.077|x_2|+3129.395|x_3|+10.028$ and $\underline{B}=625$. Moreover, the limitation imposed on s is $-0.1=-\underline{\delta}<s<\bar{\delta}=0.2$.

Let $\rho=a=r_1=1$ and $\tau=\frac{2}{7}$. Then, from Theorem 1, the HOSM controller for SM dynamics (13) can be designed as

$$u=-\frac{\overline{A}(x)}{\underline{B}}\operatorname{sign}(\xi_2)-\beta_3\phi(s)\lfloor\xi_2\rceil^{\frac{1}{7}}\quad(14)$$

with $\xi_2=\lfloor\ddot{s}\rceil^{\frac{7}{3}}+\beta_2^{\frac{7}{3}}\phi^{\frac{7}{3}}(s)(\lfloor\dot{s}\rceil^{\frac{7}{5}}+\beta_1^{\frac{7}{5}}s)$ and $\phi(s)=\frac{0.02^{\frac{16}{7}}(0.02+s^2)}{(0.2-s)^{\frac{23}{7}}(0.1+s)^{\frac{23}{7}}}$. Now let us compare our HOSM controller with a traditional one. The latter does not consider pre-set constraints and reads as

$$u=-\frac{\overline{A}(x)}{\underline{B}}\operatorname{sign}(\xi_2)-\beta_3\lfloor\xi_2\rceil^{\frac{1}{7}}.\quad(15)$$

For controller (14), the gains are set to $\beta_1=1.5$, $\beta_2=3.2$, and $\beta_3=8.6$. Controller parameter values (15) are the same as those for controller (14). The initial condition is chosen as $(x_1(0),x_2(0),x_3(0))=(0.198,0.2,0.3)$. The simulation results are given in Figure 1.

It shows time evolution of x_1 for controllers (15) and (14). As can be seen from the figure, the anticipated control has been achieved for (14). At the same time, system trajectories for controller (15) will transgress to the pre-set constraint. The simulation results clearly demonstrate the superiority of the proposed controller (14).

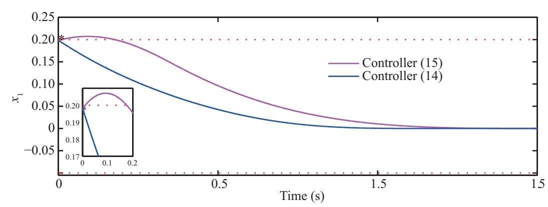

Figure 1 (Color online) Trajectories of x_1 under controllers (15) and (14).

Conclusion. This study has suggested a novel HOSM controller design with asymmetric output constraints. An asymmetric BLF capable of satisfying pre-set output constraints has been constructed. The HOSM controller has been designed by inserting the asymmetric BLF into the backstepping-like method. The results obtained in this study can be extended to MIMO systems in the nearest future.

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